Alpha and Risk-Adjusted Correlation for Economic Capital Calculations

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Abstract: In calculating a firm's overall capital need and contributions by its various business lines, the resulting numbers depend heavily upon key risk assumptions and parameters, including projected profit margin, downside volatility and correlations. This paper makes a clear distinction between *historical* (statistical) correlation in insurance losses versus *risk-adjusted* correlation in prices. Firstly, the paper demonstrates that uncertainty in the market price of risk introduces risk-adjusted price correlation. Secondly, the paper uses risk-adjusted correlation to reflect sampling errors in the estimation of correlation parameters. Finally, this paper defines alpha as discounted present value of excess profit margins in future years, as well as how to explicitly recognize alpha in calculating economic capitals.

1. INTRODUCTION

Economic capital calculations are at the heart of enterprise-wide risk management of financial institutions. It is well-known that assumed correlation parameters can have huge impacts on firm-wide capital requirement and allocated economic capitals to its business lines. Despite the high importance of such risk parameters, there are alarmingly varying industry practices regarding correlation assumptions. Consensus on appropriate values of correlation parameters remains to be elusive. To tackle this issue, this paper makes a clear distinction between *historical* versus *risk-adjusted* correlations. It presents analytical methods for adjusting correlation parameters to be used for risk aggregation and capital

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allocation. This conceptual distinction may help bridge currently diverging industry practices.

In evaluating risk projects at a given time of specific market environment, the alpha associated with a risk project refers to expected excess profit margin over some target benchmark, evaluated over a specified time horizon (say, 1 to 5 years). Treynor and Black (1973) were pioneers in explicitly reflecting "alpha" in a one-period problem of optimal asset allocation. Following Treynor and Black's earlier work, Miller (1999) advocates using "alpha" in allocating enterprise risk capitals.

For insurance liabilities, there is no actively-traded market from which one can derive "fair prices". Most pricing and reserving valuations are performed by marking-to-model. Market conditions (sentiments) at the time can have a huge impact on the pricing and reserving valuations. In the property-casualty insurance industry, large deviations from fair prices can persist for an extended time period, with alternating periods of soft markets and hard markets. The period of soft market is characterized by lower pricing, relaxed underwriting standards, more generous coverage provisions; The period of hard markets is characterized by increased prices, tightened underwriting standards, and narrowed coverage. This relatively long duration of each soft or hard market makes it essential to reflect such cyclical market behavior in the economic capital calculations.

2. REVIEW OF INDUSTRY PRACTICES

2.1. Factor-based Regulatory Capital Requirements

Banking and insurance regulations often prescribe capital charges for each type of risk. For illustration we consider three types of risks $\{X_1, X_2, X_3\}$. Let RC_j represent the required capital for risk X_j, j= 1, 2, 3.

In the banking Basel Accords, the *factor-based* capital charges are *additive* across risks:

$$RC_{Total} = RC_1 + RC_2 + RC_3 .$$

In drastic contrast to banking Basel Accords, the U.S. Insurance NAIC RBC (risk-based capital) explicitly reflects diversification benefit among various types of risks, and calculates total required capital according to a square-root rule:

$$RC_{Total} = \sqrt{RC_1^2 + RC_2^2 + RC_3^2}\,.$$

The different treatments of diversification benefit between the banking Basel Accords and insurance NAIC RBC lie in their implicit correlation assumptions: The assumed correlation parameter under Basel Accords is one (or perfect correlation), whereas the assumed correlation under the insurance NAIC RBC is zero.

Interestingly, such discrepancies also exist among major rating agencies' approaches. The S&P *factor-based* capital charges are *additive* across risks; while the AM Best BCAR model has a square-root component as in the NAIC RBC.

2.2. Company Internal Model Approach

Company internal models often represent more advanced approaches than the simple factor-based capital framework. The internal model approach can be described in three steps:

Step 1: Individual probability distributions for various types of risks (various business units, or lines of business) are quantified. By assuming a correlation matrix, these individual risk distributions are combined together to derive an aggregate loss distribution for the company over a specified time period.

Step 2: From the aggregate loss distribution one can compute the total company capital requirement at a prescribed security level, e.g., with 99% probability that the company will stay solvent over the next time period:

$$RC_{Total} = \text{The } 99^{\text{th}} \text{ percentile} - \text{Expected Loss.}$$

Remark: supposedly the expected loss shall be covered by revenues (premium or fees).

Step 3: The total required capital is then being allocated to various business units for making risk-based decisions (business planning, product pricing, risk-based performance measure, etc). See Wang (2002), Ward and Lee (2002).

