

Risk Geographies: The Next Generation of Value-at-Risk Models

Presented to the 4th ERM Symposium
April 23-25, 2006
Session RM 6

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Presentation Overview

- Key events in Value-at-Risk science
 - Most likely ruin event (MLRE)
 - Least solvent likely event (LSLE)
- Variable annuity example
 - Allowing for non-linearities
 - Implications for hedging and capital
- Conclusions

Computation Approaches

Analytical

Risk metrics approach

Driver distribution is multivariate normal

Response function is linear (or quadratic)

Analytical capital and risk calculations

Risk Geographies

Numerical probability density integration, focusing on the **driver values** with high probability and adverse **response function**.

Monte Carlo

Simulate **driver values** from **driver distribution**

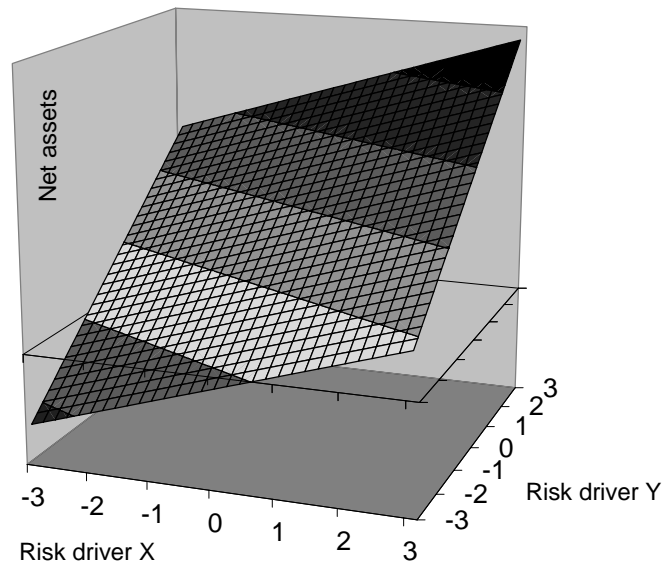
Apply the **response function** to compute net assets for each simulated vector of **driver values**

Analyse the simulated distribution of net assets.

Response Function

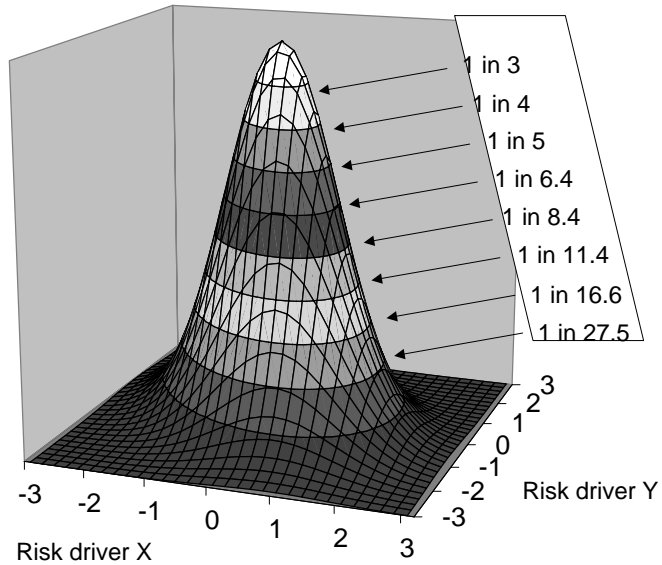
Consider a model firm whose net assets are exposed to two risk drivers – for example interest rates and stocks. In this example, we have standardised the risk drivers to have mean zero and standard deviation 1.

The response function expresses net assets as a function of risk drivers.

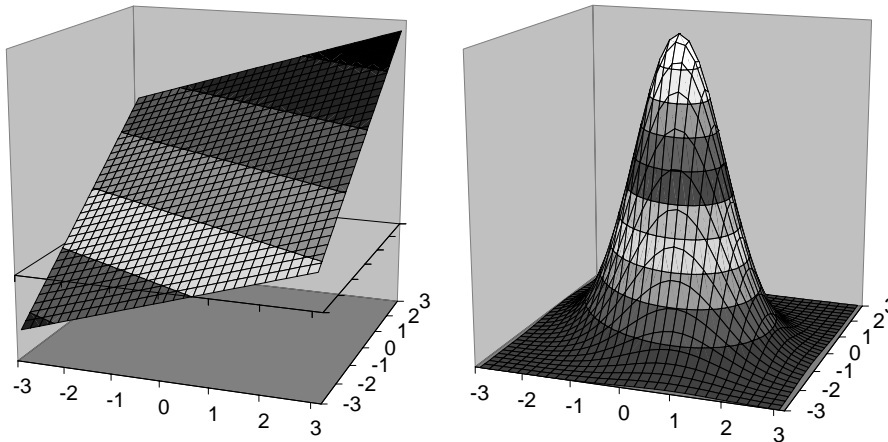


Likelihood Function

The likelihood function shows the probability density of various combinations of risk drivers. In this case, our likelihood function is a bivariate normal distribution with a characteristic bell-shape.

