A Conditional Lower Partial Moments-framework for Corporate Risk Measurement

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Abstract

The search for a unifying measure of corporate risk has so far proven elusive. In this paper I introduce a Conditional Lower Partial Moments (CLPM) – framework in which risk measures are derived by applying a penalty to liquidity shortfalls that are not covered within the firm's remaining debt capacity. The intuition underlying the CLPM-framework is that *combinations* of low liquidity and debt capacity characterize situations where the firm's investment and operating activities may be disrupted. The CLPM (a_1, t_1, t_2)-framework is a generalization of Fishburns LPM (a, t)-framework (1977) to include the debt capacity-parameter t_2. Measures of risk within this framework provide additional insights compared to currently popular liquidity-based risk measures such as Cash-Flow-at-Risk (CFaR). CLPM-measures have the attractive feature that they are sensitive not only to market conditions and exposures, but also to pro-active changes in the firm's financial and hedging policies as well as its strategic risk taking (such as capex and acquisition levels).

KEY WORDS: Risk Measurement, Liquidity, Debt Capacity, Shortfall Risk

Corporate level risk management is sometimes depicted as the task of managing the firm's capital structure and liquidity in a way that ensures that the firm is continuously able to execute all its strategic initiatives in a timely manner (see, for example, the hedging framework in Froot, Scharf and Scharfstein, 1993, 1994). In this paper I address the question if we can formulate a risk measurement framework to accompany this risk management task.

The framework starts with the very intuitive observation that the larger the firm's cash holdings, and the lower its Debt-to-Equity (or some other proxy for the firm's capital market access), the larger the firm's capacity to pursue investment opportunities and withstand cash flow volatility. Several authors have emphasized that information on the firm's debt capacity is a key input to corporate level liquidity risk management (see, for example, Mello and Parsons, 1999). Conditioning liquidity risk on the firm's debt capacity is based on the intuitive notion that *combinations* of low liquidity and solvency characterize situations where the firm's investment and operating activities may be disrupted. While these insights are hardly new, a formal risk measurement framework that weighs together these elements into a summary measure of risk has been lacking.

What this suggests is that currently popular liquidity-based risk measures, such as Cash Flow-at-Risk (CFaR), need to be qualified with a reference to the firm's debt capacity. To overcome the inherent limitations of an at-Risk type of measure, this paper draws heavily on the Lower Partial Moments (LPM)-framework (Bawa, 1977, Fishburn, 1977), in which the risk of a portfolio of financial assets is described in terms of the probability and magnitude of shortfalls relative some benchmark level of performance. The basic idea of the LPM is that a penalty is applied to shortfalls to reflect the risk tolerance of the decision-makers and the perceived costs of the shortfall. By adjusting these parameters a great deal of flexibility, which is lacking in the case of an at-Risk measure, is achieved in terms of describing the risk preferences of the decision-maker.

The LPM-framework, however, has not developed to include a reference to debt capacity. For the managers of a corporation, with its largely fixed portfolio of fairly illiquid assets, the state of the balance sheet is an important piece of information as it will indicate the firm's ability to withstand short- to medium term fluctuations in liquidity to which the successful execution of strategy may be intimately tied. Taking the corporate perspective, this paper suggests a Conditional Lower Partial Moments (CLPM)-framework for measurement of corporate level risk. The CLPM (a_1, t_1, t_2)-framework is a generalization of Fishburn's LPM (a, t)-framework (1977) to include the debt capacity-parameter t_2. The debt capacity-parameter concerns the identification and description of a liability overhang in the firm's balance sheet that could lead the firm to conduct its business sub-optimally in some states of the world. It amounts to defining corporate risk in terms of disruptive consequences that arise in situations where the firm's cash flow generation and balance sheet strength are, on a combined basis, insufficient to maintain its desired level of cash disbursements. If a firm is unable to attract new financing at non-prohibitive rates, it implies underinvestment or the scaling back of some other cash disbursement and hence suboptimal execution of strategy. Conceptually, the more severe the consequences of such uncovered liquidity shortfalls, the more they should be penalized ex-ante to reflect this cost. This paper discusses three risk measures based on different levels of the firm's risk tolerance.

