Modeling of Economic Series

Research Sponsored by the Casualty Actuarial Society and the Society of Actuaries

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Enterprise Risk Management Symposium
April 2006

Acknowledgements

We wish to thank the Casualty Actuarial Society and the Society of Actuaries for providing financial support for this research, as well as guidance and feedback on the subject matter.

Outline of Presentation

- Motivation for Financial Scenario Generator Project
- Short description of included economic variables
- An overview of the model
- Applications of the model
- Comparison of this model with another actuarial return generating model
- Conclusions

Overview of Project

- CAS/SOA Request for Proposals on "**Modeling of Economic Series Coordinated with Interest Rate Scenarios"**
  - A key aspect of ***dynamic financial analysis***
  - Also important for regulatory, rating agency, and internal management tests – e.g., ***cash flow testing***
- Goal: to provide actuaries with a model for ***projecting economic and financial indices***, ***with realistic interdependencies*** among the variables**.**
  - Provides a ***foundation*** for future efforts

Scope of Project

- ***Literature review***
  - From finance, economics, and actuarial science
- ***Financial scenario model***
  - Generate scenarios over a 50-year time horizon
- ***Document and facilitate use of model***
  - Report includes sections on data & approach, results of simulations, user's guide
  - Posted on CAS & SOA websites
  - Writing of papers for journal publication

Economic Series Modeled

- Inflation
- Real interest rates
- Nominal interest rates
- Equity returns
  - Large stocks
  - Small stocks
- Equity dividend yields
- Real estate returns
- Unemployment

Current Report Structure

Text Sections

1) Intro & Overview
2) Excerpts from RFP
3) Selected Proposal
4) Literature Review
5) Data & Approach
6) Issue Discussion
7) Results of Simulations
8) Conclusion

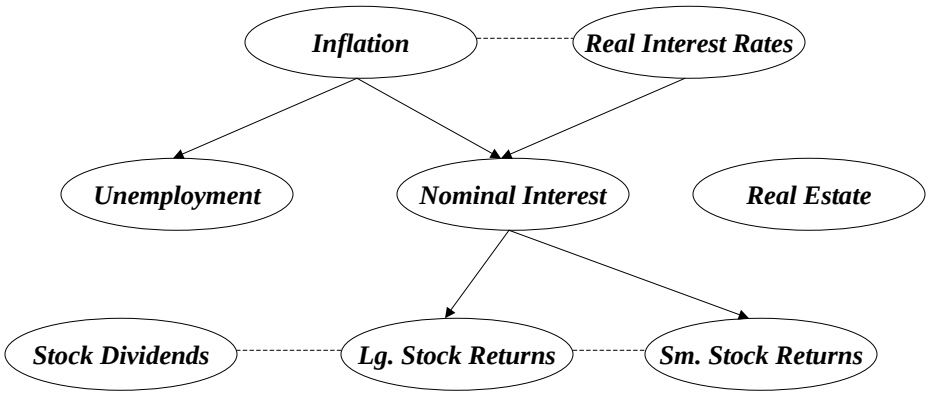
Appendices

A) User's Guide to the Model
B) Presentations of this Research
C) Simulated Financial Scenario Data
D) Financial Scenario Model

Prior Work

- Wilkie, 1986 and 1995
  - Widely used internationally
- Hibbert, Mowbray, and Turnbull, 2001
  - Modern financial tool
- CAS/SOA project (a.k.a. the Financial Scenario Generator) applies Wilkie/HMT to U.S.

Relationship between Modeled Economic Series



Inflation (q)

- Modeled as an Ornstein-Uhlenbeck process
  - One-factor, mean-reverting

$$dq_t = \kappa_q (\mu_q - q_t)\, dt + \sigma\, dB_q$$

- Speed of reversion: $\kappa_q = 0.40$
- Mean reversion level: $\mu_q = 4.8\%$
- Volatility: $\sigma_q = 0.04$

Explanation of the Ornstein-Uhlenbeck process

- Deterministic component

  If inflation is below 4.8%, it reverts back toward 4.8% over the next year

  Speed of reversion dependent on κ

- Random component

  A shock is applied to the inflation rate that is a random distribution with a std. dev. of 4%

- The new inflation rate is last period's inflation rate changed by the combined effects of the deterministic and the random components.

