2005 SOA ERM Symposium

CS D1: Risk Tolerances and Risk Metrics
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Overview

- RM Metrics
- Fuzzy Logic
- Tolerance

RM Frameworks

Different Risk Management Frameworks:

1. AAA
2. IAA
3. COSO

Help with:

1. Identification
2. Classification
3. Clarification
4. Unification

AAA Framework

AAA Risk Categories	Elements of Insurance Risk
Credit Risk	*Business Credit Risk*
	Invested Asset Credit Risk
	Political Risk
	Concentration Risk
Market Risk	*Interest Rate Risk*
	Basis Risk
	Reinvestment Risk
	Currency Risk
	Asset Concentration Risk
	Asset/Liability Matching Risk
	Asset Mkt. Value Risk
	Political Risk
	Sovereign Risk
Liquidity Risk	*Liquidity Risk*
	Asset/Liability Matching
	Asset-Affiliates
	Capital Markets
	Catastrophes

AAA Framework (cont'd)

Operational Risk	***Human Capital Risk***
	Operational Risk
	Start-Up Risk
	Distribution Channels Risk
	Fraud Risk
	Strategic Risk
	Competitive Risk
	Stress Testing/Scenarios
	Business Continuation
	Acquisition of Business
	Distribution Channel Risk
Legal Risk	***Political Risk***
	Regulatory Risk
	Legal Risk
	Distribution Channels Risk
	Sovereign Risk
	Fraud Risk
Reputational Risk	***Reputation Risk***
	Quality/Service Delivery Risk
	Fraud Risk

AAA Framework (cont'd)

Not Categorized	
Pricing/Underwriting (P/U):	***Pricing Risks***
Reserving (R)	***Reserving Adequacy***
	Rapid Growth
	New Company/Start Up
	Economic Environment

Quantitative Assessments

- For a specific risk need to quantify both:
  - Amount of potential loss
  - Probability of loss
- Product of potential loss and probability results in a **quantitative assessment** for a specific risk
- Need to identify various risk measures (or KRI's)
- Do not necessarily need large amounts of historical data

Risk Measure

- **An objective numeric metric that can be used for quantitative risk assessments**
- Examples:
  - Duration for Interest Rate risk
  - Quit rate for HR risk
  - Retained Face Amount for U/W Risk

Risk Metrics – Credit Risk

AAA Risk Category	AAA Risk Subcategory	Risk Metrics
Credit Risk	Business Credit Risk	Gross Balances
	Concentration Risk	CTE
	Political Risk (Cr)	Sovereign Exposure Limitations
	Concentration Risk	VAR Pre-defined Exposure Limits on Individual Securities
	Invested Asset Credit Risk	Ratings - Agency Ratings - Internal Expected Default Freq & LGD VAR Spread Volatility - Tail Loss in MV
	Business Credit Risk	Net Balances with business counterparties Expected loss
	Political Risk (Cr)	Sovereign Exposure Limitations
	Business Credit Risk	Tail loss

Risk Metrics – Market Risk

AAA Risk Category	AAA Risk Subcategory	Risk Metrics
Market Risk	Currency Risk	Currency exclusion Assets & Liability mismatch in different currencies using BV/MV Cost of SWAPs and changes over time CTE EV Delta Normal VaR
	Asset Market Value Risk	Time to maturity Changes to spreads EV TVaR on Equity Markets
	Asset/ Liability Matching Risk	Sovereign Exposure Limitations Asset Partial Duration Convexity EV VAR Duration Convexity
	Basis Risk	The Greeks Option Adjusted Spread

Risk Metrics – Market Risk

AAA Risk Category	AAA Risk Subcategory	Risk Metrics
Market Risk	Interest Rate Risk	Duration / Convexity Option Adjusted Spread EV CTE
	Sovereign Risk (M)	Absolute level of exchange rates Sovereign Exposure Limitations Bond credit spread Sovereign debt rating Lehman Brother's Policitical/Economic Risk Index Mismatch between co's A&Ls in foreign currency.
	Reinvestment Risk	Asset Partial Duration Calculate ending-wealth position assuming more realistic rein rates (e.g. implied gov't risk free fwd rates) and compare to beg
	Asset Concentration Risk	Sovereign Exposure Limitations

Risk Metrics – Pricing & U/W'g

AAA Risk Category	AAA Risk Subcategory	Risk Metrics
Pricing/ Underwriting Risk	Net Retention Risk	Risk Based Capital
	Underwriting Risk	Rate Adequacy level Expected U/W Ratio SDEV of U/W Ratio Prob. Of operating loss Exp. Cond. Severity of operating loss Cond. Severity of operating loss at n-th percectile
	Mortality	Actual to Expected mortality ratio DCAT or DFM sensitivity analysis U/W point systems Statistical estimation error on data and parameters NAAR EV CTE

Risk Metrics Availability

Risk Categories	**Risk Metrics**
Market	Yes
Credit	Yes
Liquidity	Yes
Insurance	Yes
Operational	?
Legal & Regulatory	?
Strategic	?