The CLPM seeks to express risk in terms of interactions between important corporate "bottom lines" such as the cash position and balance sheet ratios closely correlated with the firm's ability to attract external funding. In so doing the CLPM provides a robust alternative, or complement, to Enterprise Risk Management (ERM)-approaches (ERM is discussed in, for example, Liebenberg and Hoyt, 2003). In ERM systems a multitude of risk factors are reported with reference neither to the overall volatility in the firm's cash flows or the health of its balance sheet. Through its insistence on expressing risk as a function of such "bottom lines" that are linked to the execution of the firms strategic objectives, the CLPM provides a basis for measuring and managing corporate downside risk, thereby putting the firm in a better position to pursue the upside.

The article is outlined as follows. In the next section I discuss the Lower Partial Moments (LPM)-framework for risk measurement. The section thereafter introduces the Conditional Lower Partial Moments (CLPM)-framework as it applies to the corporate setting. Then two sections follow that present a stylized account of how to model the firm's cash balance and its debt capacity, respectively. The section thereafter develops the intuition behind the CLPM by discussing three measures of risk within this framework. In the section thereafter, some managerial implications are discussed. A final section concludes the paper.

I. The Lower Partial Moments-framework

Corporate risks have traditionally not been managed in an integrated manner. There is a wealth of so-called risk "silos" (see Miller, 1992, for a comprehensive list of risk categories). Typical risk silos include financial risk, operational risk, competitive risk and legal risk to name but a few. Each of these risk silos has tended to be analysed and managed on an isolated basis. Nor do companies tend to have a single, unified perception of their risk profile at the corporate level. Risk might, for example, be defined in terms of cumulative cash flow, earnings, self-imposed borrowing constraints, or some balance sheet ratio (Hayt and Song, 1995). As part of a general trend towards integrated risk management a number of frameworks have appeared that seek to integrate the various aspects of corporate risk (Miller, 1992, Froot, Scharfstein, and Stein, 1994, Stulz, 1996, Meulbroek, 2002).

Whereas the concept of a holistic, integrated approach to risk management has been gaining ground, less has been achieved in the area of *measurement* of corporate level risk. A unifying framework for measurement of corporate risk has, to the best of my knowledge, not been attempted in a comprehensive manner. It is clear from the outset that we are not looking for a single measure of risk that will be applicable in all situations. Given that risk is such an elusive concept and that the appropriate definition of risk can shift from context to context we stand to benefit from a versatile and robust *framework* for constructing risk measures.

So how could we approach the problem of measuring corporate risk? Some guidance may be obtained by gleaning on risk measurement in financial economics, where risk, and particularly the measurement of it, has been high on the agenda ever since Markowitz's (1952) path-breaking paper on portfolio selection. This paper, and much of the subsequent work in portfolio theory, used the standard deviation of the returns of a security as a measure of risk. The standard deviation has been criticized as a risk measure since it measures dispersion around a mean, which is not necessarily a good proxy for risk, and its rise to stardom may in fact be attributable to its advantage in terms of computational ease rather than its inherent qualities as a measure of risk (see, for example, Nawrocki, 1999).

I will instead appeal to the Lower Partial Moments-framework (LPM) on the grounds that it offers a coherent and versatile tool for creating measures of risk that are economically relevant and intuitively appealing. The LPM framework was originally developed by Bawa (1977) and Fishburn (1977) and has been applied to the problem of optimizing a portfolio of financial assets by several authors (see, for example, Harlow, 1991). In the LPM framework one establishes a critical benchmark level of performance. Risk is then measured relative to this benchmark level. A penalty is applied to outcomes that are *worse* than the benchmark, but not those in which the target level is met or outperformed. This asymmetry is a key feature of the LPM-framework. The general framework is the LPM (a, t)-model (Fishburn, 1977) where risk preferences are described by selecting the values of *a*, the penalty applied to shortfalls below target, and *t*, the target level of return. Risk is defined by the following function.