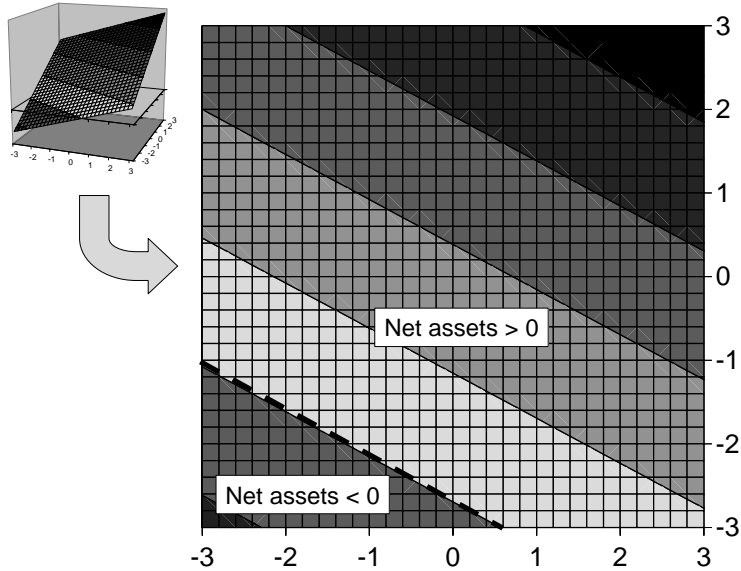


Where is Likely and Painful?



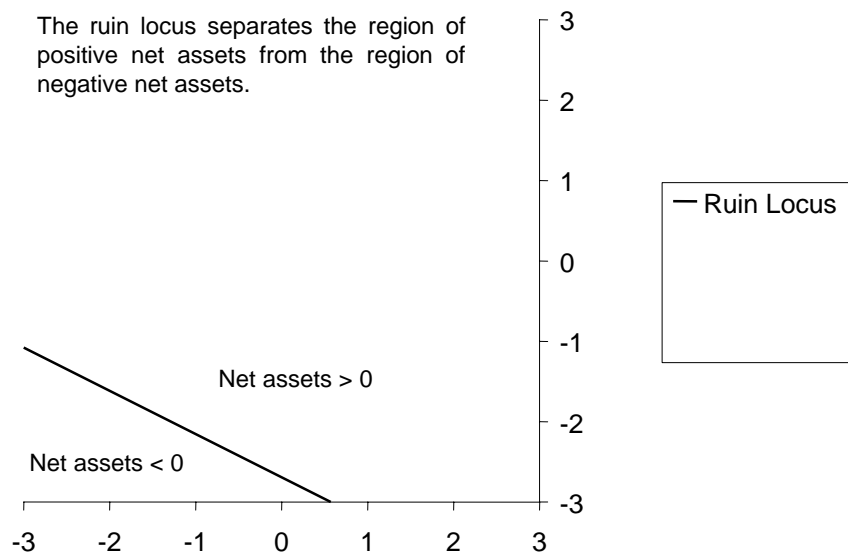
We seek to determine combinations of factor values which are simultaneously likely and painful.