Both the banking Basel II and EU Insurance Solvency II encourage companies to develop their own internal risk capital models, as a more advanced approach than the prescribed factor-based rules. Banking Basel II states that there might be incentives for companies to build internal models as they are expected to yield a lower required capital than the factor-based capital requirement. Interestingly, this implicitly admits that although the factor-based capital requirement is additive across risks, there might be room for recognizing risk diversification in a more sophisticated internal risk model. In contrast, the same cannot be said about insurance RBC. Although Insurance RBC is factor-based, the prevailing practice of using the square-root rule has already given too much credit for risk diversification. It is doubtful that much more diversification can be derived from company internal risk models than already included in the square-root rule.

2.3. Capital Allocation

It is well known that correlation parameters can have an important impact on capital allocation results. In the past few years many insurance companies have launched capital allocation projects. Using low historical correlation, the capital allocation results often show large diversification benefits being credited to some lines of business, which have created much controversy, debate, and confusion among business managers.

Some researchers have advocated various alternatives to using a percentile of the aggregate loss distribution for setting capital requirement and capital allocation. See Mildenhall (2003); Venter (2003); Vrieze, and Brehm (2003). A popular risk measure is the Conditional Tail Expectations (CTE); see Artzner et al (1999). Within the multivariate normal framework, Panjer (2002) showed that using CTE or percentile will lead to the same allocation result: the allocated capital to risk X_i is proportional to its relative contribution to the total variance:

(2.1) $$\frac{RC_i}{RC_{Total}} = \frac{\sigma_i \sum_{j=1}^{n} \rho_{ij}\sigma_j}{\sum_{k,j=1}^{n} \rho_{kj}\sigma_k\sigma_j}.$$

In this paper, our discussion of correlation parameters will be within the multivariate normal framework (although it allows for straightforward extensions to normal copulas). As a result, using either percentile or CTE would not make any material difference for capital allocation discussions.

3. HISTORICAL VERSUS RISK-ADJUSTED CORRELATION

In this paper, I propose to make a clear distinction between *historical* (observed) versus *risk-adjusted* correlation. To help articulate this conceptual distinction, here I draw an analogy in bond default risk probabilities.

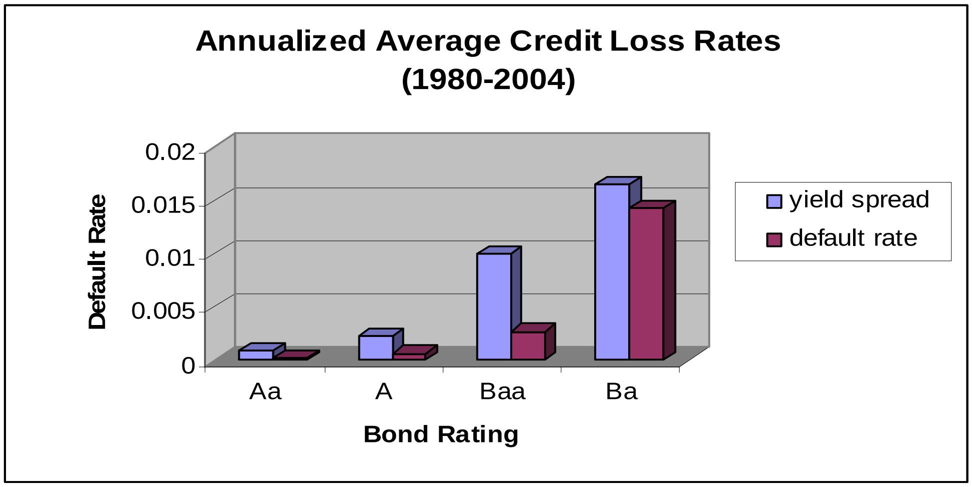
Historical versus Implied Default Probabilities

In modeling corporate bond defaults, it is widely known that historical default probability can differ dramatically from the implied default probabilities as implied from bond yield spreads. As shown in Table 3.1 and Figure 3.1, the default probabilities implied by yield-spreads are much higher than historical default probabilities. Their relative differences are most striking for high-grade bonds (with low expected default frequencies). Such differences represent the market's explicit adjustments for risk, uncertainty and illiquidity.

Table 3.1. Annualized Average Credit Loss Rates for 5-year bond (1980-2004) v.s. Bloomberg zero coupon spreads to Aaa, (ref: Berndt et al, 2005).

rating	yield spread	default rate
Aa	0.0009	0.0002
A	0.0022	0.0006
Baa	0.0100	0.0027
Ba	0.0166	0.0143

Figure 3.1 Annualized Average Credit Loss Rates for 5-year bond (1980-2004) v.s. Bloomberg zero coupon spreads to Aaa. (ref: Berndt et al, 2005)



Setting a capital charge for a risk is analogous to assigning a risk-adjusted price for the risk. In the same way that implied default probabilities differ from historical default probabilities, conceptually, we should expect to use *risk-adjusted correlation* (rather than *historical correlation*) in allocating economic capitals.