Real Interest Rates (r)

- Problems with one-factor interest rate models
- Two-factor Vasicek term structure model
- Short-term rate (***r***) and long-term mean (***l***) are both stochastic variables

$$dr_t = \kappa_r (l_t - r_t)\, dt + \sigma_r\, dB_r$$

$$dl_t = \kappa_l (\mu_l - l_t)\, dt + \sigma_l\, dB_l$$

Nominal Interest Rates

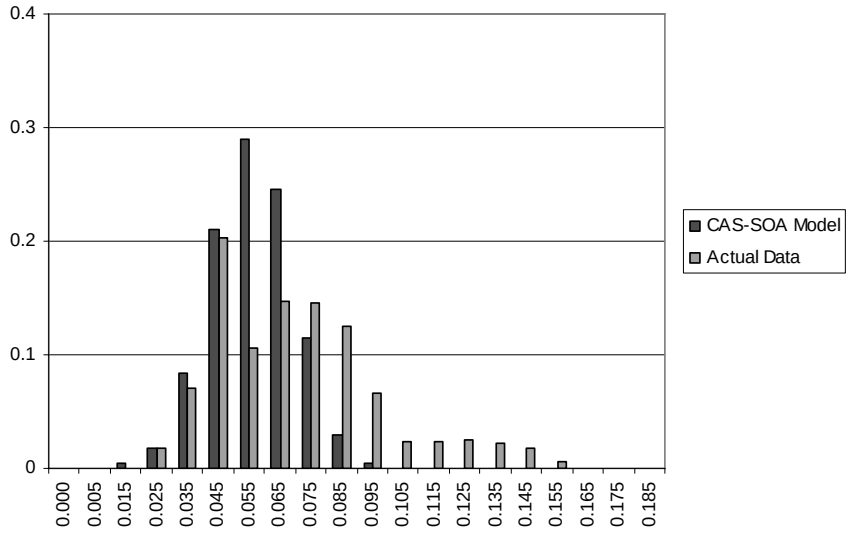
- Combines inflation and real interest rates

$$i = \{(1+q) \times (1+r)\} - 1$$

where i = nominal interest rate
q = inflation
r = real interest rate

Histogram of 10 Year Nominal Interest Rates

Model Values and Actual Data (04/53-01/06)



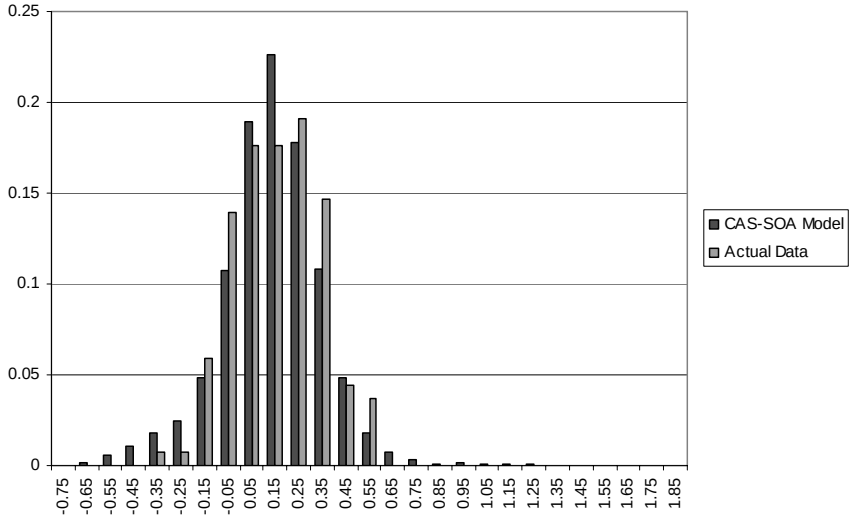
Equity Returns

- Empirical "fat tails" issue regarding equity returns distribution
- Thus, modeled using a "regime switching model"
  1. High return, low volatility regime
  2. Low return, high volatility regime
- Model equity returns as an excess return ($\boldsymbol{x_t}$) over the nominal interest rate

