

Modeling of Economic Series Coordinated with Interest Rate Scenarios

*Research Sponsored by the
Casualty Actuarial Society and the
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Outline of Presentation

- Motivation for Financial Scenario Generator Project
- Short description of included economic variables
- An overview of the model
- Applications of the model
- Comparison of this model with another actuarial return generating model
- Conclusions

Overview of Project

- CAS/SOA Request for Proposals on **"Modeling of Economic Series Coordinated with Interest Rate Scenarios"**
 - A key aspect of *dynamic financial analysis*
 - Also important for regulatory, rating agency, and internal management tests – e.g., *cash flow testing*
- Goal: to provide actuaries with a model for ***projecting economic and financial indices, with realistic interdependencies*** among the variables.
 - Provides a ***foundation*** for future efforts

Scope of Project

- ***Literature review***
 - From finance, economics, and actuarial science
- ***Financial scenario model***
 - Generate scenarios over a 50-year time horizon
- ***Document and facilitate use of model***
 - Report includes sections on data & approach, results of simulations, user's guide
 - Posted on CAS & SOA websites
 - Writing of papers for journal publication

Economic Series Modeled

- | | |
|--|--------------------------|
| ■ Inflation | ■ Equity dividend yields |
| ■ Real interest rates | ■ Real estate returns |
| ■ Nominal interest rates | ■ Unemployment |
| ■ Equity returns <ul style="list-style-type: none">• Large stocks• Small stocks | |

Current Report Structure

Text Sections

- 1) Intro & Overview
- 2) Excerpts from RFP
- 3) Selected Proposal
- 4) Literature Review
- 5) Data & Approach
- 6) Issue Discussion
- 7) Results of Simulations
- 8) Conclusion

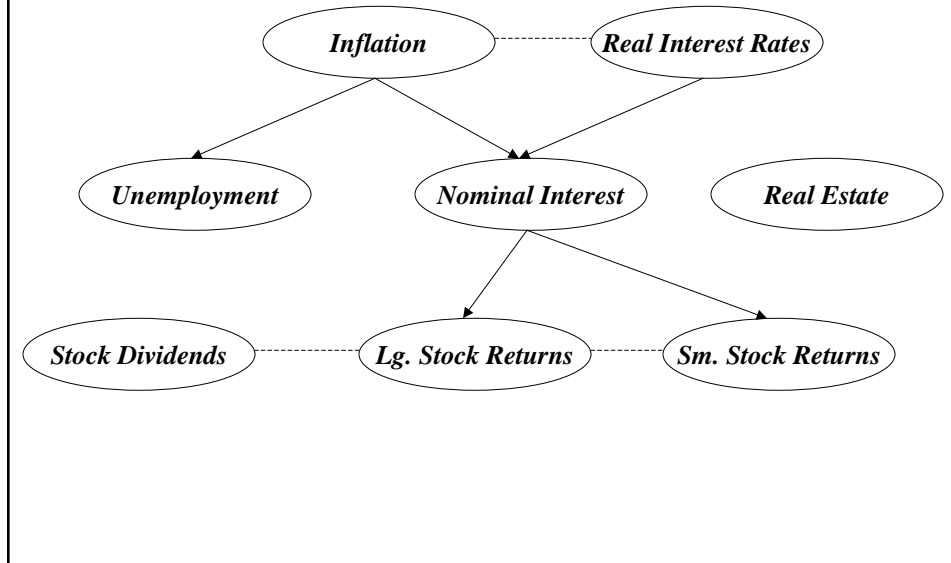
Appendices

- A) User's Guide to the Model
- B) Presentations of this Research
- C) Simulated Financial Scenario Data
- D) Financial Scenario Model

Prior Work

- Wilkie, 1986 and 1995
 - Widely used internationally
- Hibbert, Mowbray, and Turnbull, 2001
 - Modern financial tool
- CAS/SOA project (a.k.a. the Financial Scenario Generator) applies Wilkie/HMT to U.S.

Relationship between Modeled Economic Series



Inflation (q)

- Modeled as an Ornstein-Uhlenbeck process

- One-factor, mean-reverting

$$dq_t = \kappa_q (\mu_q - q_t) dt + \sigma dB_q$$

- Speed of reversion: $\kappa_q = 0.40$
- Mean reversion level: $\mu_q = 4.8\%$
- Volatility: $\sigma_q = 0.04$

Explanation of the Ornstein-Uhlenbeck process

- Deterministic component

If inflation is below 4.8%, it reverts back toward 4.8% over the next year

Speed of reversion dependent on κ

- Random component

A shock is applied to the inflation rate that is a random distribution with a std. dev. of 4%

- The new inflation rate is last period's inflation rate changed by the combined effects of the deterministic and the random components.