Fuzzy Logic

- History
- Basic Concepts
- Simple Examples

Fuzzy Logic - History

- Aristotle and previous Philosophers
  - “Laws of Thought” were posited to devised concise theory of logic and later mathematics
  - “Law of Excluded Middle”: Every Proposition either True or False
- Plato
  - Laid foundation for fuzzy logic by indicating there was a third region where these opposites “tumbled about”
- Dr. Lotfi Zadeh (1965), UC/Berkley
  - Infinite-valued logic a generalization of bi-valued
  - Published “Fuzzy Sets” paper describing mathematics of fuzzy set theory and by extension fuzzy logic

Fuzzy Logic – Basic Concepts

- Central Notion
  - Superset of conventional boolean logic extended to incl. partial truth
  - Truth values are indicated by a value on the range [0.0,1.0]
  - Absolute Falseness is mapped to 0.0
  - Absolute Truth is mapped to 1.0
  - Mathematically, let X = {x} be a set of objects with elements x
  - A fuzzy set A in X is characterized by a membership function mA(x), which maps each point in X onto the real interval [0.0,1.0]

Fuzzy Logic – Basic Concepts

- Example
  - "Jane is old"
  - If Jane's age was 75, might assign the statement a truth value of 0.8
  - Set terminology: "Jane is a member of the set of old people"
  - mOLD(Jane) = 0.80 where m is a membership function

Fuzzy Logic – Basic Concepts

- Fuzzy Systems vs. Probability
  - First glance suggests both have similar values between 0 and 1
- Probability
  - "There is an 80% chance that Jane is old"
  - Semantically, still caught in the Law of Excluded Middle
  - Supposes that Jane is or is not old
  - 80% chance of knowing which set she is in
  - Statements about likelihood of outcome: an event either occurs or not
- Fuzzy Terminology
  - Jane is "more or less" old, or some other term corresponding to value of 0.8
  - One cannot say unequivocally if event occurred or not
  - Trying to model <u>extent</u> to which an event occurred

Fuzzy Logic – Basic Concepts

- Union (OR) and Intersection (AND)
  - Clearest point of departure from probabilistic theory for sets to fuzzy sets
- Example
  - X=Bob; T is fuzzy set of Tall people and S is fuzzy set of Smart people
  - Let mS(x)=0.90 and mT(x)=0.90
  - Probability yields an intersection value of 0.9*0.9=0.81
  - Fuzzy result is MIN (mS(x),mT(x)) = 0.90
  - Probablistic statement is "If Bob is very smart, and Bob is very tall, then Bob is a quite tall, smart person"
  - Fuzzy calculation implies "If Bob is very smart, and Bob is very tall, then Bob is a very tall, smart person"

Fuzzy Logic – Basic Concepts

- Skeptics
  - Assignment of values is an imprecise operation
- Fuzzy systems lay no claim to establishing a formal procedure for assignments of values
- Only argument for a particular assignment is its intuitive strength
- Fuzzy logic proposes a formal method of operating on these values, once primitves have been established
- Fuzzy logic is a logic of fuzziness, not a logic which is itself fuzzy
- Generalization of classic logic

Fuzzy Logic – Example

- Driving down a 2 way, 6 lane street in a large city
- Speed limit is 45 mph
- Usually optimum and safest to “drive with the traffic”
- Traffic going at 48 mph
- Specific and precise instructions to “drive with the traffic” ?