Birds-Eye View: Response



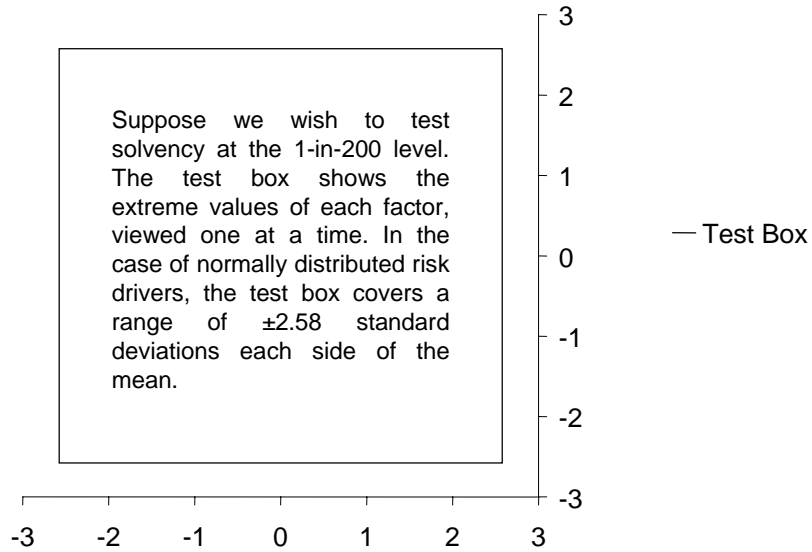
Ruin Locus

The ruin locus separates the region of positive net assets from the region of negative net assets.

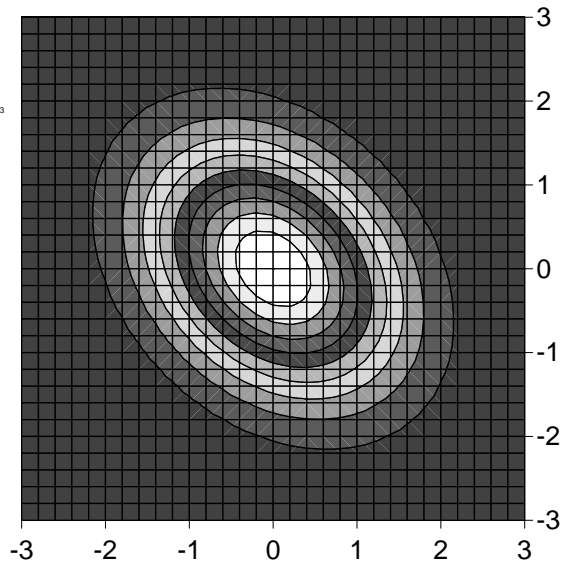
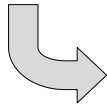
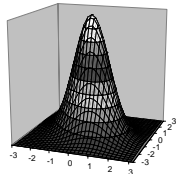


Test Box

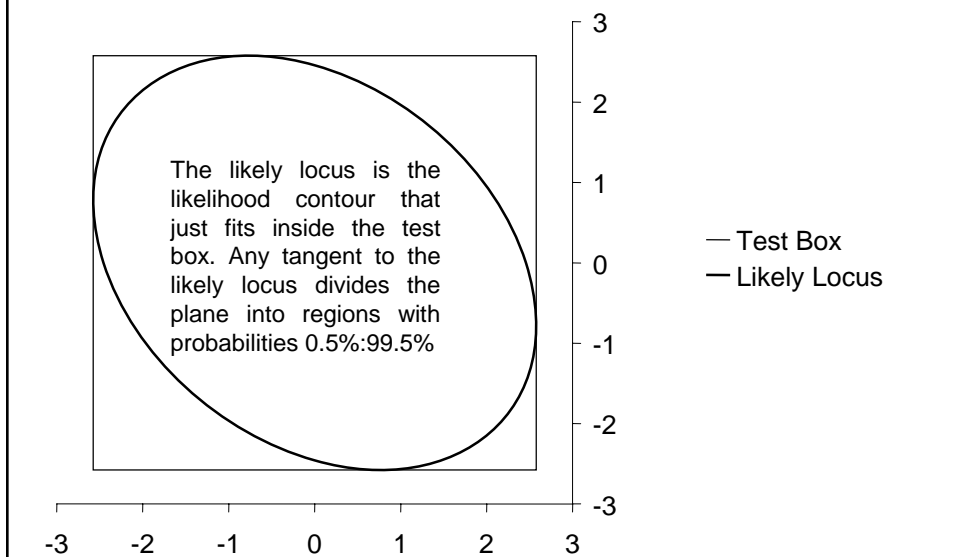
Suppose we wish to test solvency at the 1-in-200 level. The test box shows the extreme values of each factor, viewed one at a time. In the case of normally distributed risk drivers, the test box covers a range of ± 2.58 standard deviations each side of the mean.