Real Interest Rates (r)

- Problems with one-factor interest rate models
- Two-factor Vasicek term structure model
- Short-term rate (r) and long-term mean (l) are both stochastic variables

$$dr_t = \kappa_r (l_t - r_t) dt + \sigma_r dB_r$$

$$dl_t = \kappa_l (\mu_l - r_t) dt + \sigma_l dB_l$$

Nominal Interest Rates

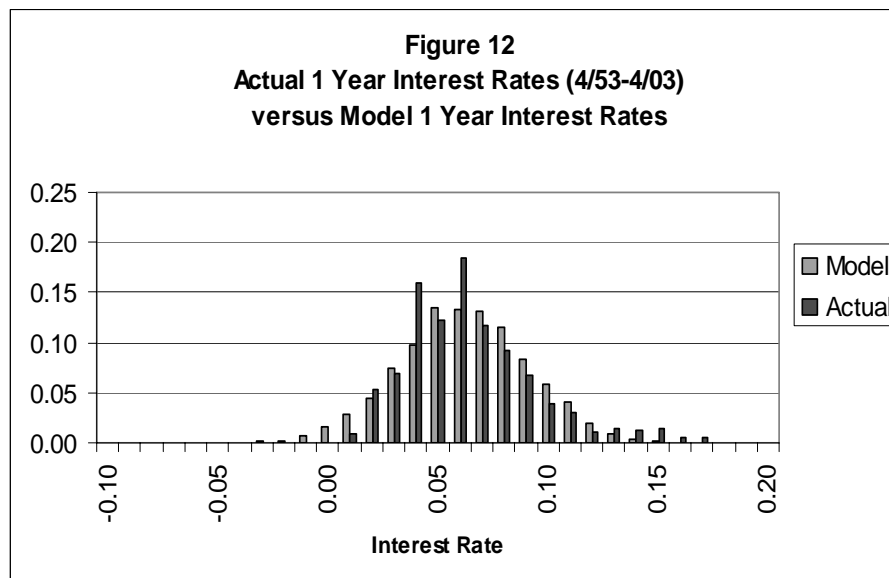
- Combines inflation and real interest rates

$$i = \{ (1+q) \times (1+r) \} - 1$$

where i = nominal interest rate

q = inflation

r = real interest rate



Equity Returns

- Empirical “fat tails” issue regarding equity returns distribution
- Thus, modeled using a “regime switching model”
 1. High return, low volatility regime
 2. Low return, high volatility regime
- Model equity returns as an excess return (x_t) over the nominal interest rate

$$S_t = q_t + r_t + X_t$$

Figure 16
Actual S&P 500 (1871-2002)
versus Model Large Stock Returns

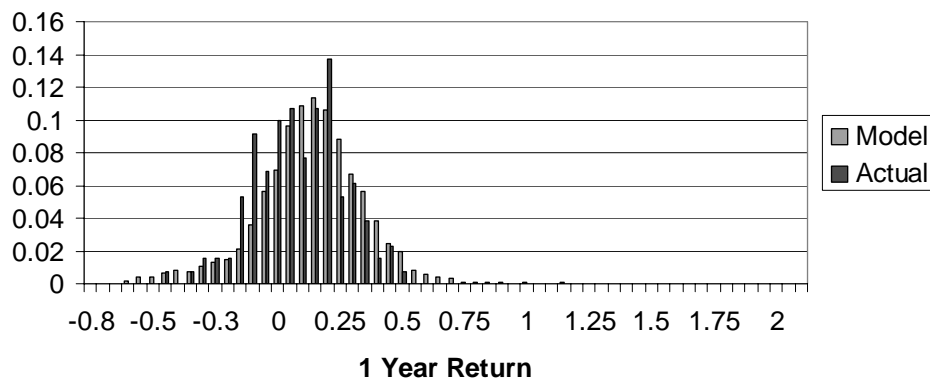
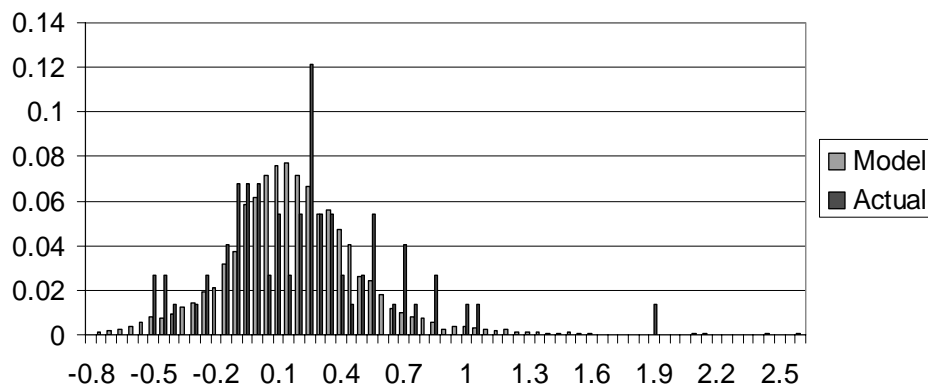


