

Measuring Operational Risk Interdependencies using Interpretive Structural Modeling

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Abstract

The typical insurance firm is subject to a wide variety of risks. Understanding and quantifying the interrelationships between individual risk elements is a significantly important but complex challenge. If we view all the risks in a firm as an integrated system, we can apply a computer-assisted learning process called Interpretive Structural Modeling (ISM) to construct a structural graph and illustrate those risk interrelationships. In this paper, we use ISM concepts and techniques to better understand an insurance company's overall risk profile. Dephi techniques can be used to "parameterize" this process according to group consensus regarding risk elements and interrelationships. An Analytical Hierarchy Process (AHP) can then be used to quantify relationships and weigh the significance of different risks. Such a modeling approach can be of great value to a firm's enterprise risk management (ERM) process.

Section 1: Introduction

The typical firm is subject to a wide variety of risks. While there are a number of ways to classify and categorize these risks, one approach is to place them into one of the following four groups: hazard, financial, operational, and strategic risks. Each of these four groups would, of course, have numerous sub-categories and factors.

Often, these various risks can reasonably be considered independent. For instance, hurricanes and earthquakes would seem, based on what we know about their respective sciences, to be unrelated natural events. But there appears to be increasing recognition that many risks are in fact interrelated. The movement toward an enterprise risk management framework, for example, acknowledges that the risks and operations of an organization largely interact, and that they should be managed together, in recognition of that fact, and within the context of the overall corporate mission and climate. Thus, understanding the quantifying the complex and extensive interrelationships between individual risk elements is a significantly important challenge.

In this paper, we demonstrate a modeling approach which can help us to elucidate and visualize risk interrelationships. Specifically, we use Interpretive Structural Modeling (ISM) to clarify these relationships. ISM is a method which can be applied to a system – such as a network or a society – to better understand both direct and indirect relationships among the system’s components. We also suggest that the Analytical Hierarchy Process (AHP) can be used to quantify relationships, weigh the significance of different risks, and thus enhance understanding of an organization’s overall risk profile.

The remainder of this paper is organized as follows. In Sections 2 and 3, Interpretive Structural Modeling and the Analytical Hierarchy Process, respectively, are summarized. Sections 4 and 5 provide simple examples of the application and benefits of these procedures, in a risk-based context. Section 6 concludes.

Section 2: Interpretive Structural Modeling

Interpretive Structural Modeling was first proposed by J. Warfield in 1973 to analyze the complex socioeconomic systems. ISM is a computer-assisted learning process that enables individuals or groups to develop a map of the complex relationships between the many elements involved in a complex situation. Its basic idea is to use experts’ practical experience and knowledge to decompose a complicated system into several sub-systems (elements) and construct a multilevel structural model. ISM is often used to provide fundamental understanding of complex situations, as well as to put together a course of action for solving a problem. (For additional detail on the generic ISM process, see, for example, Anantatmula and Kanungo (2005) and Warfield (1976). Generic ISM terminology referred to below can be found in these two, and many other, sources.)

The ISM procedure can be described briefly as encompassing the following steps. The comments below have been specifically tailored to a hypothetical application of ISM to an enterprise risk management modeling project.

- (1) **Organize an ISM implementation group:** To begin, a group of people with relevant knowledge, skills, and backgrounds is assembled. This group should consist of experts from different areas throughout the firm; this wide-ranging skill-set is critical, as ERM should ideally be embedded into the company's operations throughout the firm. A coordinator is established within the group. The coordinator's role is to promote efficient task execution, and to encourage a holistic approach to the project. The coordinator should not only be knowledgeable about the firm's different departments and operations, but also have some power to control the process and make the final decisions.
- (2) **Identify and select the relevant risks:** During this stage, group members work together to document all the risks³ to which the firm is subject.⁴ This can be done, for example, via group brainstorming. However, in firms with an effective and embedded ERM process, this list of risks may already exist as a product and tool of that ongoing ERM process (and was probably, at least in part, originally derived by group brainstorming).⁵
- (3) **Determine the Adjacent Matrix:**⁶ Through the use of the expert group (and possibly via a Delphi approach involving those and/or other experts), the directed relationships among the risk factors are hypothesized. This matrix provides an initial impression of how, in what order, and through which other factors the various risk factors might ultimately be the source of a missed objective. Here, the adjective "directed" refers to the need to specify the direction of the relationship (if any) between any two risk factors – e.g., from A to B, from B to A, in both directions between A and B, or A and B unrelated.
- (4) **Determine the Reachability Matrix:** Based on the adjacent matrix, a binary (elements are 0 or 1) matrix that reflects the directed relationships between the risk factors is created. Basically, the reachability matrix answers the question: yes or no – can we "reach" factor B by starting at factor A, where by "reach" we mean is there a direct or indirect directed relationship from A to B? (In practice, it might sometimes be possible and more convenient to construct the reachability matrix directly simply by using the experts' knowledge and Delphi techniques.)
- (5) **Decompose the risks into different levels:** Here, the reachability matrix is decomposed⁷ to create structural models. This is an algorithm-based process which provides for the grouping of risks into different levels, depending upon their interrelationships. This provides a multilevel interpretive structural model in which the relationships among risks are clarified.

