What is the Optimal Investment in a Hedge Fund?

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Phelim Boyle

Wilfrid Laurier University
and Tirgarvil Capital

pboyle at wlu.ca

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Outline

- Introduction and background
- Hedge Funds
- Nature of returns
- How much should an investor put in a hedge fund?
- Biases
- Estimation risk
- Robust optimization

Explosive Growth

- Hedge funds are increasingly important players in financial markets
- In 1990, there were just 610 funds controlling some $39 billion of assets.
- By 2000 there were 3,873 funds with $490 billion.
- Latest estimate is over 9,000 funds, with $1.3 trillion assets
- HFs account for about five percent of global financial assets
- They account for 50% of the trading on New York and London.

Who invests in Hedge Funds

Investors in HF include

1. Rich individuals
2. Institutions
3. Endowment funds
4. Pension plans
5. Funds of funds
6. Retail investors

Reasons for growth

- Advances in technology
- The derivatives revolution
- Specialization
- Growing complexity of markets (catastrophe bonds, structured products.)
- Recent poor equity market performance
- Low interest rates

Promise of Hedge Funds?

1. Hedge Funds promise extra return alpha
2. Low volatility
3. Low correlation with the market
4. Low beta

Recent returns suggest that there has been

- Reduction in alpha
- Increase in beta

Characteristics of Hedge Funds

- Lightly regulated investment pools
- Presumption is (was?) that investors are sophisticated
- Structured as limited partnership
- Managers have wide investment freedom
- Can take short positions
- Use derivatives
- Employ leverage
- Hedge Funds (until now) minimal disclosure

Characteristics of Hedge Funds

- Funds have generally a lock up period
- Manager invests own funds
- Management fees range from 1% to 2% and incentive fees are 15% to 25% per year
- Wide variety of investment styles (strategies)
  1. Funds often aim for absolute returns
  2. Low volatility
  3. Low correlation with the market

Benefits of Hedge Funds

- They provide liquidity
- Price efficiency
- Better risk distribution
- Promote globalization: provide more choice for investors

Potential risks

Main concerns

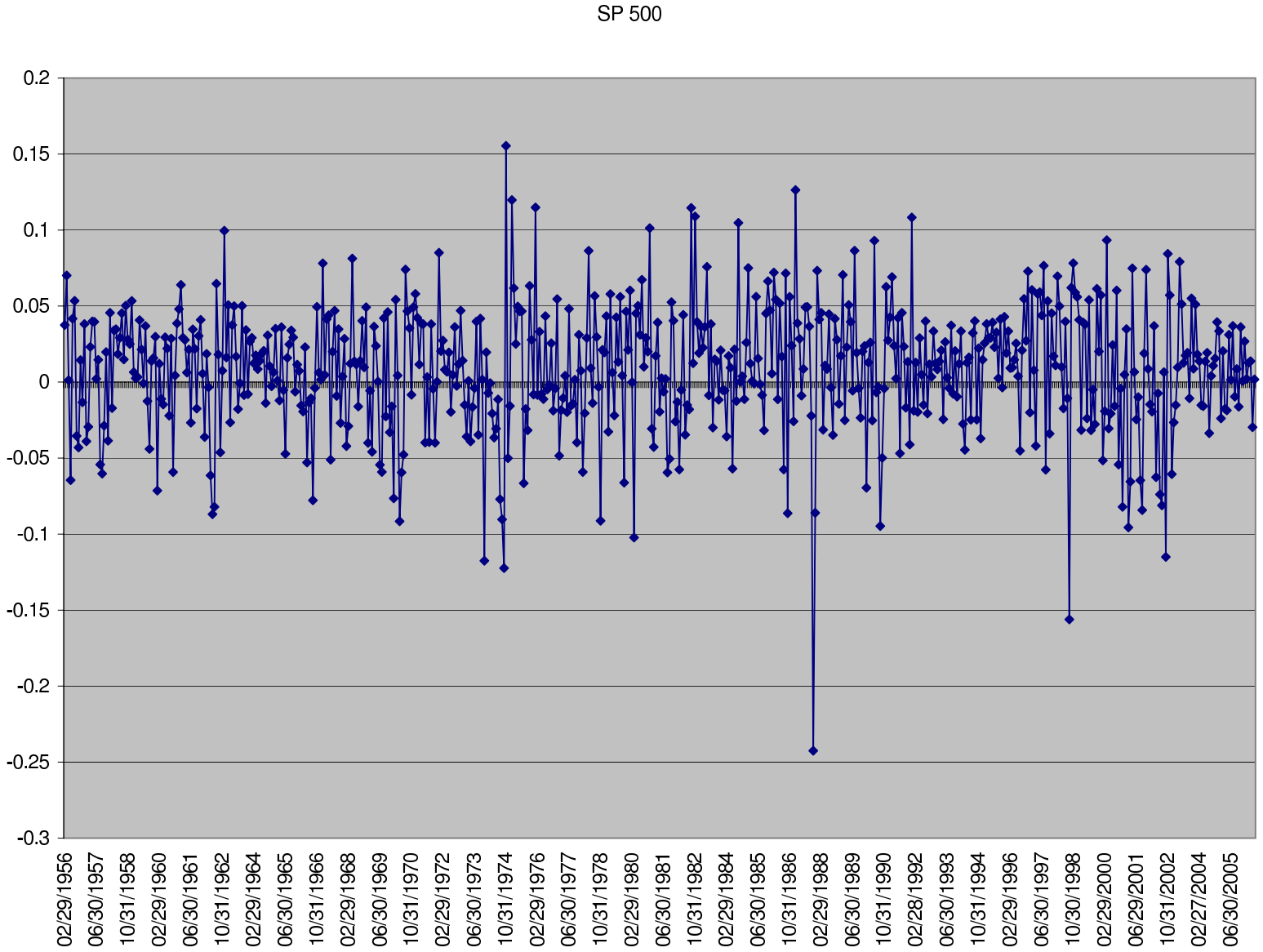
1. Hedge funds are destabilizing
2. Hedge funds lack transparency
3. Hedge funds are highly levered
4. Hedge funds are prone to herd behavior
5. Hedge funds are prone to commit fraud