For insurance companies, a straightforward statistical analysis of insurance losses may indicate very low correlation among different lines of business. This was the basis for the square-root rule in the insurance RBC approach. However, upon a deeper analysis of the nature of insurance business, for capital allocation purposes we have reasons for using higher risk-adjusted correlations than historical (statistical) correlation. For capital allocation purposes, to reflect the inherent risks and illiquidity of insurance contracts, the large transaction costs associated with maintaining or running-off the insurance operations, we should use risk-adjusted correlations that are higher than historical correlations.

4. RISK-ADJUSTED PRICE CORRELATION

Consider an insruance risk *X* and a reference portfolio *Z*. Here *X* can represent the losses from a product or business line; Z represents losses from a reference portfolio (the company portfolio, or the industry portfolio).

Firstly, with the passage of time, k =0, 1, 2, 3, …, once new information becomes available, we update our probabilistic estimates for the loss variables *X* and *Z*:

(4.1) $$\hat{\mu}_X(k) = \mu_X^0(k) + \varepsilon_X(k), \quad \text{and} \quad \hat{\mu}_Z(k) = \mu_Z^0(k) + \varepsilon_Z(k).$$

Here $\mu_X^0(k)$ represents the true mean value of the statistical distribution of the prospective loss variable *X,* and $\varepsilon_X(k)$ represents the estimation error contained in the estimated mean loss amount $\hat{\mu}_X(k)$. For the estimation error terms, we denote

$$\sigma_X^2(k) = Var(\varepsilon_X(k)),$$

$$\sigma_Z^2(k) = Var(\varepsilon_Z(k)) \text{ , and}$$

$$\rho_{X,Z}(k) = Corr\langle \varepsilon_X(k), \quad \varepsilon_Z(k)\rangle.$$

Next, we employ pricing methods to derive fair risk-adjusted prices for *X* and *Z*.

Assume that the prices for *X* and *Z* are obtained through the following equations:

(4.2) $$h_X(k) = \mu_X^0(k) + \varepsilon_X(k) + \lambda(k)\cdot\sigma_X(k), \quad \text{and}$$

$$h_Z(k) = \mu_Z^0(k) + \varepsilon_Z(k) + \lambda(k)\cdot\sigma_Z(k).$$

The quantity $\lambda(k)$ is called the market price of risk, which may well change over time as market condition changes. At any specific time, the same market price of risk is used to derive the prices of *X* and *Z* simultaneously. At time t = k, the uncertainty in the market price of risk is measured in terms of variance of λ:

$$\sigma_\lambda^2(k) = VaR(\lambda(k)).$$

In property-casualty insurance, we can estimate σ_λ over a time period that spans over at least one underwriting/pricing cycle.

For the sake of simplicity, we assume zero correlation between the estimation error $\varepsilon_Z(k)$, and the market price of risk, $\lambda(k)$, that is

(4.3) $$Cov\langle \varepsilon_X(k), \quad \lambda(k)\rangle = 0, \quad \text{and} \quad Cov\langle \varepsilon_Z(k), \quad \lambda(k)\rangle = 0.$$

Later on we can relax this constraint especially in discussing natural catastrophic insurance.

We can now derive the correlations between their risk-adjusted prices:

$$VaR\langle h_X(k)\rangle = \sigma_X^2(k)\cdot\left(1+\sigma_\lambda^2(k)\right)$$

$$VaR\langle h_Z(k)\rangle = \sigma_Z^2(k)\cdot\left(1+\sigma_\lambda^2(k)\right)$$

$$Cov\langle h_X(k), \quad h_Z(k)\rangle = \sigma_X(k)\cdot\sigma_Z(k)\cdot\left(\rho_{X,Z}(k)+\sigma_\lambda^2(k)\right)$$