Figure 17
Actual Small Stock Returns (1926-1999) versus
Model Small Stock Returns



Other Series

- ***Equity dividend yields (y) and real estate***

- Mean-reverting processes

- ***Unemployment (u)***

- Phillip's curve: inverse relationship between u and q

$$du_t = \kappa_u (\mu_u - u_t) dt + \alpha_u dq_t + \sigma_u \varepsilon_{ut}$$

Selecting Parameters

- Historical or calibration with current market prices
- Model is meant to represent range of outcomes possible for the insurer
- Default parameters are chosen from history (as long as possible)
- Of course, different parameters may affect analysis

Model Description

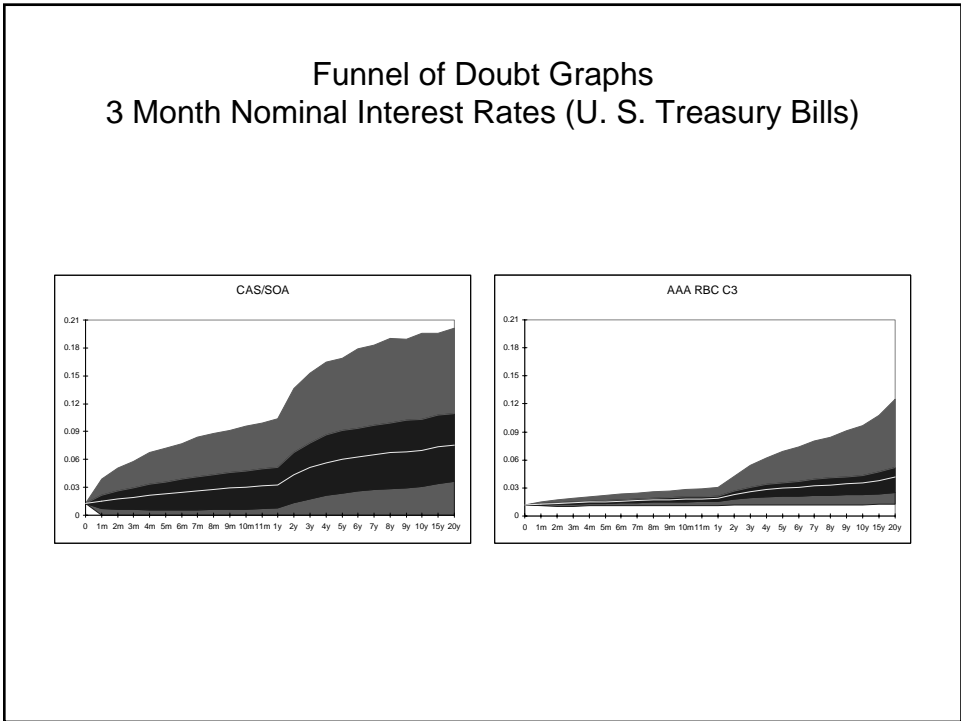
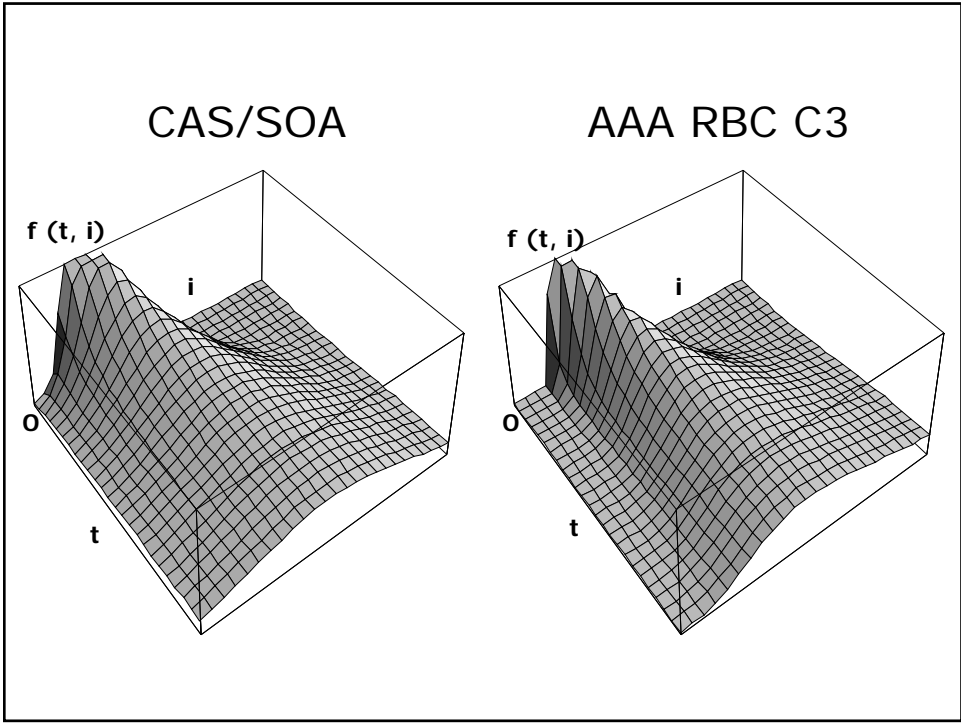
- Excel spreadsheet
- Simulation package - @RISK add-in
- 50 years of projections
- Users can select different parameters and track any variable