$$s_t = q_t + r_t + x_t$$

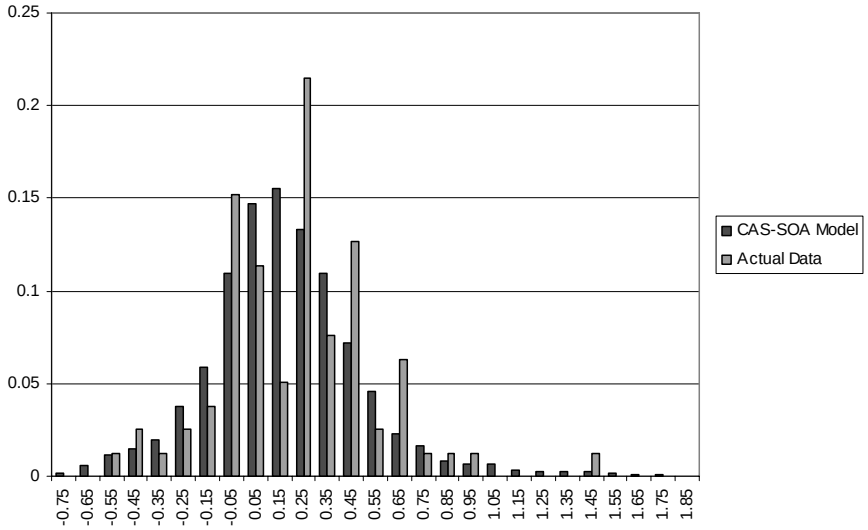
Histogram of Large Stock Return

Model Values and Actual Data (1872-2006)



Histogram of Small Stock Return

Model Values and Actual Data (1926-2004)



Other Series

- ***Equity dividend yields (y) and real estate***
  - Mean-reverting processes
- ***Unemployment*** (***u***)
  - Phillip's curve: inverse relationship between ***u*** and ***q***

$$du_t = \kappa_u (\mu_u - u_t)\, dt + \alpha_u\, dq_t + \sigma_u\, \varepsilon_{ut}$$

Selecting Parameters

- Historical or calibration with current market prices
- Model is meant to represent range of outcomes possible for the insurer
- Default parameters are chosen from history (as long as possible)
- Of course, different parameters may affect analysis

Model Description

- Excel spreadsheet
- Simulation package - @RISK add-in
- 50 years of projections
- Users can select different parameters and track any variable

Applications of the Financial Scenario Generator

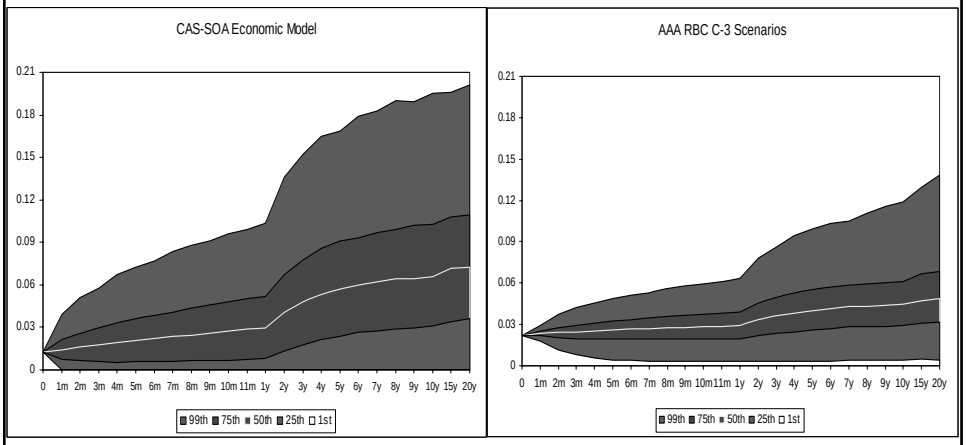
- Financial engine behind many types of analysis
- Insurers can project operations under a variety of economic conditions (Dynamic financial analysis)
- Useful for demonstrating solvency to regulators
- May propose financial risk management solutions