Fuzzy Logic – Example

- Receiving a large number of fuzzy inputs
  - Some weaving in and out going more than 48 mph
  - Some going exactly 45 mph
  - How many cars in front and how fast driving
  - Any “old clunkers” going really slow
  - Police radar on this stretch of road and how much leeway above 45
  - Trucks holding up one of the lanes
  - Side traffic from side streets
- Difficult, but humans do it everyday because that is the way the human brain works

Fuzzy Logic – Examples

- Provides opportunity for modeling of conditions which are inherently imprecisely defined
- Applications are wide-ranging
  - Linear and Non-linear control
  - Pattern Recognition
  - Financial Systems
  - Operations Research
  - Data Analysis
- Hundreds of $million of successful, fuzzy logic based applications
  - Self-focusing cameras
  - Computer programs trading successfully in financial markets
  - Anti-lock breaks
  - Subway control systems

Fuzzy Logic – Examples

- Competitive risk
  - Risk that competition in the market may lead to loss of future business resulting in reduced profits
- Steps:
  - Identify possible KRI's
  - Select linguistic variables/descriptors for each KRI (e.g. High , Medium, Low)
  - Develop membership functions for each of the KRI's
  - Create fuzzy rules to indicate how KRI's affect loss
  - Apply fuzzy logic math to develop loss distribution

Fuzzy Logic – Competitive Risk

- KRI's:
  - Pricing, Innovation, Distribution Channel, Customer Service, Losing Key Employees to Competition
- Define High, Medium, Low membership characteristics and assign values based on expert opinion
  - Examples:
    - Prices higher than most/all competitors
    - Portfolio of products is generic and can be easily duplicated
    - Single distribution channel
    - Offer less customer service than competition
    - High percentage of those quitting going to competition

Fuzzy Logic – Competitive Risk

- Set up rules based on expert opinion
  - Examples:
    - Rule 1: If Price is High and Innovation is High then risk is Medium
    - Rule 2: If Price is High and Innovation is Low
      and Distribution is Low
      and Customer Service is Low
      then risk is Very High
    - Rule 3: …
    - Rule 4: …
- Feed fuzzy variables, fuzzy rules and fuzzy set theory into a Fuzzy Logic Engine to develop loss distribution

RM Tolerance Terminology

- Risk Appetite
- Risk Tolerance
- Risk Limits
- Trigger Points
- Early Warning Indicators

Risk Appetite

- **The level of aggregate risk at which the company can successfully manage its business for an extended period of time**

Tolerance

- **Tolerance levels are acceptable levels of risk for a specific risk category as defined by company management in terms of loss amounts, error rates, or other types of units**

Risk Limits

- **Boundaries of tolerance levels above which too much risk is being taken relative to company's policies and standards**

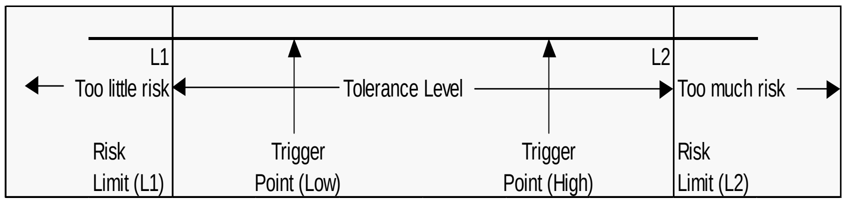
Trigger Points

- **Pre-defined levels of risk at which a trigger is activated**
- **Activation will require pre-defined steps to be followed by a pre-defined owner**
- **May be based on risk measures or Early Warning Indicators**

Early Warning Indicators

- **Readily available objective metric that provides information on direction and/or magnitude of specific risks**
- **Not necessarily risk measures as previously defined**
- **Examples:**
  - **Unrealized G/L’s for Market Risk (Equity,R/E)**

Conceptual Framework



Risk Tolerance & Judgment

- **Individuals typically neither rational nor consistent when making judgment under uncertainty**
- **Risk from perceived gains treated differently from risks concerning perceived losses**
- **Kahneman & Tversky (1979) developed Prospect Theory in an attempt to explain these common and systematic deviations from reality**

Prospect Theory

1. **People evaluate rewards and losses relative to a neutral reference point**
2. **People think about potential outcomes as gains or losses relative to this neutral reference point**
3. **People form their choices based on resulting change in asset position as assessed by an S-shaped curve**

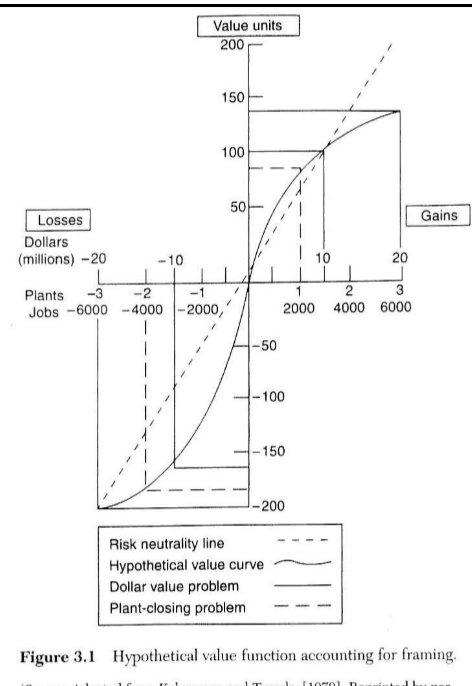


Figure 3.1 Hypothetical value function accounting for framing.