Section 3: Analytical Hierarchy Process

In the 1970's, Dr. Thomas Saaty developed the analytical hierarchy process (AHP) as an effort to reflect the human thought process.⁸ The assumption underlying AHP is that people, by nature, tend to mentally "cluster" things together according to their common

characteristics when addressing a complex decision. AHP formalizes the decision-making algorithm, and allows for consideration of both qualitative and quantitative decision elements. Essentially, AHP involves interpreting the decision process as a series of one-on-one comparisons, and then synthesizing the results, in the process establishing a clear basis upon which the final decision was made.

The steps in the analytical hierarchy process are as follows⁹:

- (1) ***Establish a structural hierarchy***: This can be done in several ways, for example by brainstorming, or, relevant for this paper, by using the previously described interpretive structural model (ISM).
- (2) ***Establish comparative judgments***: After the structural hierarchy is established, priorities among the risk factors are determined. As Atthirawong and MacCarthy (2002) say, “A set of comparison matrices of all elements in a level of the hierarchy with respect to an element of the immediately higher level are constructed so as to prioritize and convert individual comparative judgments into ratio scale measurements. The preferences are quantified by using a” pre-specified scale (in their paper, Atthirawong and MacCarthy use a nine-point scale). This is another point in the overall ISM / AHP process that can be addressed via a Delphi survey of experts. The experts can be asked their opinions regarding the relative importance and strength of interrelationships among risk factors.¹⁰
- (3) ***Synthesize priorities and evaluate consistency***: Based on the prior step, a matrix is produced for each hierarchical level. Then, for each matrix, the eigenvector and maximum eigenvalue are determined (using matrix software, if the system is large and complex enough). The eigenvector represents the relative weights, or importance, of the various risk factors. The maximum eigenvalue can be used to provide an evaluation of the overall consistency of the pair-wise comparison across the entire system.

In the following two sections, simple examples of ISM and AHP risk factor models are provided.

Section 4: Example I – Three Independent Risks

As a first example, suppose a simple situation wherein the “riskiness” (or the cost or pure premium) of a personal auto insurance policyholder is a function of the contribution of three risk factors:

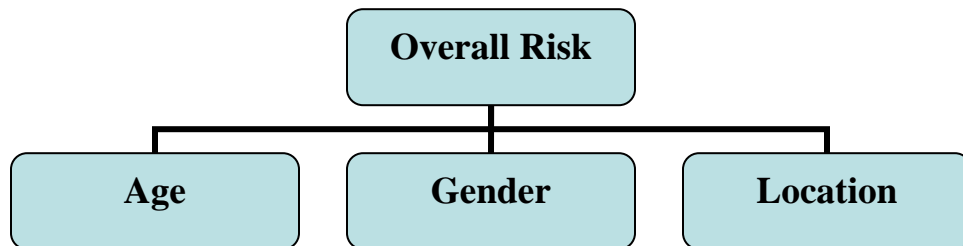
$$\begin{aligned}R_1 &= \text{age} \\R_2 &= \text{gender} \\R_3 &= \text{location}\end{aligned}$$

Here, we will assume that the first four steps of the ISM process described in Section 2 have been completed, and, because a group of fictitious “experts” feels that these three risk factors are all mutually independent, the reachability matrix \mathbf{R} is rather trivial:

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Here, the lower-right 3×3 matrix (rows/columns 2 through 4) is an identity matrix representing the three risk factors R_1 , R_2 , and R_3 . It is an identity sub-matrix because we have assumed independence among the three factors, and there is thus no directed relationship between any of them (so the off-diagonal values are all zero). The first row and column represent the “ultimate parameter” of the system – in this example, the riskiness or cost of the auto insurance policyholder. The first column is all ones because the directed relationship is from the risk factor to the overall system riskiness; the first row is all zeroes because there is assumed to be no relationship in the opposite direction (from the overall riskiness measure to the individual risk factors).¹¹

Based on this simple model structure, the hierarchy of the elements of this system is as follows:



Because of the assumption of risk factor independence and direct relationship between each risk factor and the overall level of risk, this structural model produces only one level of elements below the ultimate value.