Estimating Hedge Fund returns

- Returns are not normal.
- Even for equity markets we see bull and bear markets
- Bull market: good returns, low volatility
- Bear market: poor returns, high volatility

Regime Switching Models

Chan, Getmansky, Haas, and Lo(2005) applied these models to hedge fund returns.


SP 500
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0.1
0.05
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-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
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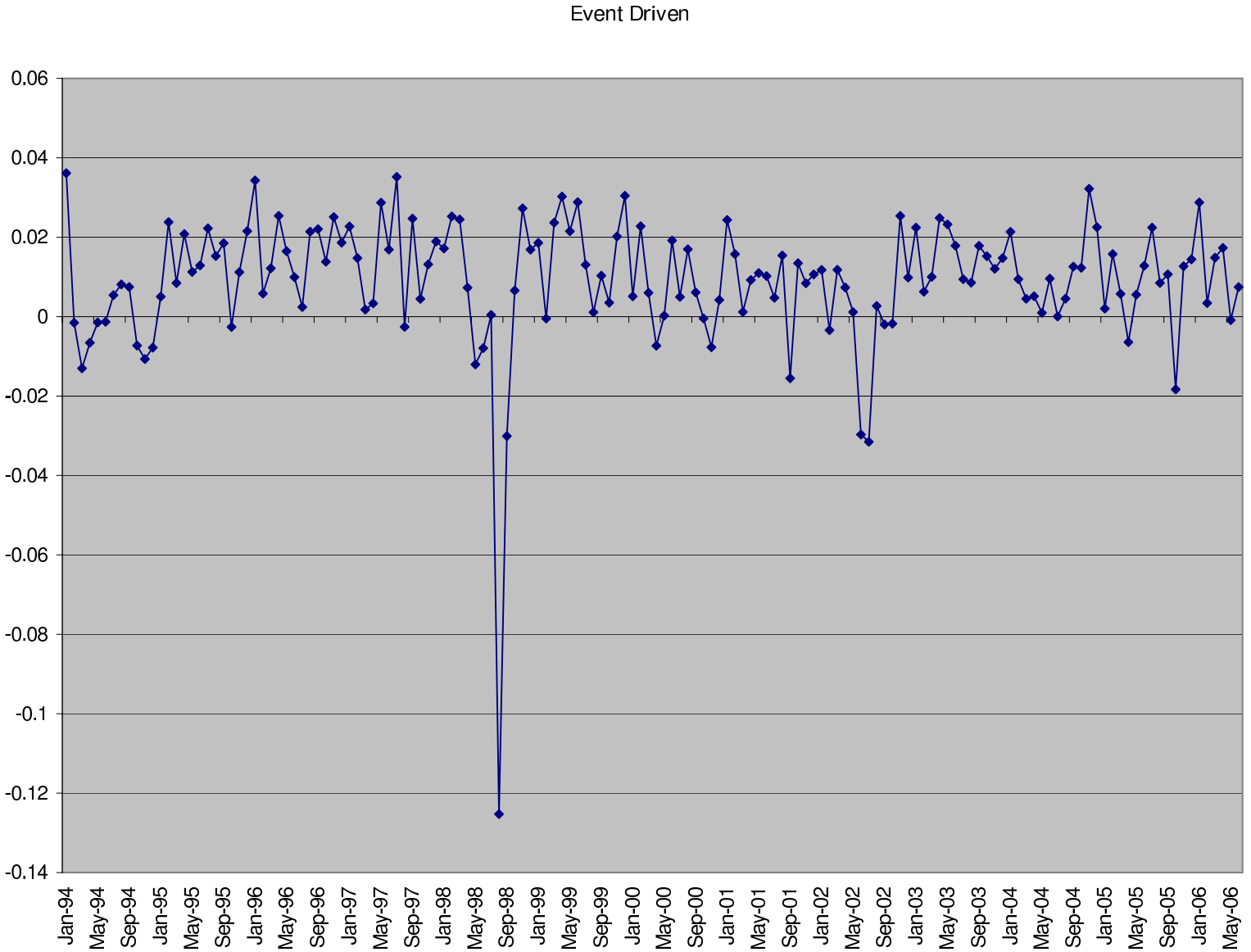
Event Driven
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0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
-0.12
-0.14
Jan-94
May-94
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May-06