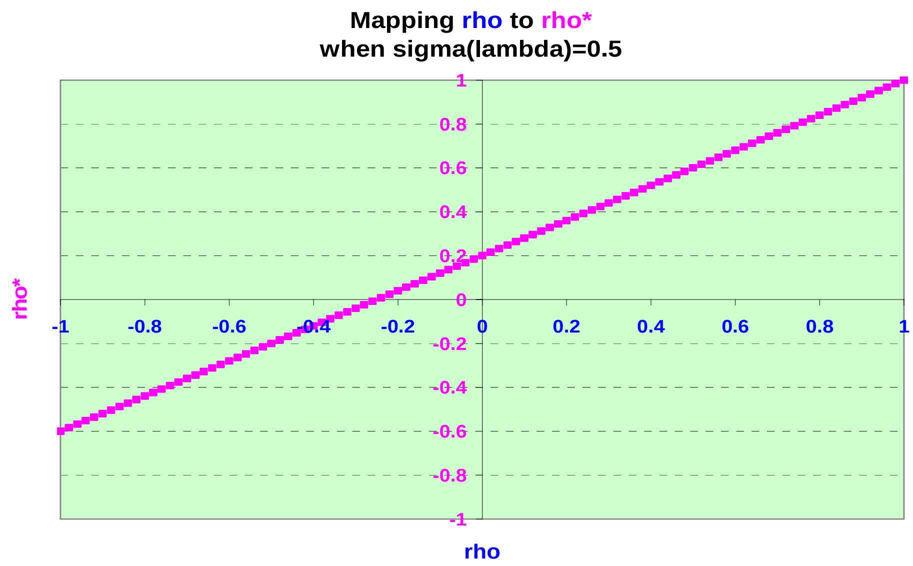
Thus we obtain the following correlation between the "prices":

(4.4) $$\rho^*_{X,Z}(k) = Corr\langle h_X(k), \quad h_Z(k)\rangle = \frac{\rho_{X,Z}(k)+\sigma_\lambda^2(k)}{1+\sigma_\lambda^2(k)}.$$

One can verify that $\rho^*_{X,Z}(k) \ge \rho_{X,Z}(k)$. In other words, uncertainty regarding the market price of risk induces a higher correlation in prices than the loss correlation. Note that

1. when $\rho_{X,Z}(k)=0$ we have $\rho^*_{X,Z}(k) = \dfrac{\sigma_\lambda^2(k)}{1+\sigma_\lambda^2(k)} > 0$. Even though business lines may appear to have low statistical correlation in terms of losses, uncertainty in the market price of risk may induce higher positive correlation in the "risk-adjusted" prices.
2. when $\rho_{X,Z}(k)=1$ we have $\rho^*_{X,Z}(k)=1$
3. when $\rho_{X,Z}(k)=-1$ we have $\rho^*_{X,Z}(k) = \dfrac{\sigma_\lambda^2(k)-1}{\sigma_\lambda^2(k)+1} > -1$.

Figure 4.1 Uncertainty in "Market Price of Risk" and "Adjusted Correlation"



5. CONFIDENCE INTERVALS FOR CORRELATION PARAMETERS

Given a sample of "n" pairs of observations: $\{(x_j, \quad y_j), \quad j=1,\Lambda\ ,n\}$, we can calculate the sample correlation coefficient:

$$(5.1) \qquad r_{xy} = \frac{\sum_{j=1}^{n}(x_j-\bar{x})(y_j-\bar{y})}{(n-1)\cdot s_x \cdot s_y} = \frac{n\cdot\sum_{j=1}^{n}x_j y_j - \left(\sum_{j=1}^{n}x_j\right)\left(\sum_{j=1}^{n}y_j\right)}{\sqrt{n\sum_{j=1}^{n}x_j^2-\left(\sum_{j=1}^{n}x_j\right)^2}\cdot\sqrt{n\sum_{j=1}^{n}y_j^2-\left(\sum_{j=1}^{n}y_j\right)^2}},$$

where $\bar{x}$ and $\bar{y}$ are sample means of s_x and s_y are sample standard deviations.

In statistics, hypotheses about the value of the population correlation coefficient ρ between variables X and Y of the underlying population, can be tested using the Fisher transformation applied to the sample correlation r.

$$(5.2) \qquad z_{xy} = FISHER(r_{xy}) = \frac{1}{2}\ln\frac{1+r_{xy}}{1-r_{xy}},$$

which is called Fisher transformation.

If (X, Y) has a bivariate normal distribution with correlation coefficient ρ, then the Fisher transform (5.2) is approximately normally distributed with mean

(5.3) $$\frac{1}{2}\ln\frac{1+\rho}{1-\rho}$$

and standard deviation $1/\sqrt{n-3}$. This property forms the basis for a common way of constructing a confidence interval for ρ. For the Pearson correlation coefficient ρ, the 100α% confidence interval is

(5.4) $$\left\langle FISHER^{-1}\left(z_{xy}-\frac{z_{1-\alpha/2}}{\sqrt{n-3}}\right),\quad FISHER^{-1}\left(z_{xy}+\frac{z_{1-\alpha/2}}{\sqrt{n-3}}\right)\right\rangle,$$

where

- z_{ij} is the Fisher z-transform of the correlation coefficient:
- $z_{1-\alpha/2}=\Phi^{-1}\left(1-\frac{\alpha}{2}\right)$ is the 100(1-α/2) percentile of the standard normal variable;
- The inverse Fisher transform is:

(5.5) $$FISHER^{-1}(x)=\tanh(z)=\frac{\sinh(x)}{\cosh(x)}=\frac{e^{z}-e^{-z}}{e^{z}+e^{-z}}=\frac{e^{2z}-1}{e^{2z}+1}.$$

Table 5.1. For a sample size of 15, the 95% confidence interval for estimated historical correlation

Corr	**Lower Corr***	**Upper Corr***
(1.00)	(1.00)	(1.00)
(0.80)	(0.93)	(0.49)
(0.60)	(0.85)	(0.13)
(0.40)	(0.76)	0.14
(0.20)	(0.65)	0.35
0.00	(0.51)	**0.51**
0.20	(0.35)	0.65
0.40	(0.14)	0.76
0.60	0.13	**0.85**
0.80	0.49	0.93
1.00	1.00	1.00

Interestingly, even if the sample correlation coefficient is zero, the 95% confidence interval upper limit still indicates a strong "0.51"! The large range of feasible "ρ" cautions us not to put too much confidence in historical correlation parameters, and even calls for an explicit adjustment of the historical correlation.

Based on the Fisher transform for constructing confidence intervals for the correlation parameter, we give the following mathematical formula for making an explicit adjustment in the correlation parameter for sampling errors:

(5.6) $$\rho^* = \tanh\left(\frac{1}{2}\ln\frac{1+\rho}{1-\rho} + \frac{1}{\sqrt{n-3}}\cdot\Phi^{-1}\left(1-\frac{\alpha}{2}\right)\right) \text{ or}$$

$$\rho^* = FISHER^{-1}\left(FISHER(\rho) + \frac{1}{\sqrt{n-3}}\cdot\Phi^{-1}\left(1-\frac{\alpha}{2}\right)\right).$$

The ρ* in (5.6) adjusts for estimation error in the correlation coefficient. This adjustment is especially important when the correlation parameter is estimated from small sample data, as it is the case in property-casualty insurance whereas we have mostly 10 to 20 years of historical observations.

Figure 5.1

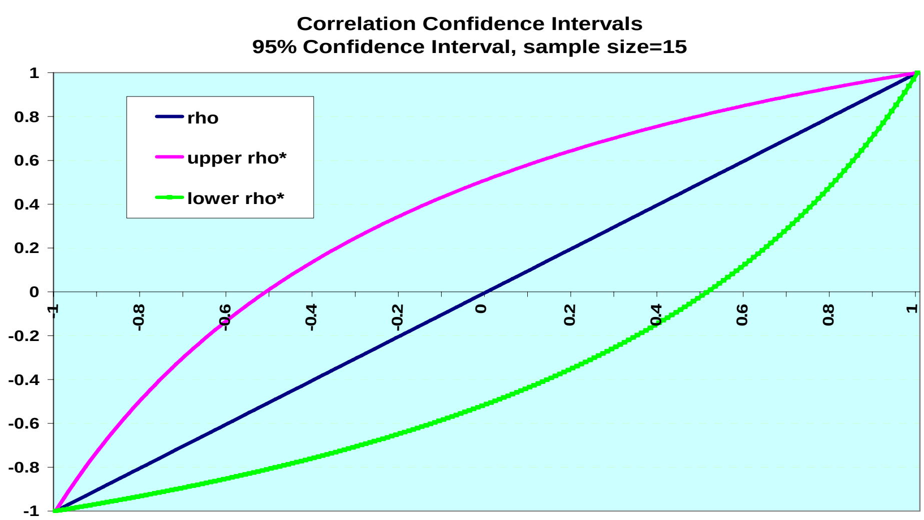
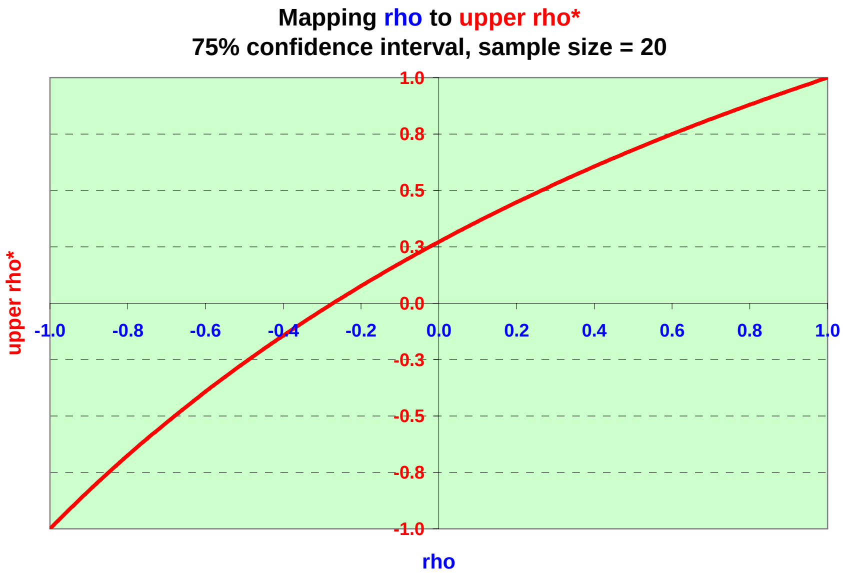