Now we proceed to implement the AHP algorithm to this simple situation. Starting with the structural model above, we now use our fictitious expert resources to specify the pairwise comparison matrix for the single hierarchical level of our three risk factors.

Suppose that this matrix C , based on a Delphi survey, is as follows:

$$C = \begin{bmatrix} 1.00 & 2.00 & 4.00 \\ 0.50 & 1.00 & 2.00 \\ 0.25 & 0.50 & 1.00 \end{bmatrix}$$

Each element in this comparison matrix reflects the experts' view as to the relative importance of each pair of risk factors (with respect to their impact on the overall risk value). For example, the value of 2 in the (1,2) spot indicates the opinion that factor R_1 is twice as important as R_2 . The opposite side of this pair-wise comparison coin is the 0.50 value in the (2,1) spot, indicating the opinion (consistent with that in the prior sentence) that R_2 is one-half as important as R_1 .

We now proceed to find the eigenvector and the maximum eigenvalue of this system. Recall that eigenvalues and eigenvectors are related according to the following matrix equation:

$$(C - \lambda I)\bar{v} = 0 \quad \text{or} \quad C\bar{v} = (\lambda I)\bar{v}$$

where C is an $n \times n$ matrix, \bar{v} is an $n \times 1$ column vector, and λI is an $n \times n$ matrix with eigenvalues $\lambda_1, \dots, \lambda_n$ along the diagonal and zeroes off the diagonal. Applying this structure to the comparison matrix C above, we get that the eigenvector

$$\bar{v} = \begin{bmatrix} 0.571 \\ 0.286 \\ 0.143 \end{bmatrix}$$

which reflects the relative weights of the three risk factors.¹² Solving the system also produces the eigenvalues:

$$\lambda_1 = \lambda_2 = \lambda_3 = 3,$$

so that $\lambda_{\max} = 3$. As described in Atthirawong and MacCarthy (2002), this value can now be used to determine a consistency index:

$$CI = \frac{\lambda_{\max} - n}{n - 1}.$$

In our example, $CI = 0$. In theory, for the relative weights (the eigenvector) to be valid, the CI value should be below a certain critical value which depends upon n ; in our example, zero is clearly less than any positive critical value, and so this model would be judged acceptable.

Section 5: Example II – More Complex Risk Interrelationships

Of course, this ISM / AHP approach increases in value the larger and more complex the risk system. As a second example, we maintain the auto insurance policyholder framework, but we increase the complexity of the first example slightly, and add several

risk factors, introducing some indirect factor relationships. Suppose our list of risk factors is doubled in number:

- R₁ = age
- R₂ = gender
- R₃ = location
- R₄ = marital status
- R₅ = distance driven
- R₆ = socioeconomic class

Furthermore, we assume that a Delphi survey indicates the following reachability matrix **R**:

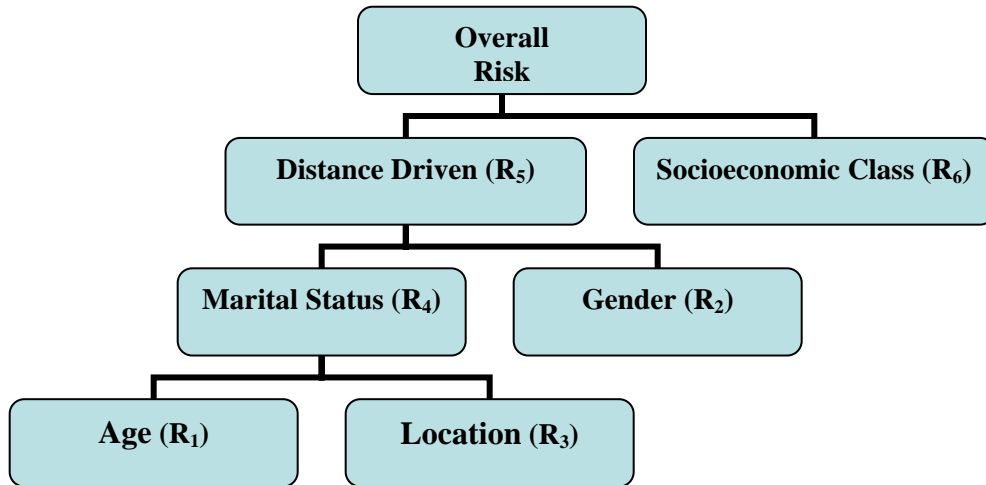
$$R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

where, analogous to Example I, the first row and column refers to the overall risk level, and rows/columns 2 through 7 reflect, in order, the six hypothesized risk factors. For example, the value of one in the (2,5) spot of the reachability matrix above suggests an opinion that age does affect marital status (but not vice versa, as the value in the (5,2) spot is zero). (We emphasize that this matrix, and all the values associated with both the examples in this paper, are for illustrative purposes only; we are not suggesting that these are “correct” or conform to reality.)