Regime Switching Model

- We suppose stock (and HF) return process lies in one of two regimes.
- Changes in regime are determined by an unobserved state variable
- Model it by a Markov Chain, Hamilton (1989), (1990),(1994)
- Data shows periods of low volatility and periods of high volatility
- Changes in regime are caused by factors that we do not model.
- At the current time we do not know which regime we are in.
- Afterwards we can identify which regime we were in with some degree of confidence

S and P parameter estimates 1956-2006 Monthly

Parameter	Mean (standard deviation)	Annualized %
$\widehat{\mu_1}$	0.0124(0.002)	14.88
$\widehat{\mu_2}$	-0.0049(0.014)	-5.88
$\widehat{\sigma_1}$	0.0323 (0.002)	11.36
$\widehat{\sigma_2}$	0.0608(0.007)	21.06
$\widehat{p_{1,2}}$	0.0432(0.013)	na
$\widehat{p_{2,1}}$	0.1411 (0.101)	na

Here $p_{1,2}$ is the probability of a transition from regime one to regime two. Similarly $p_{2,1}$ is the probability of a transition from regime two to regime one. .

Plotting the regimes

The next graph shows our estimates of the regimes.

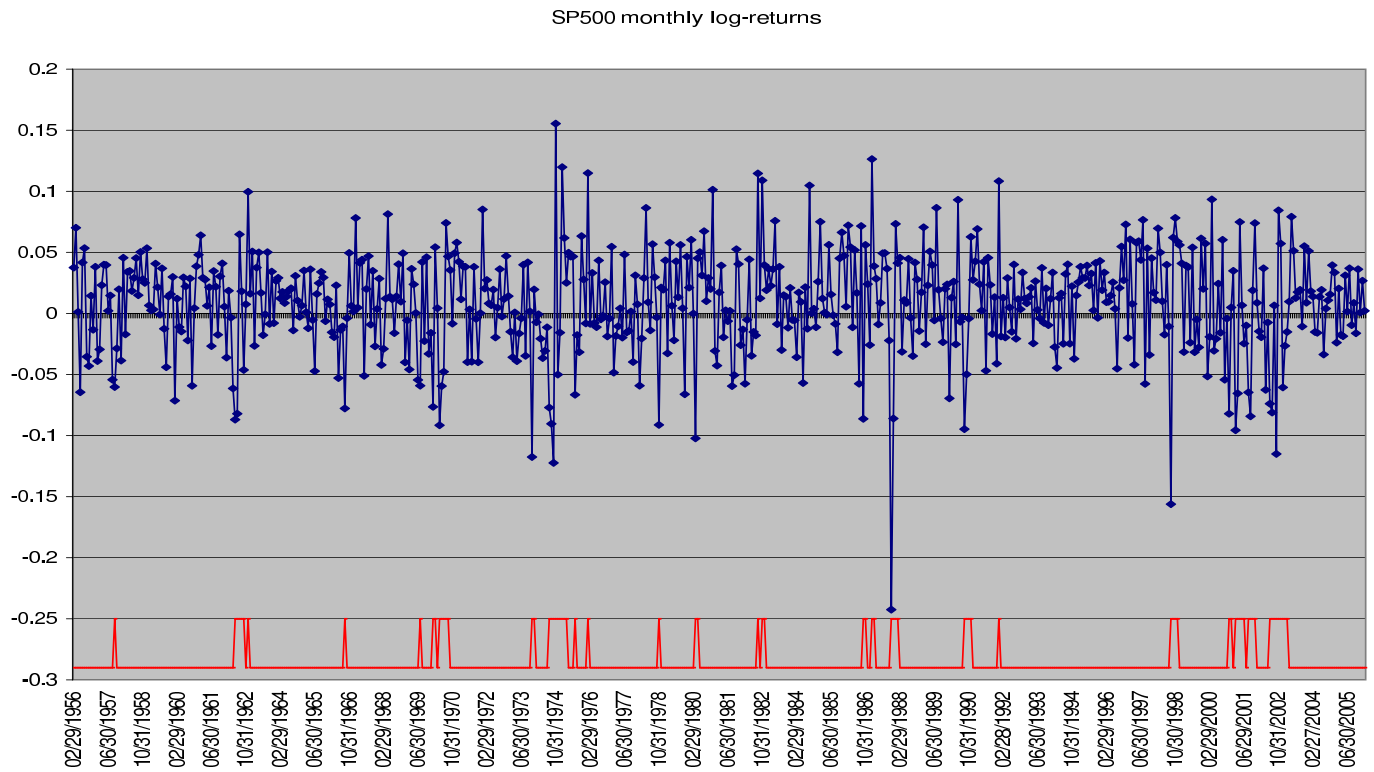


Figure 1: Monthly returns of the S&P 500 together with the estimation of each regime. The regimes are shown at the bottom of the graph.

Plotting the regimes

In the next graph we group according to the regimes. We see that the distribution within the regime appears to be normal.