Figure 5.2



Remark 5.1. When more than two risks are involved, one should check for the positive definite property after adjusting the correlation parameters.

$$\begin{pmatrix} 1 & \rho_{12} & \rho_{13} \\ \rho_{12} & 1 & \rho_{23} \\ \rho_{13} & \rho_{23} & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & \rho_{12}^* & \rho_{13}^* \\ \rho_{12}^* & 1 & \rho_{23}^* \\ \rho_{13}^* & \rho_{23}^* & 1 \end{pmatrix}$$

6. IMPLICATIONS IN CAPITAL ALLOCATION

Recall that within the multivariate normal framework, the capital allocation to risk X_i is a covariance-based allocation; see equation (2.1). Using risk-adjusted correlation parameters, the covariance allocation method yields a different allocation:

$$\frac{RC_i^*}{RC_{Total}^*} = \frac{\sigma_i \sum_{j=1}^{n} \rho_{ij}^* \sigma_j}{\sum_{k,j=1}^{n} \rho_{kj}^* \sigma_k \sigma_j}.$$

Example 6.1 Consider three risks X_1, X_2 and X_3 with

$(\sigma_1, \quad \sigma_2, \quad \sigma_3) = (1, \quad 1, \quad 8)$, and $\begin{pmatrix} 1 & \rho_{12} & \rho_{13} \\ \rho_{12} & 1 & \rho_{23} \\ \rho_{13} & \rho_{23} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

(1) Using different volatilities for the market price of risk, we get different risk adjusted correlations, which in turn yield different capital allocation results:

Attribution	$\sigma(\lambda)$=0	$\sigma(\lambda)$=0.5	$\sigma(\lambda)$=1
to X_1	1.5%	3.8%	6.6%
to X_2	1.5%	3.8%	6.6%
to X_3	97.0%	92.4%	86.7%

Note that $\sigma(\lambda)$=0 corresponds to the case of using historical correlation without adjustment; the proportions of allocated capitals to risks X_1 and X_2 are very small. As the volatility of λ increases, the proportions of allocated capital to risks X_1 and X_2 also increase.

(2) Suppose that the correlation parameters are estimated from 20 observations, and use the upper bound of the 75% confidence interval for the estimated correlation parameters, we get the following allocated capitals:

Attribution	Unadjusted Correlation	Upper bound of 75% confidence interval
to X_1	1.5%	4.6%
to X_2	1.5%	4.6%
to X_3	97.0%	90.8%

7. PROJECTING ALPHA IN UNDERWRITING/PRICING CYCLE

Historically, the market price of risk over time exhibited cyclical behavior, rather than a random pattern. We use a simplistic model for the cyclical profit margins as

(7.1) $$PM_j = A \cdot \sin\left(\frac{j}{2b\pi}\right), \; j = 0,1,2,\ldots$$

where "b" is the length of a complete underwriting cycle, and "A" is the amplitude of the cycle. We can calibrate the parameter "A" so that the underwriting year excess profits exhibit a volatility that matches the observed underwriting year loss ratio volatility.

Discounted Cash Flow Formula for Calculating "Alpha"

Recall that risky assets can be valued according to the Discounted Cash Flow equation:

$$DPV = \sum_{j=1}^{N} \frac{FV_j}{(1+R)^j},$$

where

- FV_j is the nominal value of cash flow amount at the end of future period $(j-1, j]$;
- R is the interest rate for discounting

Applying the Discounted Cash Flow valuation approach, we define the prospective "alpha" as "Discounted Excess Profit Margin":

(7.2) $$\alpha = \sum_{j=1}^{N} \frac{EPM_j}{(1+R)^j}$$

where

- "N" is the maximum number of future years to be considered (for instance "N" can be between 2 and 5 years).
- EPM_j is the projected profit margin at the end of future period $(j-1, j]$;
- R is the interest rate for discounting, which should reflect the level of uncertainty in the projected profit margins.

Fair Profit Margin Implies A Benchmark Amount of Economic Capital

For risk *X* and reference portfolio *Z*. Assume that the market prize of risk for the reference portfolio Z is λ_Z^0. Based on CAPM-type of approach, we can derive a market price risk for risk *X* as:

$$\lambda_X^0 = \lambda_M^0 \cdot \rho_X^* .$$

This reinforces the importance of the risk-adjusted correlation parameter ρ_X^*. Base on previous discussions, there are multiple justifications for using a value of ρ_X^* that is significantly greater than zero (or even close to one).