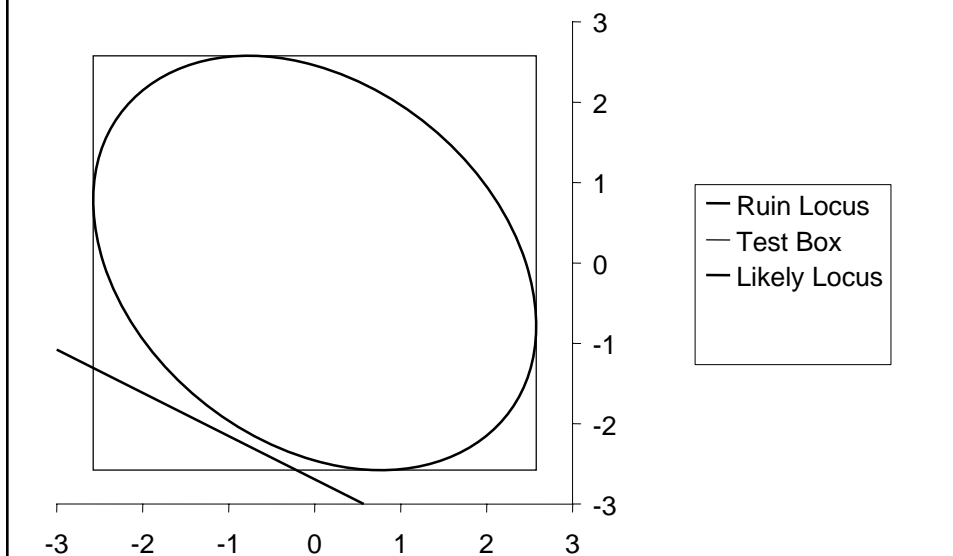
Bird's Eye View: Likelihood



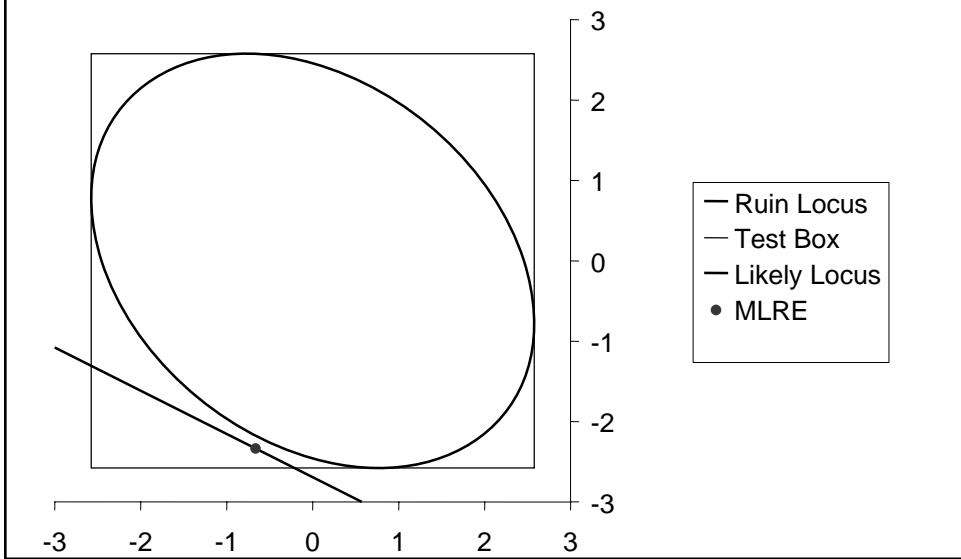
Likely Locus



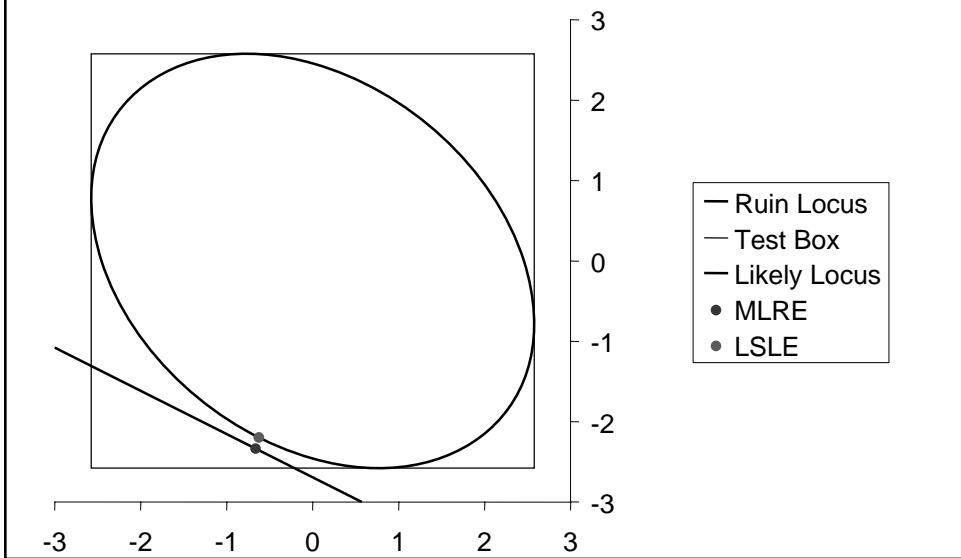
Likely Locus and Ruin Locus



Most Likely Ruin Event



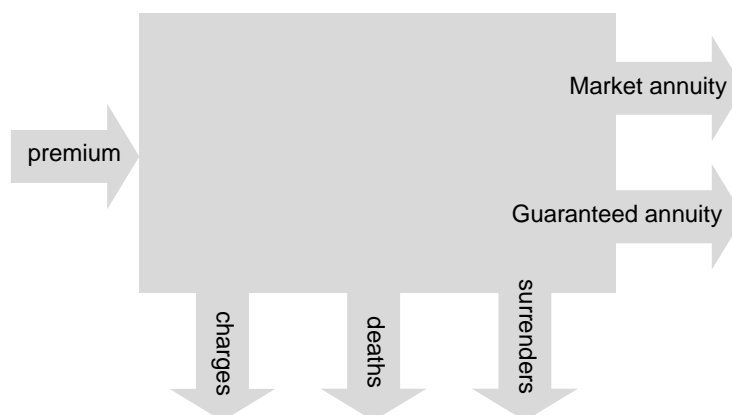
Least Solvent Likely Event



Risk Management

- LSLE is a transparent and powerful summary of the risks facing an organisation as a whole or its constituent business units
- Evaluate internal hedging opportunities by computing LSLE at business unit level and comparing to top level LSLE
- Reduce capital requirements by looking for strategies that mitigate losses in the top level LSLE
 - e.g. hedging
 - reinsurance
 - investment strategy
 - contingent capital
 - then you have to start over and compute a new MLRE!

Variable Annuity Example



Variable Annuity Example

- Single premium, fund invested in equities with flat annual % charge
- Guarantee grows at constant rate each year
- Surrenders receive fund value but no guarantee
- Policyholder death pre-retirement receives
 - $\max\{\text{fund value, premium} + \text{accrued guarantees}\}$
- Guaranteed annuity rate, applies to guaranteed fund
- Survivors to retirement receive annuity equal to one of
 - $\text{actual fund value} * \text{market annuity rate}$ (default option)
 - $\text{guaranteed value} * \text{guaranteed annuity rate}$ (elective)