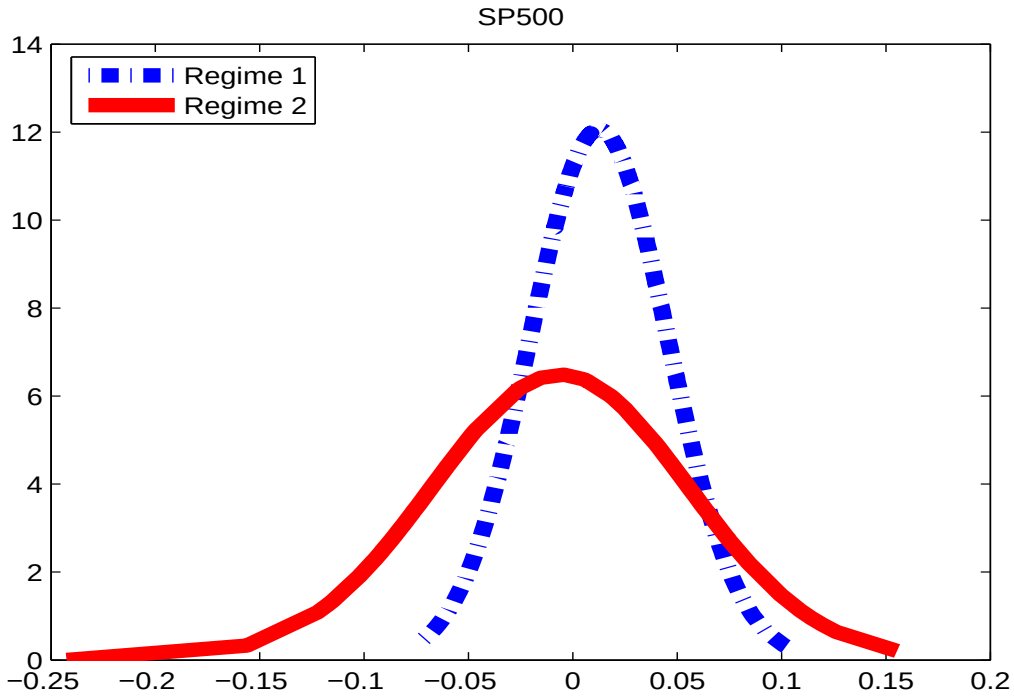


Figure 2: Densities of S&P 500 observations grouped according to their regimes.

Fitting Hedge Fund returns to RS Model

We can fit regime switching models to each of the hedge fund strategies. Our data runs from January 1994 until June 2006. Parameters in the next table .

Table . Maximum Likelihood Estimates of the Univariate RSLN model for the CSFB hedge fund indexes. The estimates in this table are computed using monthly returns from January 1994 to June 2006.