CAS/SOA vs. AAA

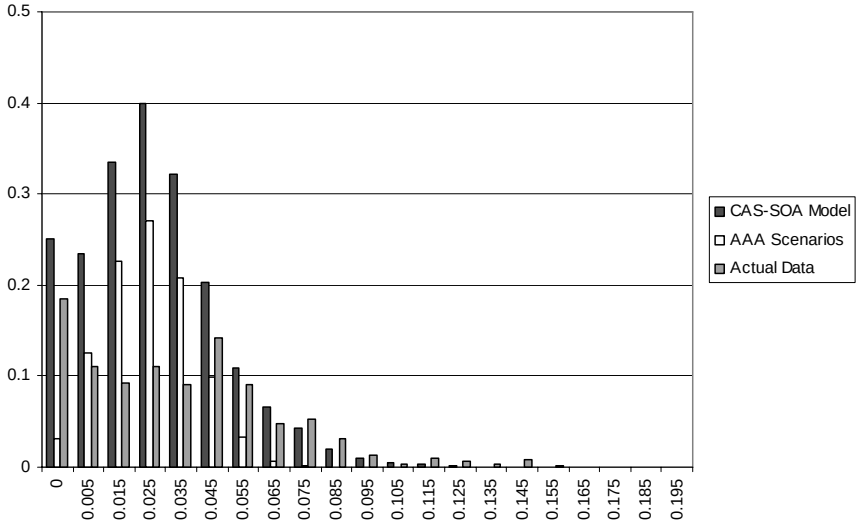
- AAA models provides guidance for Risk-Based Capital (RBC) requirements for variable products with guarantees
- Focus is on
  - Interest rate risk
  - Equity risk
- 10,000 Pre-packaged scenarios available
- Model and scenarios are available at:

http://www.actuary.org/life/phase2.asp

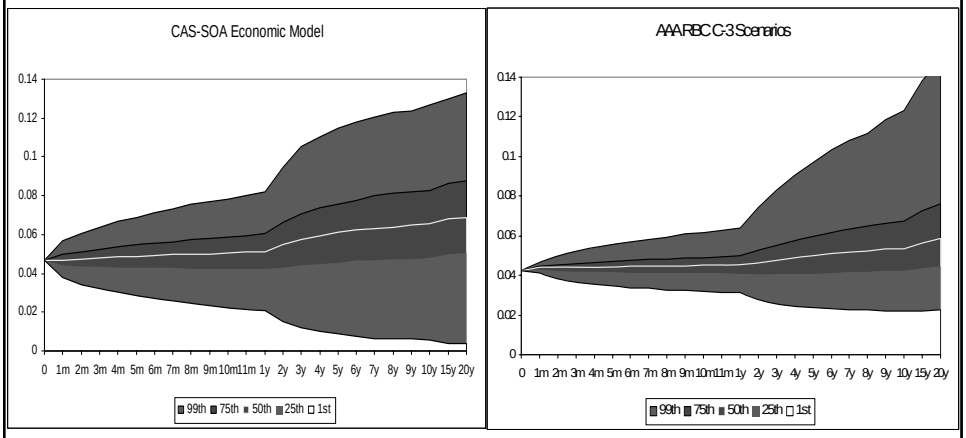
Funnel of Doubt Graphs
3 Month Nominal Interest Rates (U. S. Treasury Bills)



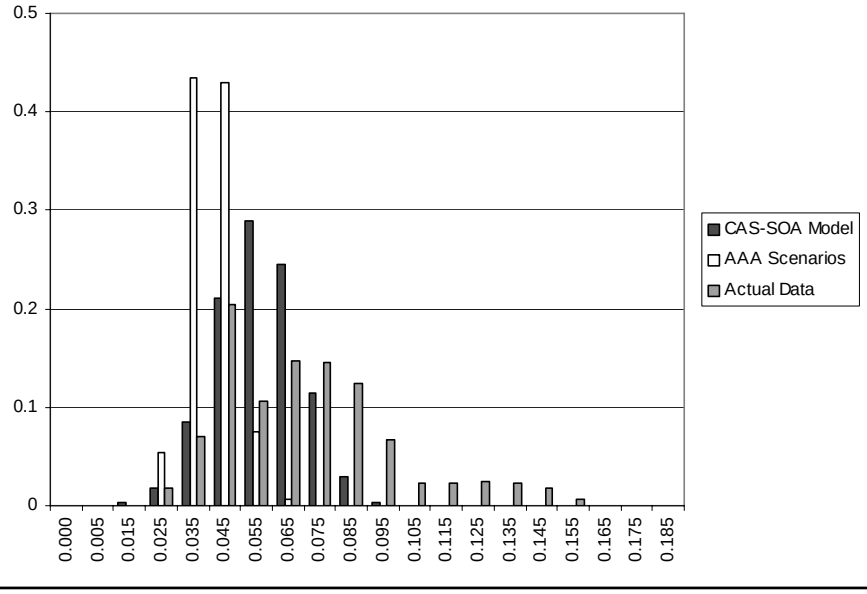
Histogram of 3 Month Nominal Interest Rates
Model Values and Actual Data (01/34-01/06)