(*Source:* Adapted from Kahneman and Tversky [1979]. Reprinted by permission of the Econometric Society.)

Prospect Theory - Outcomes

1. **Framing of problem can dramatically change the perceived neutral point**
2. **Response to loss is more extreme than to gain**
   - **Pain of losing $X > Pleasure of winning $X**
3. **Tendency to overweigh prob. of low-prob. events and underweigh prob. of high-prob. events**
   - **Different from expected-utility theory where risky option is weighted by its probability**
   - **Kahneman and Tversky incorporate systematic deviations from rationality in probability assessments for their decision model**

Prospect Theory - Outcomes

- **Descriptive power has inspired excitement for the last 20 years in fields of :**
  - » **Decision Theory**
  - » **Psychology**
  - » **Marketing**
  - » **Law**
  - » **Medicine**
  - » **Finance**
  - » **Organizational Behaviour**
  - » **Economics**
- **Helped researchers understand errors and inconsistencies in human judgment**

Framing of Risky Decisions

- **Tversky and Kahneman (1981) asked 150 subjects the following:**
  - **Consider 2 decisions concurrently:**
    - **Decision 1**
      - **a) a sure gain of $240**
      - **b) a 25% chance to gain $1000 and 75% chance to gain nothing**
    - **Decision 2**
      - **c) A sure loss of $750**
      - **d) A 75% chance to lose $1000 and 25% chance to lose nothing**
  - **In decision 1, 84% chose (a) and 16% (b) ➔ averse**
  - **In decision 2, 87% chose (d) and 13% (d) ➔ risk seeking**

Framing of Risky Decisions

- **Tversky and Kahneman (1981) asked 86 subjects not exposed to previous problem the following:**
  - **Chose between:**
    - **e) A 25% chance to win $240, and 75% chance to lose $760**
    - **f) A 25% chance to win $250, and 75% chance to lose $750**
  - **All 86 chose (f) over (g) since it dominates**
  - **Why is this interesting?**

Framing of Risky Decisions

- **Interesting because:**
  - **Adding choices (a) and (d) = (e)**
  - **Adding choices (b) and (c) = (f)**
  - **The sum of the undesirable choices dominates sum of the desirable choices**
  - **Framing of the combined problem in 2 parts results in a reversal of preferences**
    - **Violates both consistency and coherence**

Framing of Risky Decisions

- **Subdivision of decision making is likely to enhance the potential for inconsistency and non-rational choice**
- **To arrive at a coherent strategy for making judgments under uncertainty, need to become more aware of this bias and develop procedures for identifying and integrating risky decisions across organizations**

Anchoring

- **Location of reference point is critical**
- **Example:**
  - **Have 100 shares in XYZ from 2 years ago when the value was $20/share**
  - **Shares dropped $10 last 2 years**
  - **Currently drilling for oil in a promising area**
  - **Geologist analysis suggests that if they hit then stock expected to go back up to $20, but if the well is dry then stock will to fall to $0**
  - **Do you want to sell now for $10?**

Anchoring - Example

- **What is your reference point?**
  - **Amount you gain (ie. amount for stock above $0)**
  - **Amount you lose (ie. amount for stock below $20)**
  - **If you cognitively adopt $0 as your reference point then you will take the sure “gain” and sell now**
  - **If your reference point is $20 then you will be risk seeking and will hold onto the stock rather than accept the sure “loss”**

Questions

- **Any Questions?**

Setting Risk Tolerance Levels
Session CS D1

Richard Goldfarb, Ernst & Young

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Overview of Common ERM Framework

- Step 1: Estimate Profit/Loss Distributions by Risk Source, Business Unit, etc.
- Step 2: Aggregate P/L Distributions into Firmwide P/L Distribution
- Step 3: Estimate Required Capital Based on Firmwide Capital Standard
  - Linked Directly to Risk Tolerance
  - Often Based on Target Probability of Default
  - Often Tied to Target Credit Rating
  - e.g. "Required Capital is the amount needed to ensure a probability of default less than the probability of default for AA-rated bonds."
- Step 4: Attribute Capital to Each Risk Source or Business Unit
- Step 5: Calculate Selected Metrics
  - Return on Risk Adjusted Capital
  - Economic Value Added

How Do Firms Establish the Firmwide Capital Standard in Step 3?