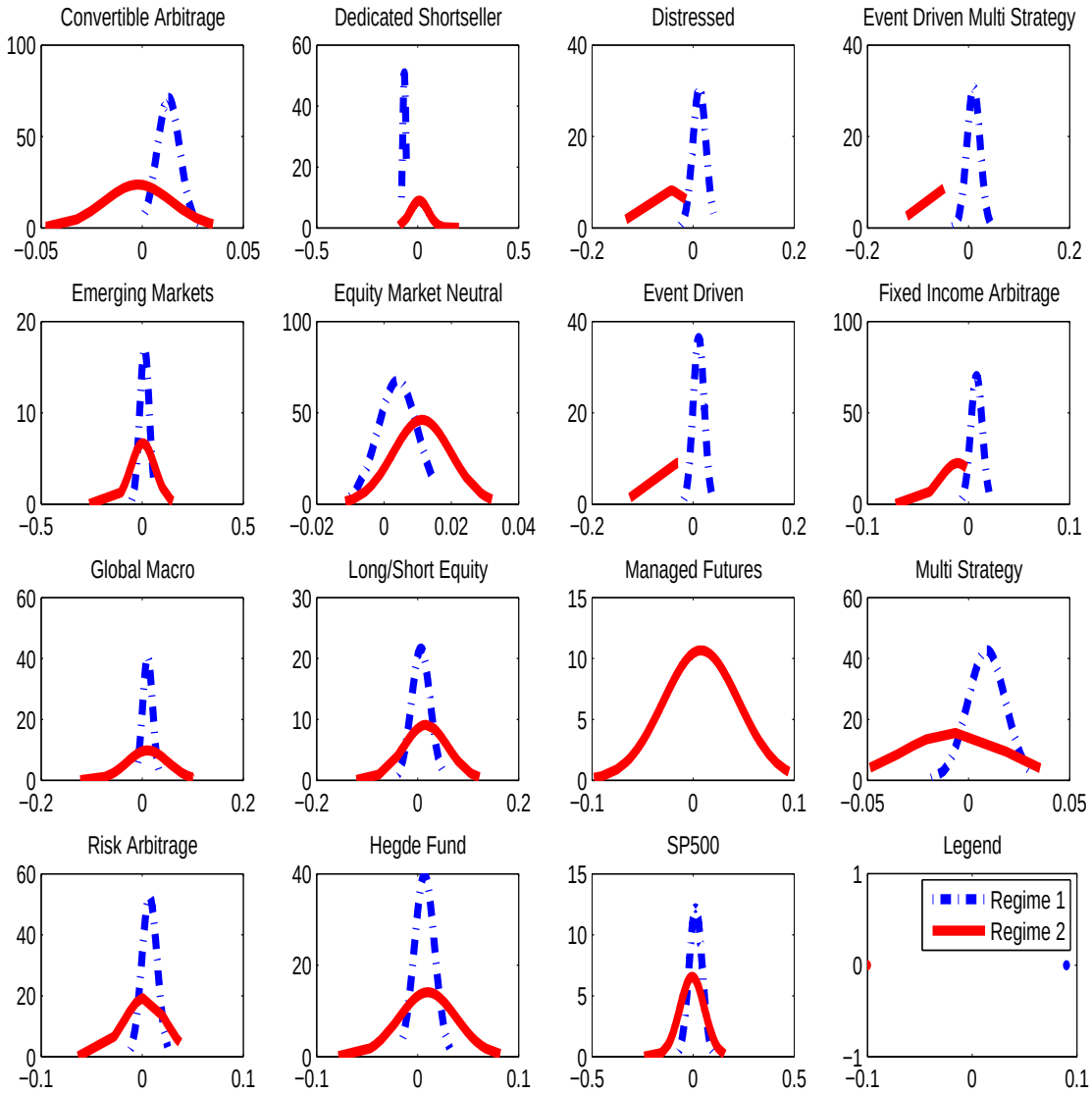
Index	p_{12} %	p_{21} %	Annual μ_1 %	Annual μ_2 %	Annual σ_1 %	Annual σ_2 %	Log-L
CSFB Indexes							
Hedge Funds	0.84	0.72	8.37	12.64	3.48	9.84	386.06
Convertible Arbitrage	11.09	17.03	16.76	-2.52	1.92	4.19	469.37
Dedicated Shortseller	76.25	11.26	-55.98	10.26	2.76	15.58	248.79
Emerging Markets	1.16	0.93	14.60	4.04	8.04	20.76	267.15
Equity Market Neutral	3.28	3.02	5.00	14.45	2.04	3.00	516.55
Event Driven	1.67	46.73	13.97	-39.66	3.84	15.00	445.74