$$F_a(t) = \int_{-\infty}^{t} (t-x)^a dF(x) \qquad a > 0 \tag{A}$$

One key advantage of the LPM-framework compared to the symmetric standard deviation is that it allows risk to be measured as deviations on the downside relative to a critical benchmark level. By viewing only shortfalls relative target as risk, the LPM achieves greater intuitive appeal. The standard deviation, in fact, penalizes positive deviations as much as negative ones, whereas people in general to not tend to think of performing better than a benchmark as “risk” (see, for example, Kahneman and Tversky, 1979). Another advantage of the LPM is that it relieves the analyst of relying on the normal distribution, since it places no restrictions on the probability distribution assigned to the variable in question (Harlow, 1991). A final strength of the LPM is the fact that risk preferences can readily be adapted by regulating how heavily shortfalls from the selected target level are penalized. This is done by raising each target shortfall by a power *a* (the risk aversion coefficient). The general principle is that the higher the value of *a*, the higher the penalty placed on a given shortfall (and by extension the higher the risk aversion of the decision-maker). For most practical purposes, the risk coefficient will take on values between 0 and 2 (more on the interpretation of these coefficients later).

II. Moving beyond Cash Flow-at-Risk

Does the LPM have any added value when we switch from a portfolio of financial assets to the context of a non-financial corporation? To answer this question, we first need to consider which variable is being analyzed. In the original LPM analysis the variable being analyzed was by default the return of some asset, or portfolio of assets. For a non-financial firm, though, there is a wealth of potential target variables that in some way relate to the firm's steering model; cash flow, earnings, the Debt-to-Equity-ratio, etc.

Even though other target variables are conceivable, this paper emphasizes liquidity. Intuitively, a firm must have cash available to meet cash obligations as they fall due and to offset fluctuations in working capital. There is also a strategic dimension to liquidity in that cash must continuously be available to fund the firm's strategic investments. According to this line of thinking, formalised by Froot, Scharf, and Scharfstein (1993), corporate risk management should explicitly aim to ensure that liquidity never falls short of the level needed to implement the firm's optimal investment. Another objective for corporate risk management identified in the literature is to avoid costs of financial distress (Stulz, 1996), which are ultimately related to the prospect of the firm having insufficient liquidity to service debt obligations.

One approach that has been pursued is to build a Cash Flow-at-Risk-model (CFaR), in which one calculates the firm's maximum shortfall of cash flow, associated with a certain probability, relative to a certain forecast. Approaches to CFaR-modelling can be found in Risk Metrics (1999), Stein et al (2001) and Andrèn et al (2005). Such modelling efforts can be helpful in managing the firm's operating cash flow and provide a sense of the firm's overall liquidity risk over a certain time period. One clear advantage of CFaR, compared with the traditional "silo"-approach to risk management, is that it sums up the firm's various risk exposures into a single measure of risk.

While these are important improvements, I will argue that CFaR falls short of being the best representation of corporate risk. One limitation is that CFaR cannot really distinguish successfully between shortfalls that come at a great cost to the firm from those that have little or no consequence. In other words, CFaR is rather inflexible with respect to the utility function, or degree of aversion to shortfalls, of the decision-maker. In consequence, it systematically fails to incorporate the cost of risk. Nor is CFaR, as it stands, easily made compatible with normative theories as to how risk management can create value. In the CFaR literature the cash flow concept used is operating cash flow, which almost by definition does not convey information as to whether such cash flow shortfalls might disrupt strategic plans, or whether the firm risks having insufficient funds to meet debt obligations.

Applied to the corporate setting, the Lower Partial Moments-framework addresses some of these criticisms. The capital requirement to implement strategic plans or meet debt payments are attractive target levels around which to measure risk. The firm's ability to tolerate shortfalls can easily be reflected by choosing the appropriate value for the risk aversion coefficient *a*. However, having evolved in the context of portfolio optimization the LPM has not developed to include a reference to the balance sheet of the investor. The main reason for this is that when the volatility of value is the object of analysis, there is no real distinction between value and cash flow. The value of an asset is the cash flow that would be realised if it were sold. An investor's liabilities can therefore be set in relation to the probability distribution of value directly. For the managers of a corporation, however, with its largely fixed portfolio of fairly illiquid assets, the state of the balance sheet is an important piece of information as it will indicate the firm's ability to withstand short- to medium term fluctuations in liquidity to which the successful execution of strategy may be intimately tied.