- Alternative Methods
  - Set Minimum Required Capital to Achieve Long Term Business Plan
  - Set Minimum Required Capital to Maintain Target Credit Rating
  - Determine Capital Required (now) to Ensure Target Probability of Default Over Selected Horizon
- Target Probability of Default
  - Subjectively Set – Based on Management's Risk Tolerance
  - Objectively Set – Tied to Corporate Bond Default Probabilities at Selected Credit Rating Level
- But this raises important questions:
  - What methods can be used to estimate bond default probabilities?
  - What is a reliable source for these probabilities?
  - What are the bond default probabilities by rating?
  - How stable are these measures of default probability?
  - What other methodology choices affect the results?

Bond Default Probabilities by Rating – Alternative Approaches

- Historical Default Probabilities
  - Altman's Studies
  - Moody's Historical Default Statistics
  - S&P's Historical Default Statistics
- Inferred from Current Market Prices of Corporate Bonds
- Merton Model (Moody's/KMV)
  - Uses Equity Prices to Estimate Net **Asset** Volatility
  - Assumes Specific Default Point for Assets
  - Estimates Probability of Default Based on Distributional Assumptions and These Parameters
- Other Methods
  - Econometric Models
  - Altman's Zeta Model

I'll focus the next several slides on the Historical Default Probabilities

Historical Default Probabilities

- Altman's Default Studies
  - Forms cohorts by *original* rating at issuance
  - Separates the default probability from the default severity (loss given default)
  - Estimates annual default probabilities *n*-years from issuance
- Moody's and S&P Studies
  - Form cohorts based on rating at the *beginning of the year*
  - Separate default probability from default severity
  - Moody's and S&P use slightly different definitions of default
    - See Next Slide

Moody's and S&P Definitions of Default

Moody's Default Definition	**S&P's Default Definition**
Missed or delayed disbursement of interest and/or principal, **including delayed payments made within a grace period**.	First occurrence of a payment default on any financial obligation. **An exception occurs when an interest payment missed on the due date is made within the grace period.**
Issuer files for bankruptcy (Chapter 11, or less frequently Chapter 7, in the US) or legal receivership occurs.	
Distressed exchange occurs where: (i) the issuer offers bondholders a new security or package of securities that amount to a diminished financial obligation (such as preferred or common stock, or debt with a lower coupon or par amount), or (ii) **the exchange had the apparent purpose of helping the borrower avoid default.**	Distressed exchanges are considered defaults whenever the debt holders are coerced into accepting substitute instruments with lower coupons, longer maturities, or any other diminished financial terms.
Source: Moody's *Default & Recovery Rates of Corporate Bond Issuers,* Jan 2004	Source: Standard & Poor's *Rating Performance 2002*

Bond Default Probabilities by Rating

Estimated Default Probabilities
By Rating Class

S&P Rating	Moody's Equivalent	Default Probability (Subsequent year)	Coverage Level
AAA	Aaa	0.01%	99.99%
AA	Aa3/A1	0.03%	99.97%
A	A2/A3	0.11%	99.89%
BBB	Baa2	0.30%	99.70%
BB	Ba1/Ba2	0.81%	99.19%
B	Ba3/B1	2.21%	97.79%
CCC	B2/B3	6.00%	94.00%
CC	B3/Caa	11.68%	88.32%
C	Caa/Ca	16.29%	83.71%

Source: James, *RAROC Based Capital Budgeting and Performance Evaluation: A Case Study of Bank Capital Allocation,* 1996, Wharton Working Paper 96-40. Author cited Bank of America as his source.

Stability of the Default Probabilities

- The following tables show reported default probabilities from S&P and Moody's reports for selected years.
- Notice the AA/Aa default probabilities have ranged from 0.00% to 0.03%.

Default % - Data 1981 through

		1997	2000	2002	2003
S&P	AAA	0.00	0.00	0.00	0.00
	AA	0.00	0.01	0.01	0.01
	A	0.05	0.04	0.05	0.05

Default % - Data 1970 through

		1996	1999	2001	2002	2003
Moody's	Aaa	0.00	0.00	0.00	0.00	0.00
	Aa	0.03	0.03	0.02	0.02	0.02
	A	0.01	0.01	0.02	0.02	0.02

- Notice that the 0.03% standard is difficult to justify.
- Whatever you use, it is likely to be a VERY small number.
  - How reliable are 99.97% or 99.99% Value at Risk estimates?
  - Is this difference significant?