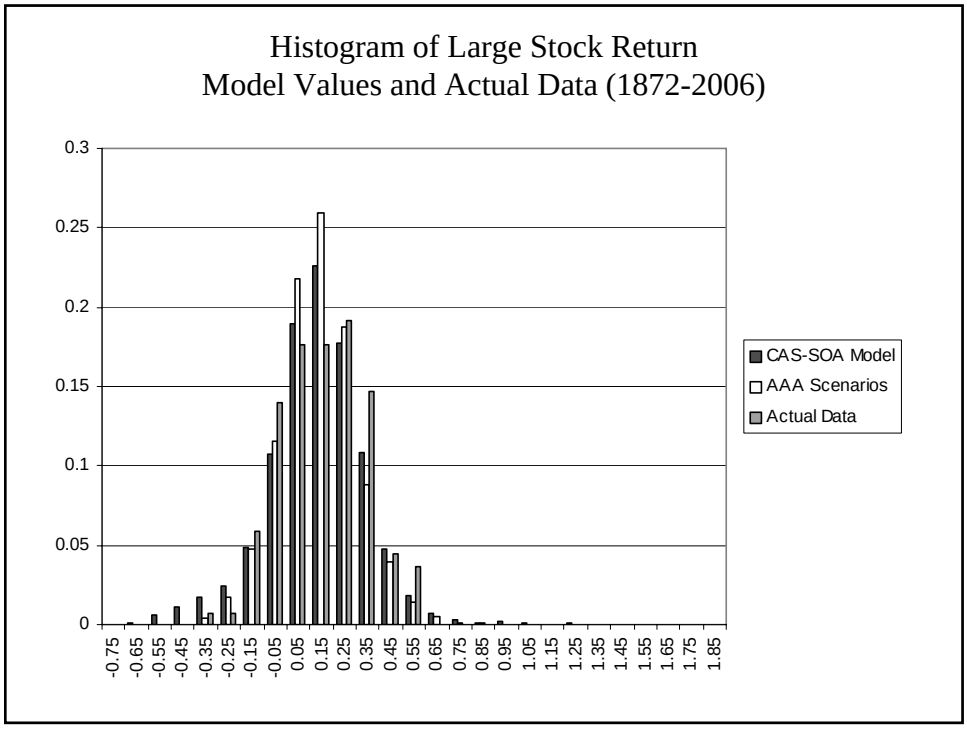


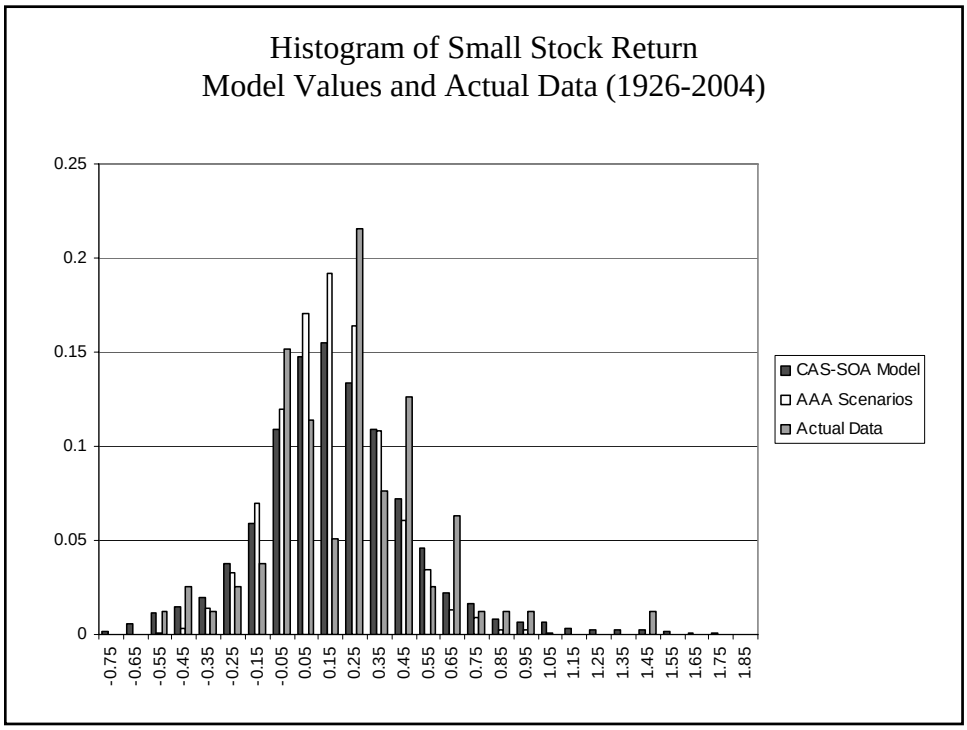
Funnel of Doubt Graphs
10 Year Nominal Interest Rates (U. S. Treasury Bonds)



Histogram of 10 Year Nominal Interest Rates
Model Values and Actual Data (04/53-01/06)




Histogram of Large Stock Return
Model Values and Actual Data (1872-2006)
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.75 -0.65 -0.55 -0.45 -0.35 -0.25 -0.15 -0.05 0.05 0.15 0.25 0.35 0.45 0.55 0.65 0.75 0.85 0.95 1.05 1.15 1.25 1.35 1.45 1.55 1.65 1.75 1.85