Risk Geographies Output

	Base	Stresses	No Hedge		With Hedge	
			LSLE	Percentile	LSLE	Percentile
Interest	4.50%	±1.50%	3.43%	3.3%	5.36%	93.1%
Equity	100%	± 35%	86%	14.4%	117%	89.5%
Volatility	15%	± 8%	21%	97.0%	9%	2.5%
Mortality (pre-ret)	100%	± 30%	97%	40.5%	101%	52.1%
Mortality (post-ret)	100%	± 30%	96%	37.0%	100%	48.8%
Surrender rate	5%	± 5%	5%	42.0%	6%	73.4%
Take-up	50%	± 30%	64%	88.8%	48%	43.8%
Net Assets	6.37		0.73		5.67	
ECAP			5.64		0.70	

Techniques Comparison

	Risk metrics	Risk. Geog	Monte Carlo ²
Fast run time	●	●	●
Convex response	●	●	●
Add extra drivers	●	●	●
Mid-year effects	●	●	●
Lumpy response	●	●	●
Heavy tails	●	●	●
Copulas	●	●	●
Parameter error	●	●	●
Sampling error	●	●	●

User Benefits

- Accurate assessment of ruin probability
- Robust calculations of economic capital
- Interactive desktop tool illuminates key assumptions and their importance
- Clear analysis of costs and benefits of hedging, reinsurance and other risk management initiatives, extending existing cash flow and valuation models
- Intuitive scenario-based risk communication
- Respond confidently to questions about operational risk, heavy tails and correlations under stressed situations
- Step-by-step transition from the correlation approach to Risk Geographies

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