Methodology Issues

- Treatment of Withdrawn Rating Category
  - Both S&P and Moody's statistics track bonds in which the ratings are withdrawn ("NR").
  - Most practitioners exclude the NR data from their default probability calculations, though S&P argues that this overstates the default probabilities.
- Single Year vs. Multi-Year Average
  - Some practitioners prefer to use longer term (5-year and 10-year) default statistics and then estimate an average *annual* default rate.
  - This approach can result in substantial variation in annual default probability estimates, as shown below:

Moody's 1970-2003 Aa Default Rates – 1, 5 and 10-Year Horizons

	1-Year	5-Year	10-Year
Aa Default Rate	0.02%	0.24%	0.68%
Annual Default Rate	0.02%	0.05%	0.07%

Methodology Issues (*continued)*

- Single Year vs. Multi-Year Default Probabilities
  - One-year horizons are common, but some practitioners use multi-year horizons
  - This requires multi-year default probabilities
  - Two Alternative Sources
    - Historical data from cohorts evaluated over multiple years
    - Annual rating transition matrices
- S&P Historical Multi-Year Default Probabilities

Cumulative Default Probabilities (%)

	Yr. 1	Yr. 2	Yr. 3	Yr. 4	Yr. 5	Yr. 6	Yr. 7	Yr. 8	Yr. 9	Yr. 10
AAA	0.00	0.00	0.04	0.07	0.12	0.21	0.31	0.49	0.56	0.63
AA	0.01	0.04	0.11	0.20	0.33	0.48	0.68	0.85	1.00	1.18
A	0.05	0.15	0.30	0.50	0.75	1.01	1.29	1.55	1.88	2.20
BBB	0.37	1.06	1.80	2.84	3.84	4.83	5.66	6.42	7.11	8.00
BB	1.45	4.38	7.98	11.39	14.45	17.64	20.23	22.51	24.88	26.72
B	6.59	15.03	22.46	28.47	33.02	36.91	40.44	43.73	45.93	48.30
CCC/C	34.14	44.07	50.54	55.65	61.35	63.93	64.94	65.58	68.78	71.46

Single Year vs. Multi-Year Default Probabilities (*continued)*

- Rating Transition Matrices
  - S&P and Moody's also publish **transition matrices** that show changes in ratings as well as defaults. For example:

Average One-Year Transition Probabilities, NR-Adjusted, 1981-2003

From/To	AAA	AA	A	BBB	BB	B	CCC/C	D
AAA	92.08	7.09	0.63	0.15	0.06	0.00	0.00	0.00
AA	0.62	90.83	7.76	0.59	0.06	0.10	0.02	0.01
A	0.05	2.09	91.37	5.79	0.44	0.16	0.04	0.05
BBB	0.03	0.21	4.10	89.38	4.82	0.86	0.24	0.37
BB	0.03	0.08	0.40	5.53	83.25	8.15	1.11	1.45
B	0.00	0.08	0.27	0.34	5.39	82.41	4.92	6.59
CCC/C	0.10	0.00	0.29	0.58	1.55	10.54	52.80	34.14

Source: S&P Rating Performance Report 2003

  - These matrices can be compounded for multiple periods to produce *n*-year default probabilities. For instance, the 5-year transition probabilities would be estimated as:

Estimated Five-Year Transition Probabilities (%)

From/To	AAA	AA	A	BBB	BB	B	CCC/C	D
AAA	66.54	25.03	6.53	1.40	0.34	0.12	0.02	0.04
AA	2.21	63.41	27.23	5.38	0.83	0.53	0.10	0.26
A	0.29	7.40	66.75	19.90	3.34	1.26	0.25	0.77
BBB	0.15	1.40	14.19	60.67	14.21	5.08	0.92	3.42
BB	0.12	0.48	2.99	16.18	44.39	20.63	3.07	12.15
B	0.04	0.32	1.25	3.10	13.55	42.69	6.13	32.91
CCC/C	0.17	0.13	0.80	1.70	4.36	13.16	5.56	74.12

Rating Transition Matrices (*continued)*

- Notice that the 5-year default probabilities implied by the transition matrices differ from the actual 5-year default probabilities in many instances:

	Implied	Historical
AAA	0.04	0.12
AA	0.26	0.33
A	0.77	0.75
BBB	3.42	3.84
BB	12.15	14.45
B	32.91	33.02
CCC/C	74.12	61.35

- Some authors suggest adjusting the transition matrix to better match the historical long-term default. See Ong, *Internal Credit Risk Models: Capital Allocation and Performance Measurement*.