Table . Maximum Likelihood Estimates of the Univariate RSLN model for the CSFB hedge fund indexes. The estimates in this table are computed using monthly returns from January 1994 to June 2006.

Index	p_{12} %	p_{21} %	Annual μ_1 %	Annual μ_2 %	Monthly σ_1 %	Monthly σ_2 %	Log-L
CSFB Indexes							
Distressed	1.76	58.17	15.95	-46.10	4.56	16.68	421.24
Event-Driven Multi-Strategy	1.18	45.24	12.59	-46.44	4.56	15.96	426.21
Risk Arbitrage	7.61	27.58	8.99	3.10	2.64	7.20	468.83
Fixed Income Arbitrage	6.70	39.59	9.99	-12.23	1.92	6.12	513.62
Global Macro	0.78	0.68	13.60	13.65	3.36	14.04	354.66
Long/Short Equity	0.95	2.98	7.86	21.15	6.36	15.24	340.48
Managed Futures	67.49	17.07	-6.74	9.80	4.32	12.96	295.49
Multi-Strategy	2.56	24.40	11.33	-8.11	3.24	8.64	454.46

Plotting the regimes

In the next graph we group data according to the regimes. We see that the distribution within the regime appears to be normal for most HFs.


Convertible Arbitrage
Dedicated Shortseller
Distressed
Event Driven Multi Strategy
Emerging Markets
Equity Market Neutral
Event Driven
Fixed Income Arbitrage
Global Macro
Long/Short Equity
Managed Futures
Multi Strategy
Risk Arbitrage
Hegde Fund
SP500
Legend
Regime 1
Regime 2

Overview

Question: How much should investor put in a hedge fund

- Introduction
- Classical investment problem
- Important for investors
- Problems with data
- Fit econometric model
- Regime switching
- Parameter estimation
- Procedure to decide optimal allocation
- Three asset classes: market portfolio, hedge fund and risk free asset
- Results

Asset Allocation

- Mean variance approach Simple but has drawbacks
  1. Static one period model
  2. Sensitive to expected return assumption
  3. Only first two moments
  4. Often assumes returns are iid

Hence, we do not use mean variance approach.

Asset Allocation Decision

- We assume investor can invest in the S&P , the hedge fund and T bills
- Use Event Driven Hedge Fund
- Investor maximises expected utility
- Utility function indexed by *relative risk aversion*
- Calibrate risk aversion parameters as follows.