The fair profit margin for risk X is then $\lambda_X^0 \cdot \sigma$, which in turn implies an amount of economic capital for taking risk *X* :

(7.4) $$EC_X(k) = \frac{\lambda_X^0 \cdot \sigma_X(k)}{TEROE},$$

where

- *TEROE* is a target excess rate of return (over the risk free rate). For instance, we may assume that TEROR = 10%.
- λ_X^0 is the long-term target average market price of risk for the given line of business, which has already reflected the risk-adjusted correlation with reference portfolios. For instance, we may assume that $\lambda_X^0 = 0.3$.

The economic capital assigned to risk *X*, when adjusted for "alpha", should be

(7.5) $$EC_X(k) = \frac{\lambda_X^0 \cdot \sigma_X(k)}{TEROE} - \alpha .$$

Example 7.1: Assume that Underwriting Year loss ratio volatility for commercial auto liability is σ=6%; after adjusting for multi-year developments the annualized volatility is 4.2%. We also assume a simple cyclical model for the excess profit margin as

$$EPM_j = A \cdot \sin\left(\frac{j}{2b\pi}\right),\ j = 0, 1, 2, \ldots$$

where "*b*=10 years" is the length of a complete underwriting cycle. We calibrate the parameter "*A*=0.0855" so that the underwriting year profit margins exhibit a volatility of σ=6%.

We then use a Discounted Cash Flow formula to calculate alpha value in year *k*:

$$\alpha(k) = \sum_{j=1}^{5} \left(\frac{1}{1+R} \right)^{j} \cdot EPM_{k+j}, \quad \text{for } k = 0, 1, 2, \ldots$$

Assume that β=1, *TEROR* = 10%, *R*=100%, and λ_X^0 = 0.3, we get the following:

$$EC = \left[\frac{0.3}{0.1} \right] \times 4.2\% = 0.126 .$$

Before reflecting alpha value, the "Economic Capital Factor" is 0.126 (to be applied to the expected loss portion of the premium).

After adjusting for alpha, the 'Economic Capital Factors" would depend on the current phase of the underwriting cycle:

$$EC(k) = \left[\frac{0.3}{0.1} \right] \times 4.2\% - \alpha(k) = 0.126 - \alpha(k)$$

time k	α (k)	EC(k)
0	0.029	0.097
1	0.059	0.067
2	0.066	0.060
3	0.048	0.078
4	0.011	0.115
5	(0.029)	0.155
6	(0.059)	0.185
7	(0.066)	0.192
8	(0.048)	0.174
9	(0.011)	0.137
10	0.029	0.097

Figure 7.1

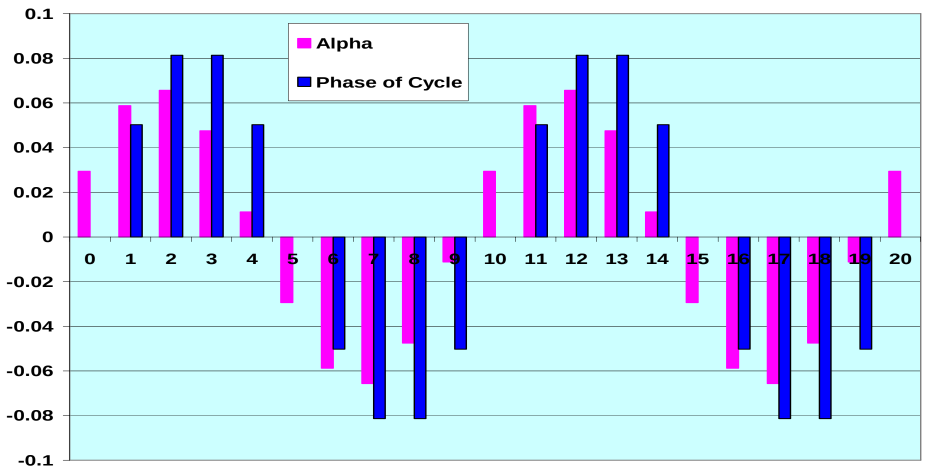
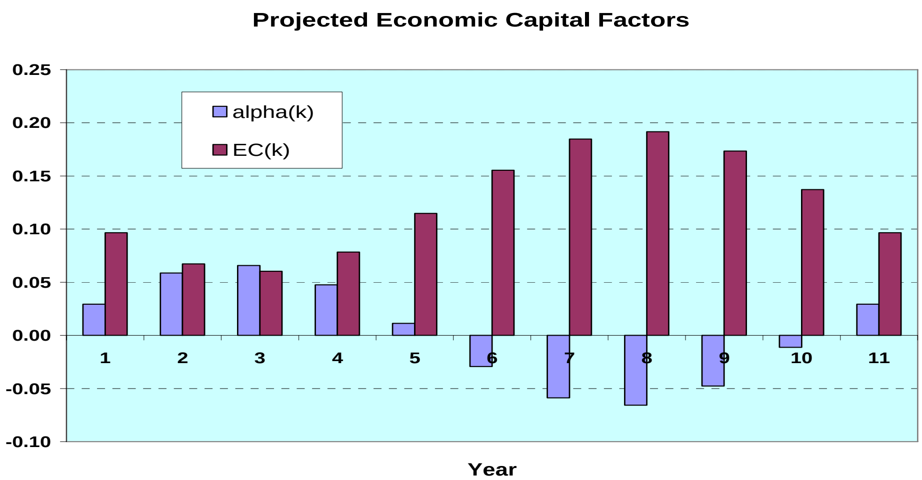