In this paper I will argue that a truly useful summary risk statistic at the corporate level should also make reference to any existing spare debt capacity. The endogenous relationship between risk management activities and debt capacity is often emphasized in the risk management literature (see, for example, Smith and Stulz, 1985). In a risk management context, the need to incorporate information on debt capacity is most clearly articulated by Mello and Parsons (1999): "*Determining the external financing capacity is critical. Many firms have significant reserves in the form of unused debt capacity, and a shortfall in the internal supply of funds will not disrupt the investment strategy*". For example, a firm whose conservative financial policy has landed its Debt-to-Equity ratio at 0,3 may find the prospect of a funding need to be less than frightening. An otherwise identical firm whose aggressive use of debt has put its Debt-to-Equity ratio at 2 may not be as keen to exposure to refinancing risk. This suggests that we can obtain more informative risk measures by qualifying the firm's liquidity risk using information on its spare debt capacity.

I will therefore introduce a Conditional Lower Partial Moments (CLPM)-framework in which risk measures are derived from the interaction between the firm's liquidity and its balance sheet. The general CLPM-framework is written CLPM (a_1, t_1, t_2). T_1 refers to the critical, or target, level for the firm's cash balance[1]. T_2, in turn, refers

[1] This could be zero, but also some positive number that reflects the fact that a firm may want a minimum buffer of cash at all times for working capital needs.

to the critical, or target, level for the proxy for the firm's balance sheet constraint. a_1, finally, is the risk coefficient as applied to the shortfalls below the target level for cash that are not covered within the firm's remaining debt capacity. The difference between the LPM and CLPM resides in the debt capacity parameter t_2. In the CLPM risk measures are defined by:

$$F_a(t_1 t_2) \;=\; \int_{-\infty}^{t_1} (t-x)^a \, dF(x) \quad (t-x) = 0 \in \int_{t_2}^{\infty} (y-t_2)\, dy \quad a > 0 \qquad \text{(B)}$$

In this framework the unit of analysis x is the firm's cash balance. This is a stock variable whose development will incorporate information on both the firm's operating cash flow and its demand for liquidity. Risky outcomes pertain to the part of the distribution for the cash balance where the target level is not met. Such outcomes can be thought of as a funding need. If those can be covered with new external funding the firm can still execute its strategy. If not, it will have to scale back its cash disbursements. The latter outcomes are considered "risk". To establish the fraction of times this occurs the CLPM makes reference to a proxy for the firm's debt capacity y. Whenever this proxy does not exceeds it critical level t_2 the model assumes refinancing and hence optimal execution of strategy.

We note that in expression B there are three basic types of outcomes. The first is where $x-t_1$ is positive. This is a non-risk outcome since the firm has internal funds to cover all its cash commitments (cash doesn't go below zero). The second outcome is when $x-t_1$ is negative and $y-t_2$ is negative. This registers as a non-risk event since the balance sheet constraint has not exceeded its critical level and external financing is assumed. The third outcome is where $x-t_1$ is negative and $y-t_2$ is positive. The firm has an external funding need, but since the constraint is exceeded it fails to obtain the desired funding. In the CLPM a risk event can thus be thought of as a dummy that takes the value one if both the cash balance and the proxy for debt capacity breach their respective target levels *simultaneously*. Looking at liquidity and solvency in a joint framework thus means that we move towards defining risk as the probability of a certain combination of events, rather than the below-target part of any particular probability distribution alone. Table 1 summarizes the main approaches to corporate risk measurement.

Table 1 Main approaches to corporate risk measurement

Risk framework	Concept	Comments
Standard deviation	Measures the degree of dispersion around the mean	• Symmetric perception of risk • Relies on normal distribution
Cash Flow at Risk	Measures the maximum loss associated with a certain statistical confidence level	• Asymmetric, ie treats losses different than gains • Somewhat inflexible with respect to risk tolerance • Based on operating cash flow
Lower Partial Moments	Measures risk as the deviations below a target level penalized by a risk aversion coefficient a	• Capable of incorporating target levels consistent with corporate risk theory • Adopts easily to varying levels of risk aversion • Makes no reference to debt capacity
Conditional Lower Partial Moments	Same as above but also makes reference to a second probability distribution to separate risky shortfalls from non-risky shortfalls	• Incorporates information on debt capacity

III. Modelling corporate cash holdings

The first leg in the CLPM-framework is to model the firm's cash balance. In this section I present a stylized account of how a firm's pre-external financing liquidity can be modelled. It will focus on two distinctly separate components of corporate liquidity: the firm's internal supply of funds, which will here be taken to mean its operating cash flow plus its ingoing cash balance, and its demand for liquidity. The latter is defined as the sum of the forecasted cash disbursement the firm expects to incur in the course of conducting its business in pursuit of shareholder value. A key component of the firm's demand for liquidity is its expenditure on investments that (presumably) have positive expected NPV. However, an non-exhaustive list also includes the dividend, interest payments, and instalments on loan, pension payments, plus firm specific cash outlays that the firm has committed itself to making. For example, an oil firm may consider planned removal costs for a retired oil platform as a cash commitment.