Calibration (use Merton Ratio)

- Assume just two assets S&P and T Bills
- Assume risk free rate $= 5\%$ and S&P vol is 15%

Use Merton ratio to connect percentage in stocks with risk aversion

Merton ratio is

$$\frac{\mu - r}{\sigma^2 RRA}$$

where RRA is relative risk aversion

Relative risk aversion	Optimal percent in S&P Risk premium =5%	Optimal percent in S&P Risk premium =3%
2	111	67
3	74	44
4	56	33
5	44	27

Procedure

We assume

- Investor makes buy and hold decision (no rebalancing)
- Three asset classes
- Investor maximizes expected utility of terminal wealth
- Holding period 12 months
- S&P and hedge fund follow regime switching model: one global regime
- Find optimal strategy (no short selling) assuming starting in regime one
- Find optimal strategy (no short selling) assuming starting in regime two

Optimization

We assume there are three available asset classes. These are

1. The core equity portfolio with rate of return r^e

2. The hedge fund with rate of return r^h

3. The risk free asset with rate of return r.

Assume an investor has initial wealth w_0. The investor's end of period wealth will be

$$w = w_0[x_1(1+r^e)+x_2(1+r^h)+x_3(1+r)] = w_0[1+r+x_1(r^e-r)+x_2(r^h-r)],$$

since $x_1 + x_2 + x_3 = 1$. We assume there is no short selling so that all the weights are non negative.

Investor's problem

The investors problem is to maximize

$$E\ [u(w_0[1 + r + x_1(r^e - r) + x_2(r^h - r)]\ )\ |\ .]$$

subject to the constraints.

- Assets have a bivariate regime switching distribution
- Conditional on sojourn times the risky assets have a bivariate lognormal distribution.

Hedge Fund(Event Driven) and S&P: start Regime One

We assume

- Three asset classes
- No rebalancing
- No short sales

Relative Risk Aversion	Core equity portfolio	Hedge Fund (Event Driven)	Risk free asset
2	0.58	0.42	0.00
3	0.37	0.63	0.00
4	0.26	0.74	0.00
5	0.19	0.81	0.00

Hedge fund looks attractive esp to the risk averse investor.

Hedge Fund(Event Driven) and S&P: start Regime Two

We assume

- Three asset classes
- No rebalancing
- No short sales

Relative Risk Aversion	Core equity portfolio	Hedge Fund (Event Driven)	Risk free asset
2	0.51	0.49	0.00
3	0.33	0.66	0.01
4	0.25	0.49	0.26
5	0.21	0.38	0.41

Hedge fund still looks attractive but risk free asset picks up

What about bias

There is strong evidence that hedge fund returns are biased (upwards)

- The Event Driven returns in our data base
  Annual return = 11.2% , Volatility = 5.6%
- De los Rios and Garcia find for event driven strategies the annual (median) return after adjusting for backfill and survivorship bias is 8.1%
- Other researchers have used 3.0% -4.5% deduction for the bias

Hedge Fund(Event Driven) and S&P: start Regime One

We assume

- Three asset classes
- No rebalancing
- No short sales
- HF returns biased deduct 3% pa

Relative Risk Aversion	Core equity portfolio	Hedge Fund (Event Driven)	Risk free asset
2	1.00	0.00	0.00
3	1.00	0.03	0.00
4	0.85	0.15	0.00
5	0.67	0.33	0.00

Hedge Fund(Event Driven) and S&P: start Regime One

We assume

- Three asset classes
- No rebalancing
- No short sales
- HF returns biased deduct 4.50% pa

Relative Risk Aversion	Core equity portfolio	Hedge Fund (Event Driven)	Risk free asset
2	1.00	0.00	0.00
3	1.00	0.00	0.00
4	1.00	0.00	0.00
5	0.90	0.10	0.00

Hedge Fund(Event Driven) and S&P: start Regime One

Just to summarize. Assume RRA is 4. Let us vary the deduction from the Hedge Fund expected return.

RRA =4

Deduction from HF expected return	Core equity portfolio	Hedge Fund (Event Driven)	Risk free asset
0	0.26	0.74	0.00
3.00 %pa	0.85	0.15	0.00
4.50% pa	1.00	0.00	0.00

Hedge fund allocation is very sensitive to the assumed expected return.