Although not completely necessary, it is sometimes helpful to rearrange the rows and columns of the reachability matrix to better see the inherent structure. This can be done by attempting to identify sub-matrices along the diagonal. For example, the above reachability matrix can be rearranged to the following:

$$R = \begin{matrix} & R_7 & R_6 & R_5 & R_4 & R_2 & R_3 & R_1 \\ \begin{matrix} R_7 \\ R_6 \\ R_5 \\ R_4 \\ R_2 \\ R_3 \\ R_1 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} & \end{matrix}$$

where R_7 is the overall risk level. Here, one can see that there are three pairs of risk factors that “go together,” by virtue of the number of direct relationships identified in each row, and by the indicated sub-matrices. Thus, by applying the ISM algorithm, it can be shown that, based on the above reachability matrix, the structural hierarchy of this risk model has the following form:



This depicts visually both the direct and the indirect relationships between the risk factors and the overall risk of the system. Considering these relationships can help an ERM professional understand the impact on the overall system of different risk management techniques applied to one or more individual risk factors.

At this point, the relative importance of each pair of risk factors inhabiting each of the three levels below “Overall Risk” can be determined via the AHP approach. Each of the three levels would be analyzed separately. For example, if for the second level, a Delphi survey of marital status and gender yielded a comparison matrix of

$$C = \begin{bmatrix} 1 & \frac{1}{3} \\ 3 & 1 \end{bmatrix},$$

the resulting eigenvector would be

$$\bar{v} = \begin{bmatrix} 0.25 \\ 0.75 \end{bmatrix},$$

indicating (not surprisingly, given the very simple nature, the small size of the system, and the hypothesized values in the C matrix) relative weights of $\frac{1}{4}$ and $\frac{3}{4}$ for marital status and gender, respectively. This could be repeated for the other two levels.

As with Example I, the underlying purpose of Example II is pedagogical; the calculations are slightly more challenging (although the principles are the same) with a much larger and more complex system. More importantly, the potential value of this ISM / AHP approach increases with the size and complexity of the hypothesized system. Greater clarity and quantitative understanding of the various risk factor interrelationships can be a significant aid to enterprise risk management.

Section 6: Conclusion and Future Research

In this paper, we have described and demonstrated the application of processes, interpretive structural modeling and the analytical hierarchy process, to a firm's risk factors. It is believed that the additional analytical and visual insight into risk factor interrelationships provided by these algorithms can be of substantial benefit to the enterprise risk management process. It is hoped that this paper will stimulate additional interest in this area.

Future research will involve ISM and AHP application to a much larger dataset of causal factors.

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Endnotes

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³ "Identification of risks" is often shown as the initial step of any risk management, ERM, or dynamic financial analysis process. But risk identification must be preceded by determination or acknowledgement of the firm's goals and objectives. Only by knowing the objectives of an organization can one determine the possible sources which might prevent meeting those objectives.

⁴ More generically in ISM parlance, the "risks of the firm" are a specific case of the "elements of the system."

⁵ At this point, ISM theory refers to determining a "contextual relationship" among the elements. In an ERM application, that overall context is clearly the firm-wide interrelationships among risks – risks which might potentially prevent the firm from fulfilling its objectives.

⁶ Also referred to as the "Structural Self Interaction Matrix" in Anantatmula and Kanungo (2005).

⁷ Referred to as "Level Partitioning" in Anantatmula and Kanungo (2005).

⁸ See Saaty (1980).

⁹ See Atthirawong and MacCarthy (2002).

¹⁰ Note that there is a difference between the objectives of the Delphi studies in the ISM and AHP procedures. In the former, experts are being asked whether there is a directed relationship between risk factors; in the latter, after structuring them hierarchically, the experts are being asked to rank the strength of the interrelationships according to a pre-determined scale.

¹¹ Mathematically, the $(i,j)^{\text{th}}$ element of the matrix is 1 if there is a directed relationship (either direct or indirect) from element i to element j , and 0 if there is no directed relationship from i to j .

¹² In this very simple framework, calculating the elements of the eigenvector is easy: take the geometric average of the values in each row of C – e.g., for row 1, $(1 \text{ times } 2 \text{ times } 4)$ to the one-third power – and divide each by the sum of those geometric averages across all three rows.