Obtaining a model of a firm's internal supply of funds presupposes a model of the firms operating cash flow. The core of any risk corporate level risk framework will always be a model describing the volatility of its operating cash flow. It is important to specify the nature and amount of variability emanating from the firm's business activities. Such a model will be particularly useful if it contains information on the firm's exposure to hedgeable macroeconomic- and market risk factors (Andren, Jankensgård, and Oxelheim, 2005). Conceptually, a firm's operating cash flow G at a future point in time t can be written as[2]:

$$G_t = a + b_1X_t + b_2Y_t + b_3Z_t + b_4U_t + \varepsilon \qquad \text{(C)}$$

In equation C, X is a vector of exchange rates to which the firm's cash flow is exposed. That is, the coefficient b_1 is a statistical estimate of how the firm's operating cash flow would respond to a one-unit change in an exchange rate to which it has an exposure. For example, let us assume that the cash flow of a US-based firm is exposed to the USD/EUR exchange because it exports to the European market. Based on data from the last six years, it estimates b_1 to be 150Mn USD. We interpret this in the following way: if the USD/EUR increases 10 cents, meaning a weaker USD, the firm's cash flow tends

[2] For a comprehensive framework for estimating corporate exposure to macroeconomic and market risk, see Oxelheim and Wihlborg (1987, 1997, 2006).

to respond with approximately a 15Mn increase to reflect the fact that its EUR-revenues are worth more in USD-terms. Similarly, the vector Y represents cash flow exposure to interest rates, Z to inflation rates, and U to commodity prices, and the coefficients b_2 through b_4 indicate the magnitude of these exposures. The error term can be thought of as representing all other sources of volatility in the firm's cash flow. It represents the "average" uncertainty in cash flow as a result of legal risks, operational risks, shifts in demand, etc, in short anything that is not a macroeconomic- or market risk included in the model. The information in model C can be translated into a probability distribution for the firm's operating cash flow at time t by observing that

$$\sigma^2_G = b_1^2 \sigma^2_{Xt}{}^2 + b_2^2\, \sigma^2_{Yt}{}^2 + b_3^2\, \sigma^2_{Zt}{}^2 + b_4^2\, \sigma^2_{Ut}{}^2 + W + \varepsilon_G^2 \qquad \text{(D)}$$

In equation D the W-term is a vector of covariances terms between the risk factors in the model. We thus obtain an estimate of the volatility in a firm's cash flow by looking at its exposure to risk factors and the volatility of those risk factors (where the correlation between these risk factors is taken into account), and by adding the volatility of the error term (for details see Andren, Jankensgård, and Oxelheim, 2005).

Interestingly, we find that the general model for the firm's demand for liquidity can be written as a function of basically the same risk factors as its operating cash flow. An obvious reason for this is that many of the firm's cash commitments may be denominated in the same currency as its operating revenues. For example, an exporting firm based in Sweden with mainly USD-revenues typically chooses to have at least parts of its debt in USD. This means that some of its loan repayments and interest payments will be in the same currency as its main exchange rate exposure. More subtly, though, a firm's demand for investment funds may be correlated with one or more of the risk factors that drive its operating cash flow. Consider an (unhedged) oil-producing firm. If the oil price falls, its operating revenues will also fall. But so may the value of investment opportunities in oil fields, thereby reducing the firms need to invest (Froot, Scharfstein, Stein, 1993). An interest rate may influence on the demand for a firm's products, especially in capital intense industries (Oxelheim and Wihlborg, 1997). The same interest rate may also influence on the firm's investment opportunity set to the extent it affects the firm's weighted cost of capital. That is, if higher interest rates mean lower revenues, there may also mean a lower need to invest if, at these higher interest rates, fewer of the firm's investment opportunities have positive NPV's. Ignoring tax issues and possible non-linear relationships, we can therefore write the firms demand for liquidity, L, at time t as follows:

$$L_t = a + c_1X_t + c_2Y_t + c_3Z_t + c_4U_t \qquad \text{(E)}$$

The exposure coefficients c_i have the same interpretation as in equation C. They indicate how much the firm's need for liquidity would change as a result of changes in its macroeconomic environment and in key product prices to which it is exposed.