Applications of the Financial Scenario Generator

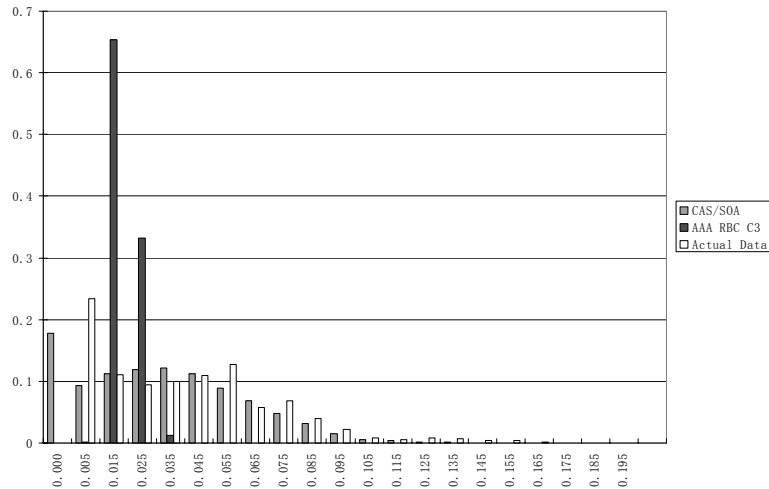
- Financial engine behind many types of analysis
- Insurers can project operations under a variety of economic conditions (Dynamic financial analysis)
- Useful for demonstrating solvency to regulators
- May propose financial risk management solutions

CAS/SOA vs. AAA

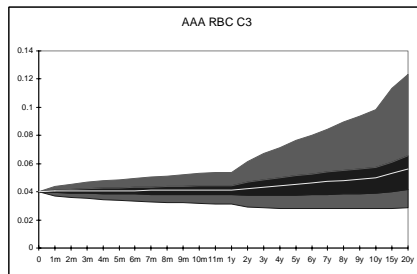
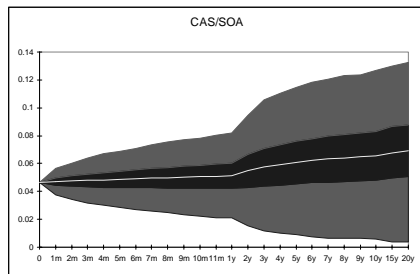
- AAA models provides guidance for Risk-Based Capital (RBC) requirements for variable products with guarantees
- Focus is on
 - Interest rate risk
 - Equity risk
- 10,000 Pre-packaged scenarios available
- Available at:
<http://www.actuary.org/life/phase2.htm>