CAS-SOA Model
AAA Scenarios
Actual Data


Histogram of Small Stock Return
Model Values and Actual Data (1926-2004)
0.25
0.2
0.15
0.1
0.05
0
-0.75 -0.65 -0.55 -0.45 -0.35 -0.25 -0.15 -0.05 0.05 0.15 0.25 0.35 0.45 0.55 0.65 0.75 0.85 0.95 1.05 1.15 1.25 1.35 1.45 1.55 1.65 1.75 1.85
CAS-SOA Model
AAA Scenarios
Actual Data

Quantification of Model Fit

- Kolmogorov-Smirnov test

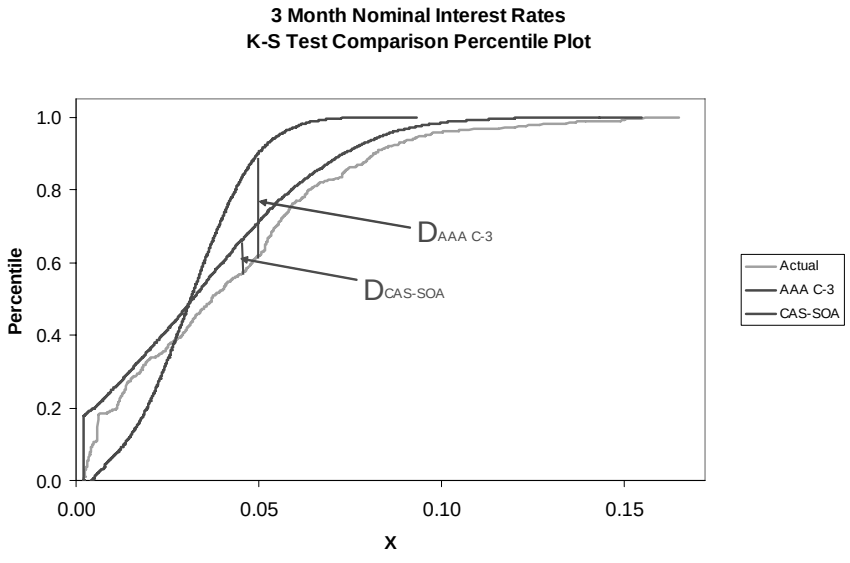
  Tries to determine if two datasets differ significantly