Now we are ready to write our model for cash (pre new external funding). We define it as

$$C_t = C_{t-1} + G_t – L_t \qquad \text{(F)}$$

Combining the information in equations C through E we find that the variance in the firms cash balance at time t can be written:

$$\sigma^2_{Ct} = (b_1\text{-}c_1)^2\,\sigma^2_{Xt} + (b_2\text{-}c_2)^2\,\sigma^2_{Yt} + (b_3\text{-}c_3)^2\,\sigma^2_{Zt} + (b_4\text{-}c_4)^2\,\sigma^2_{Ut} + W + \varepsilon_G^2 \qquad \text{(H)}$$

Based on the information in equations F and H we have an estimate of both the forecasted level of cash at time t and the uncertainty in that forecast[3]. This information can be used to construct pure liquidity risk measures. For example, risk could be defined as the fraction of times in which cash goes below zero:

$$RISK = P\,(C_t < 0) \qquad \text{(G)}$$

However, with no further information we cannot say if such a funding need will be problematic or nor. A non-zero probability only says that the firm is exposed to re-financing risk. Whether this might pose a problem or not will depend on many factors.

[3] We see from equation H that the expected volatility in cash will be reduced the higher the tendency for the operating cash flow and its cash commitments to co-vary. In other words, management can lower risk to the extent it is able to minimize the difference between the bi and ci coefficients. This is consistent, for example, with the traditional risk management strategy of matching the currency denomination of debt liabilities with that of its net operating cash flow.

Most likely, however, the state of the firm's balance sheet will be an important determinant. This suggests that a more informative risk measure would qualify risk measure I using some indicator of the strength of the firm's balance sheet.

IV. Risk Management: Know thy constraints

The second leg of the CPLM-framework is a proxy for the firm's debt capacity[4]. Debt capacity is defined as the firm's ability to raise new debt at a cost that does not exceed the return the firm would be expected to earn on those funds on a risk-adjusted basis. A firm's debt capacity is exhausted if the firm is credit rationed and simply cannot get new loans, or if its balance sheet is so weak that adding new debt would significantly increase the cost of financial distress in the future. The analysis in this section concerns the identification of a liability overhang in the firm's balance sheet that could lead it to conduct its business sub-optimally in some states of the world.

Debt capacity is a concept largely similar to that of financial constraints. The literature on financial constraints, beginning with Fazzari, Hubbard, and Petersen (1988) investigates the links between a firm's cash flow and investment on the premise that such a relationship is evidence of a cost wedge between internal and external sources of funds. This literature draws on theoretical models of capital market imperfections to generate its predictions. In these models external capital is more expensive than internal funds for two basic reasons: asymmetric information (Myers, 1984) and agency problems (Myers, 1977, Jensen and Meckling, 1976, and Jensen, 1986). Such imperfections can lead to credit rationing and a situation where the firm is dependent on internal funds to invest.

The literature on capital structure has long recognized that contracting problems in the financial markets increase in the level of debt (see, for example, Myers, 1977). The firm's Debt-to-Equity-ratio would therefore appear like a natural candidate to proxy for the firm's capital market access (or Debt-to-Total Assets, depending on which is deemed

4 The discussion in this section assumes that the firm's marginal source of external funding is debt. In principle a company should also be able to access the equity market to shore up its risk capacity. In practice, however, many companies are extremely reluctant to issue equity when performance is poor. Many companies also have concentrated ownership, implying that equity financing is a complicated business, and the resulting high cost wedge between internal cash flow and equity may leave share issues as the last-resort alternative. This means that equity-issuance can be considered a negative consequence of volatility that possibly could have been avoided, rather than a neat risk management tool to use when in financial difficulties.

most closely correlated with the firm's capital markets access). As a firm's leverage increases, so does the potential for information and agency problems with respect to the investor community. As a consequence the part of its cost of capital that is not related to its fundamental credit risk will also increase. For risk calculations one would therefore have to identify a critical level for the Debt-to-Equity-ratio at which the firm's balance sheet constraints become operative.