Figure 7.2



Remark 7.1: In real life, year over year changes in profit margin do not completely follow a random generator, but rather like a slow motion display of the roller coast of cycle swings. This characteristic makes it plausible to estimate "alpha" in a timely manner. This highlights the importance of explicitly incorporating "alpha" in the economic capital calculations for property-casualty insurers.

8. A REAL-LIFE CASE OF NATURAL CATASTROPHE INSURANCE

Natural catastrophe losses, by definition, are low frequency and high severity events. A catastrophic loss such as Hurricane Katrina in year 2005 can simultaneous impact the estimated mean loss and the market price of risk in subsequent years.

Recall that

$$h_X(k) = \mu_X^0(k) + \lambda(k) \cdot \sigma_X(k) + \varepsilon_X(k).$$

Immediately following a major catastrophic loss in year *k*, both the revised estimate of the mean $\hat{\mu}_X(k+1)$, and the market price of risk, $\lambda(k+1)$, would jump upwardly simultaneously:

$$Cov\langle \varepsilon_X(k+1), \quad \lambda(k+1) \rangle > 0 \text{ and } Cov\langle \varepsilon_Z(k+1), \quad \lambda(k+1) \rangle > 0.$$

This is exactly the time that there is significantly positive "alpha" in the increased insurance prices. Explicit recognition of "alpha" would offset (to some extent) the increased capital requirement for insurers continue to provide catastrophic insurance coverage.

Here we have a real-life drama unfold post the 2005 Hurricane Katrina, which marked the largest ever natural catastrophic losses in the U.S. history (up to the time of writing). As aftermath of Hurricane Katrina, the insurance industry experienced a major capacity crunch, and insurance prices in Gulf coast areas increased significantly for reduced limit coverage. To make things worse, Risk Management Solutions (a major commercial catastrophe modeling firm) released a revised CAT Model version 6.0 which significantly raised Possible Maximum Loss (PML) estimates. Borrowing a description by Don Mango, "the increased PMLs had ripple effects, from more stringent rating agency stress tests to put additional pressure on reinsurer capacity constraints, to cedents looking to buy more limit, and to prices going up." A discontinuous market shock has had detrimental effects across the insurance industry and adversely affected the general public and business community in need of catastrophe insurance coverage. If rating agencies gave due consideration of "positive alpha" contained in the 2006 high-flying insurance prices, it would have alleviated some of the "capacity crunch" and would have helped dampen the severity of price jumps.

9. AREAS FOR FURTHER EXTENSIONS AND RESEARCH

In this paper we make a clear distinction between historical correlation and risk-adjusted correlation. Conceptually this is analogous to the distinction between historical default probability versus implied default probability. We demonstrated that uncertainty in the market price of risk have an effect of increasing correlation in the prices. We also discussed the sampling errors involved in the estimation of correlation parameters and construction of confidence intervals for the estimated correlation parameter. This paper presents analytical methods for transforming historical correlation to risk-adjusted correlation. The insurance RBC square-root rule might have given too much diversification benefits. In contrast, the banking Basel II additive capital charges do not allow for diversification benefits. By using risk-adjusted correlations, we can find a middle ground that better reflects the true risk contributions from individual risks.

This paper advocates explicit recognition of "alpha" in economic capital calculations. The benefits of making explicit adjustment of alpha include dampening the amplitude of underwriting cycle. The effectiveness of using "alpha" depends upon how timely and accurately we can measure the alpha in the lines of business.

One area of further research is to investigate the implications of using risk-adjusted correlation when using other capital allocation methods, such as those discussed in Phillips, Cummins and Allen (1998), Meyers & Read (2001), Sherris (2004), and others.

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