Estimation risk

- Several papers have examined impact of estimation risk
- Garlappi,Uppal and Wang(2007) examine it in a mean variance context
- Assume $\mathbf{x}$ is the vector of portfolio weights
- Assume $\mathbf{\Theta}$ is the set of possible parameter values in the model
- Investor carries out a two step optimization procedure
  1. First minimizes expected utility over $\mathbf{\Theta}$ given $\mathbf{x}$
  2. Then maximizes this over $\mathbf{x}$

Problem

$$\max_{\mathbf{x}} \min_{\mathbf{\Theta}} E\left[u(\ \cdot, \mathbf{x}, \mathbf{\Theta})\ \right]$$

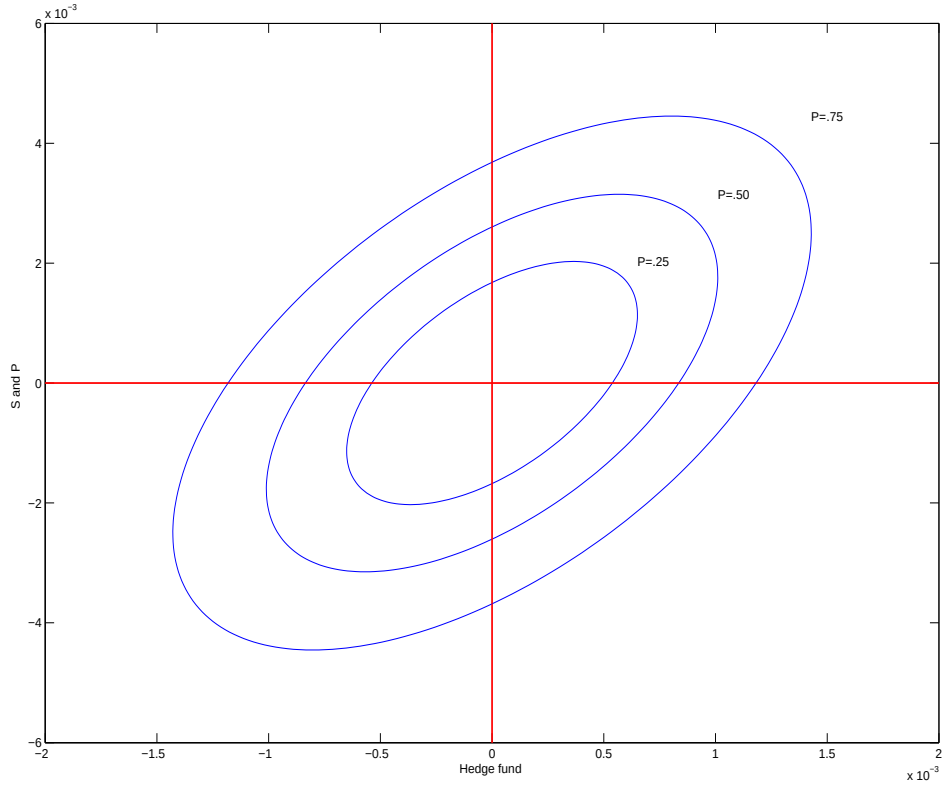


Figure 3: Confidence regions for the expected returns (in regime one) of the two asset classes: the hedge fund and the S&P. The inner ellipse corresponds to $p = .25$, the middle ellipse to $p = .50$ and the outer ellipse to $p = .75$,

Allowing for parameter uncertainty

- Next table shows the impact of parameter uncertainty on the optimal allocations.
- The case $p = 0$ corresponds to no uncertainty.
- The table shows the optimal allocation across the three asset classes assuming we start in regime one.
- We assume that the risk aversion is 4 for all cases reported below
- There is a 3% pa deduction in the hedge fund (Event Driven) returns.

Results with estimation risk

RRA =4

Uncertainty in parameters	Core equity portfolio	Hedge Fund (Event Driven)	Risk free asset
0.00	0.85	0.15	0.00
0.25	0.52	0.48	0.00
0.50	0.43	0.57	0.00
0.75	0.28	0.72	0.00

First line $p = 0$ assumes all parameters known with certainty.

Optimal allocation is very sensitive to estimation risk.

Compare the case with lot of uncertainty (75% to the case with no uncertainty zero): the optimal allocations are reversed.

Sensitivity to other parameters

Univariate case S&P and bonds. Assume RRA =4

Parameter	Sensitivity
μ_1	7.48
μ_2	2.16
σ_1	-.65
σ_2	-.56
p_{12}	-.78
p_{21}	0.18

Summary

- Optimal portfolio selection problem
- Simple model for hedge fund and core equity returns
- Bias and parameter uncertainty

Extensions

- More complex dynamics
- Portfolio rebalancing
- Fuller analysis of estimation risk