A leverage ratio per se does not tell us specifically whether a particular debt covenant is actually binding or not, however (Matsunaga, Shevlin, and Shores, 1992). A debt covenant is a provision included in debt contracts that restricts the firm's activities after the bond is sold, the purpose of which is to mitigate the bondholder-stockholder conflict (Smith and Warner, 1979). A covenant targeting subsequent financial policy may stipulate that the firm is restricted from issuing additional debt, for example by subjecting it to aggregate dollar limitations (Ibid). Covenants in existing debt contracts may constitute a very real constraint on a firm's borrowing, so it might sometimes be possible to rely on clearly formulated debt covenants that make subjectivity less necessary when estimating debt capacity.

Another ratio-based proxy for the firm's balance sheet constraints might come in the form of a particular ratio on which the firm's credit rating is perceived to hinge[5]. This presumes that it can be assumed that the firm would be downgraded if it fails to meet the rating agencies' target level, and that this event would materially impair its capital market access. A somewhat more tangible constraint may be derived from the size of a firm's existing credit facility. It may be reasonable to assume that a firm can expect to draw credits under existing arrangements but fail to attract new funding beyond that.

As the above discussion shows, there are various ways to frame the constraints imposed on a firm's investment and operating activities by the state of its balance sheet. In the remainder of this section I will use the Debt-to-Equity ratio as an example. This necessitates a model of both the firm's debt and its equity. Starting with debt, we can write the firm's debt, denoted D, at time t as:

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5 One objection to this practice might involve the use of ratios based on book values. Financial theory would have it that the real determinant of a firm's borrowing capacity is the future cash flows its assets are expected to generate. In principle this is true, although in practice there is a strong tendency for creditors, rating agencies, and managers alike to focus on ratios derived from the income statement and balance sheet when assessing creditworthiness (Fridson and Alvarez, 2002).

$$D_t = D_{t-1} - R_t + d_1 X \qquad (H)$$

In equation H, R denotes repayments on outstanding debt. The outgoing debt position is basically the ingoing debt minus any loan repayments made in period t. In addition, the outgoing debt position will be exposed to revaluation effects if the firm has loans in foreign currency. For example, a Swedish based firm with loans in USD will find that it has a larger debt liability to service if the USD appreciates against SEK period t. Again, this will typically occur under the strategy of matching the debt denomination with the currency exposure of the firm's operating income. As for the firm's equity, denoted E, a simple model can be written as:

$$E_t = E_{t-1} + P_t\,(1-\lambda) + e_1 X_t + e_2 Y_t + e_3 Z_t + e_4 U_t \qquad (I)$$

In equation I, P stands for the firm's Net Profit and λ denotes its payout ratio. Basically, equity grows by the amount of earnings the firm is capable of generating, less dividends paid. The vectors e_1 through e_4 represent the exposure of equity to changes in the firm's macroeconomic environment. These effects come about for two reasons. First, the firm's Net Profit is exposed to macroeconomic- and market risk in much the same way as its operating cash flow. Second, changes in a number of macro-exposed items hit equity directly[6].

Provided the information in equations H and I we can obtain a forecast of the firm's Debt-to-Equity ratio at time t. By incorporating the variances of the risk factors we can also obtain the variance of this ratio:

$$\sigma^2_{D/Et} = (e_1 - d_1)^2\,\sigma^2_{Xt} + (e_2)^2\,\sigma^2_{Yt} + (e_3)^2\,\sigma^2_{Zt} + (e_4)^2\,\sigma^2_{Ut} + W + \varepsilon^2 \qquad (J)$$

Given the variance of the Debt-to-Equity ratio, we can define risk measures in terms of outcomes where the firm's Debt-to-Equity breaches its critical level. We call this level $D/E_{Target..}$ One possible definition of risk is then:

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[6] A good example of this is the effects that result from translating foreign subsidiaries back to the presenting currency at two different points in time when the exchange rate has moved. Another example is changes in the present value of cash flow hedges that do not qualify for hedge accounting.