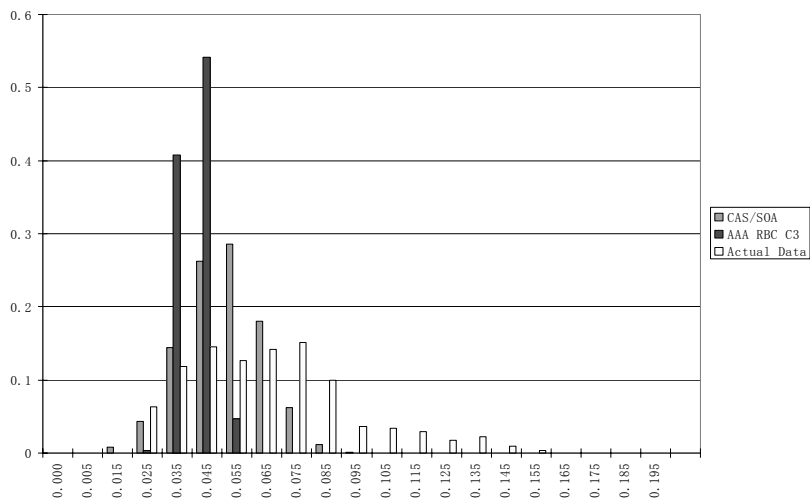
Histogram of 3 Month Nominal Interest Rates Model Values and Actual Data (01/34-05-04)



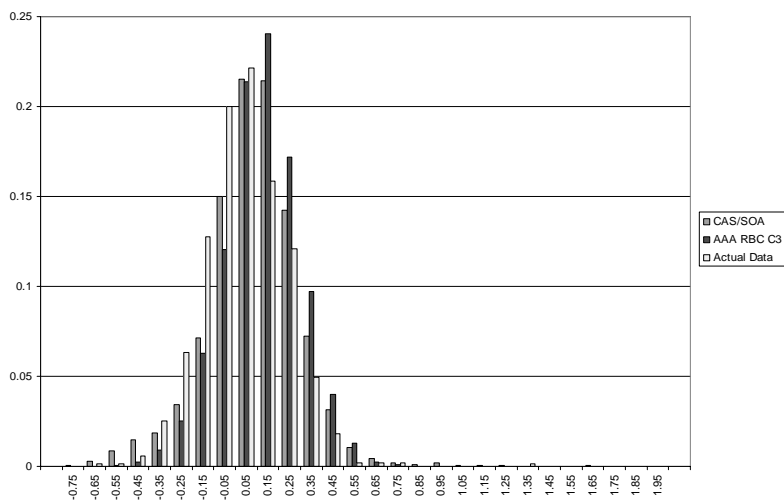
Funnel of Doubt Graphs 10 Year Nominal Interest Rates (U. S. Treasury Bonds)



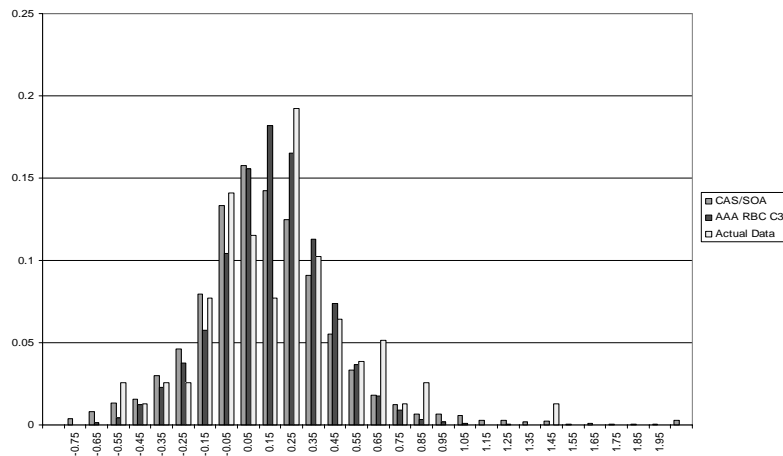
Histogram of 10 Year Nominal Interest Rates Model Values and Actual Data (04/53-05/04)



Histogram of Large Stock Return Model Values and Actual Data (1872-2004)



Histogram of Small Stock Return
Model Values and Actual Data (1926-2003)



How to Obtain Model

Posted on the following sites:

- <http://casact.org/research/econ/>
- <http://www.soa.org/ccm/content/areas-of-practice/finance/mod-econ-series-coor-int-rate-scen/>

Or contact us at: kahlgrim@ilstu.edu
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