Additional Considerations

- Link Between Credit Rating and Probability of Default
  - S&P's 2003 Rating Performance Report:

    "It is important to note that Standard & Poor's ratings do not imply a specific probability of default; however, Standard & Poor's historical default rates are frequently used to estimate these characteristics."
- When management decides to target a "AA Rating", do they mean they want their *current* rating to be AA, or do they mean they want a high degree of confidence that their rating will <u>stay</u> AA over some time horizon?
  - If it is the former, then the historical default probabilities might be useful.
  - If it is the latter, then they are not.

Summary

- Setting a Risk Tolerance, VaR Confidence Level or Target Default Probability is a critical element of ERM frameworks.
  - Practitioners are often uncomfortable with subjective values for this key input.
  - They want something objective that they can point to.
- Many practitioners rely on the .03% AA Default Probability as the target default rate assumption. This might be reasonable, but keep in mind the following:
  - There is no direct support for .03% based on historical default statistics
  - Statistics change slightly from year to year
  - Moody's and S&P historical data are not identical
  - Many methodology choices can impact the chosen estimate:
    - How to handle withdrawn ratings
    - Single year averages or implied annual probabilities from multi-year default statistics
    - Multi-year historical averages vs. multi-year transition matrices
- It is important to consider the difference between targeting a *current* default probability and ensuring a particular rating over some selected horizon.

Risk Metrics' Evolution

David L. Ruhm

2005 ERM Symposium

Session CS D1

Evolution of "risk metric" statistics

- Stages:
  - Sharpe ratio (1960's)
  - Probability of ruin (1980's and earlier)
  - Value at risk (1990's)
  - Tail conditional expectation (2000's)
  - Tail frequency x severity (2000's)
  - Risk coverage ratio (2000's)

Evolution of “risk metric” statistics

- Fact: Each statistic *answers a specific question* about risk exposure.
- Opinion: Risk statistics have advanced to become *progressively more meaningful*.

The Sharpe ratio

- Question: How much expected return is gained per unit of risk assumed, measured by standard deviation?
- Statistic: $(E[R] - r_f) / \sigma_R$

The Sharpe ratio

- Advantages:
  - Relates risk and return in one statistic
  - Not dependent on capital amount
  - Ties to financial theory
- Disadvantages:
  - Downside risk may be more important
  - Std dev is less useful for non-normal distrib’ns
  - Heavy-tailed exposure not sufficiently captured

Probability of ruin

- Question: What is the probability that results will be so adverse as to exhaust all of the capital?
- Statistic: p(Income < - Capital)

Probability of ruin

- Advantages:
  - Useful information to have
- Disadvantages:
  - Far in tail, estimation accuracy often low
  - Ignores more probable near-ruin results
  - Does not relate risk to return (risk-only metric)

Value at risk (VaR)

- Question: How much money could be lost, up to the 99% probability level?

- Statistic: 1st %ile of income

Value at risk (VaR)

- Advantages:
  – Useful information to have
- Disadvantages:
  – Far in tail, estimation accuracy will be low
  – Ignores more probable near-ruin results
  – Ignores severity beyond cutoff
  – Cut-point is not economic, but statistical
  – Does not relate risk to return (risk-only metric)

Tail Conditional Expectation

- Question: How large is the loss from the average catastrophic scenario?
- Statistic: Average of top 10% of loss events
- Abbreviations: TCE, CTE, TVaR

Tail Conditional Expectation

- Advantages:
  - Specifies capital to cover the average disaster
  - Captures a larger group of risk events
  - Better estimation accuracy than ruin measures
- Disadvantages:
  - Cut-point is a percentile, not an economic point
  - Leaves out risk from more common loss results
  - Does not relate risk to return (risk-only metric)

Tail Frequency x Severity

- Question: How large is the average loss from operations, and how likely is a loss?
- This presentation uses \$0 as the cutoff, but other risk cutoff-points are possible
- Statistic:

$$p(\text{Operating income} < \$0) \times (\text{Average op loss})$$
$$= p(\text{Operating income} < \$0) \times (TCE_{p(\text{Income} < \$0)})$$

Tail Frequency x Severity

- Advantages:
  - Cut-point is at losses, economically meaningful
  - Simple, explainable and logical
  - Measures across undesirable “risk” events
  - Generalizes on Tail Conditional Expectation
- Disadvantages:
  - Does not relate risk to return (risk-only metric)