  Uses the maximum vertical difference between percentile plots of the data as statistic D

- Chi-square test

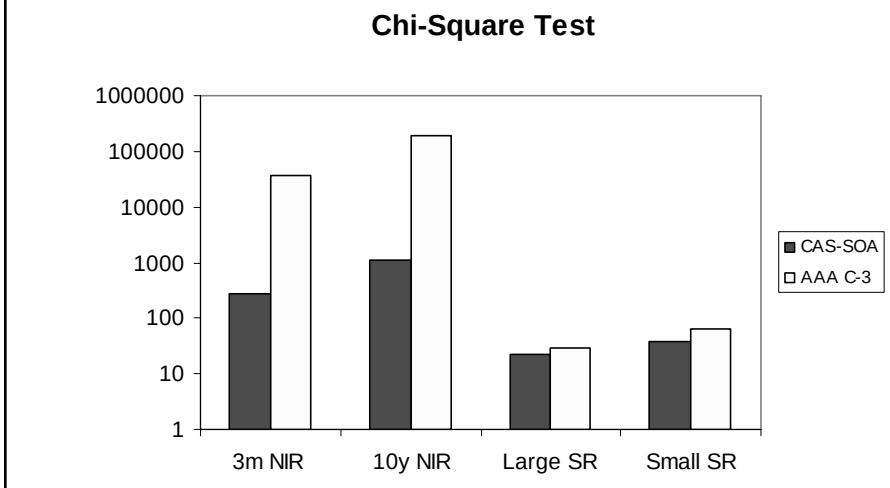
  Take the squared difference between observed frequency (**O**) and the expected frequency (**E**), and then divided by the expected frequency

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$



Kolmogorov-Smirnov Comparison of Two Data Sets

	CAS-SOA Model					AAA C-3 Model			
	3m NIR	10y NIR	Large SR	Small SR		3m NIR	10y NIR	Large SR	Small SR
1	0.1626	0.3138	0.0525	0.1306	1	0.2733	0.5265	0.0943	0.1987
2	0.1748	0.3152	0.0587	0.1280	2	0.2935	0.5375	0.0796	0.1943
3	0.2010	0.3276	0.0822	0.1430	3	0.3109	0.5575	0.0992	0.2030
4	0.1750	0.3186	0.0917	0.1550	4	0.2992	0.5395	0.1062	0.2050
5	0.1670	0.3316	0.0678	0.1290	5	0.2882	0.5325	0.0992	0.1827
6	0.1780	0.3267	0.0772	0.1610	6	0.2892	0.5285	0.0902	0.1900
7	0.1880	0.3240	0.0632	0.1590	7	0.2916	0.5340	0.1102	0.2073
8	0.1710	0.3246	0.0678	0.1220	8	0.2622	0.5110	0.0942	0.2133
9	0.1950	0.3252	0.0636	0.1323	9	0.2972	0.5385	0.1026	0.2093
10	0.1683	0.3059	0.0656	0.1431	10	0.2932	0.5212	0.0930	0.1954
Average	**0.1781**	**0.3213**	**0.0690**	**0.1403**	**Average**	**0.2899**	**0.5327**	**0.0969**	**0.1999**



Summary of Differences

- Kolmogorov-Smirnov test

  Statistic D of CAS-SOA model is smaller than that of AAA C-3 model

- Chi-square test

  For nominal interest rate, the Chi-square value of CAS-SOA model is smaller than that of AAA C-3 model

  For small stock returns, both models are rejected at significant level of 0.025 while accepted at level of 0.1

  For large stock returns, both models are rejected at significant level of 0.05 while accepted at level of 0.1

How to Obtain Models

CAS-SOA model is posted on the following sites:

- http://casact.org/research/econ/
- http://www.soa.org/ccm/content/areas-of-practice/finance/mod-econ-series-coor-int-rate-scen/

Or contact us at: kahlgrim@ilstu.edu
s-darcy@uiuc.edu
gorvett@uiuc.edu

- AAA model is posted at:

http://www.actuary.org/life/phase2.asp