Tail Frequency x Severity

- Relationship to TCE: Example
  - Suppose the following:
    - p(operating loss) = 10% “Frequency of risk events”
    - $TCE_{90\%}$ = \$300m “Average severity of risk events”

  - Tail Freq x Sev = 10% x \$300m = \$30m
  - For a fixed p(op loss) , Tail Freq x Sev ~ TCE
  - Like TCE, with an economic cutoff point

Risk Coverage Ratio

- Improves on Tail Freq x Sev by including the risk/return relationship
- Question: How much expected return is gained per unit of risk assumed, measured by Tail Freq x Sev?
- Statistic: E[Op income] / (Tail Freq x Sev)

Risk Coverage Ratio

- Advantages:
  - Measures the risk/return relationship
  - Uses Tail Freq x Sev to quantify risk
  - Like Sharpe Ratio, w/downside risk measure
  - Related to other measures (DRS, $\Omega(r)$)
- Disadvantages:
  - More complicated to explain than other metrics
  - Less catastrophe-oriented

Risk Coverage Ratio

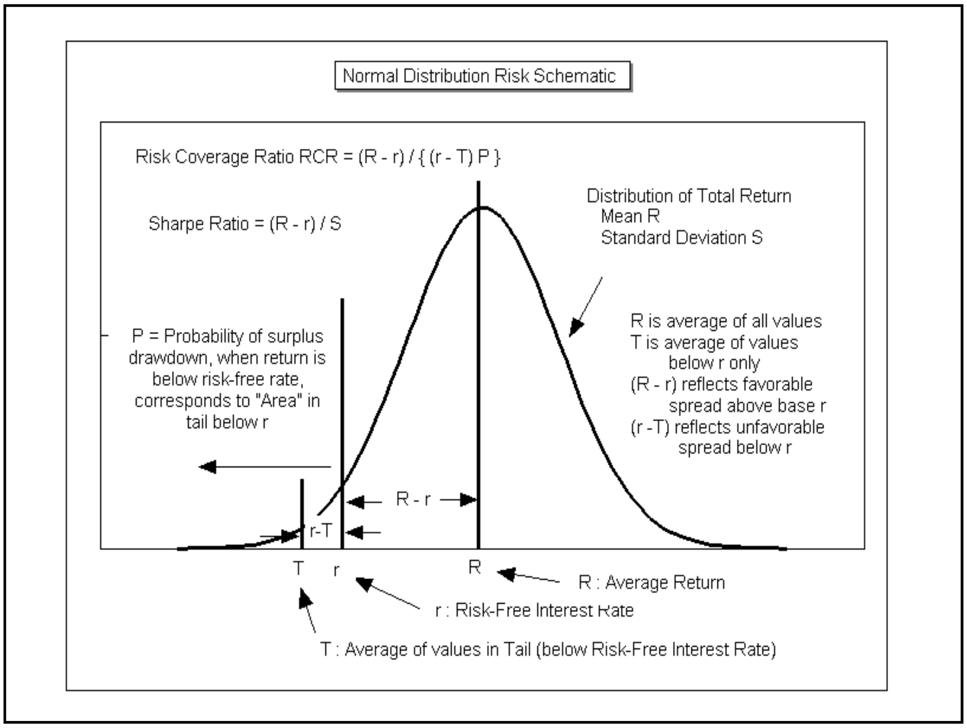
- Relationship to Sharpe Ratio:
  – RCR can be expressed in terms of return:

  – RCR = (E[ROE] – r_f) / (Tail Freq x Sev),

  where the tail cutoff for Tail Freq x Sev is r_f, the yield obtainable absent any risk-generating operations

Summary comments

- Risk measures look different from one another, but all of them boil down to answering the same small set of questions:
  – Which distribution (income, ROE, etc.)
  – Where's "risk" in the distribution (cutoff-point)
  – What statistic to measure with (std dev, avg severity)
  – Measure risk-only or the return/risk relationship
- Different combinations of answers lead to the variety of risk metrics that are possible


Normal Distribution Risk Schematic
Risk Coverage Ratio RCR = (R - r) / { (r - T) P }
Sharpe Ratio = (R - r) / S
Distribution of Total Return
Mean R
Standard Deviation S
R is average of all values
T is average of values
below r only
(R - r) reflects favorable
spread above base r
(r -T) reflects unfavorable
spread below r
P = Probability of surplus
drawdown, when return is
below risk-free rate,
corresponds to "Area" in
tail below r
R - r
r-T
T
r
R
R : Average Return
r : Risk-Free Interest Rate
T : Average of values in Tail (below Risk-Free Interest Rate)