$$RISK = P\ (D/E_t > D/E_{Target}) \qquad (K)$$

This risk measure is the probability that the firm's target for the Debt-to-Equity-ratio is breached. Given our assumptions we interpret it as the probability that the firm will end up in a situation where it will not be able to obtain further debt funding. Again, while this may be an interesting measure per se, breaching the target ratio may not be linked to any immediate consequences if the firm at the time the breach occurs has no funding need. That is, a high leverage per se does not imply any negative consequence as long as the firm generating sufficient cash flow to maintain its fixed obligations. This again suggests that more meaningful measures of risk can be obtained by studying the interface between the firm's cash balance and debt capacity.[7]

V. Three Measures of Risk based on the CPLM-framework

The third leg in the CLPM is to apply a penalty to risk outcomes to reflect the cost of risk. This section deals with this issue by way of a discussion of three measures of risk within the CLPM. To recap, by incorporating information on the firm's balance sheet constraints into our model of cash holdings we seek to distinguish between those liquidity shortfalls that have no bearing on the execution of the firm's strategic initiatives from those that do. We call the latter an *uncovered liquidity shortfall (ULS)*, as opposed to those shortfalls that are covered within the limits of the firm's remaining debt capacity. An overriding idea is to apply a penalty to ULS's to reflect the *cost* of risk. A key component of a CLPM-framework, as with the LPM, is thus the risk aversion coefficient a. We write this as $CLPM_a$. As noted earlier, this coefficient will take on its

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[7] There are important links between the two, which has consequences for a firm's risk management strategy. As an example, consider an oil-producing firm whose revenues decrease as the oil price falls. If the price falls far enough, the firm may incur an impairment loss as the value of the oil assets on the company's books is written down. Both the loss of revenues and the impairment will serve to reduce Net Profit and consequently the equity base. This will be reinforced if the firm has a sticky dividend policy, so that the size of the dividend is detached from the current level of Net Profit. If the Debt-to-Equity is assumed to be a proxy for the firm's capital market access, then its ability to attract new debt funding will be lower too in this scenario. This is consistent with the notion, familiar to many a financial manager, that obtaining finance is more troublesome the more you need it.

value according to how severe a shortfall below target is considered to be. Put simply, the highly risk averse decision-maker will set *a* relatively high to reflect the perceived cost (either in monetary or emotional terms) of the shortfall.

If *a* is set to zero, a shortfall below the target level will not be penalised at all. A shortfall is then just registered as an event; either it "happens" or it does not. Based on the probability distribution of the variable in question we are able to establish the fraction of times a shortfall is expected to occur. This gives us the probability of a shortfall. In the CLPM-framework applying the risk coefficient zero renders the **Probability of an Uncovered Liquidity Shortfall (PULS).** We interpret this as the probability of cash going so far below zero (our assumed target level) that the firm's remaining debt capacity is insufficient to accommodate it.

$$CLPM_0 = P\left(C_t < 0 \mid D/E_t > D/E_{Target}\right) \qquad \text{(L)}$$

Among the properties of this risk measure we note that it will tend to be (but does not have to be) equal to the lower of the two probabilities involved, reflecting that both target levels must be breached and that the most "distant" of these probabilities will tend to determine the probability of the event occurring.

The drawback of a shortfall probability as a measure of risk is that it does not differentiate between large and small shortfalls. In most circumstances will it matter how large the shortfall is, with large shortfalls naturally being more undesired than smaller ones. A given value of PULS will not convey information on whether the shortfalls are clustered just below the target level or if there is significant tail risk. As a consequence, a firm that defines risk as PULS may find that it sometimes produces counterintuitive results. If PULS>50% our expectation at the outset is that the negative event occurs, and hedging some exposure will in such a case make it *more* likely that the targets are breached since more of the probability mass is concentrated around the expected level, which implies larger likelihood of failure to meet target levels. That is, a hedge will contribute towards "locking in" an outcome in which the target levels are not met.

It turns out that this drawback can be overcome if we set the risk aversion coefficient to 1. In a scenario in which a shortfall occurs we raise this shortfall by one. Raising a number by one of course renders the number itself. On an expected basis, this is equivalent to weighting each shortfall with its probability of occurrence. If $a = 1$, the

CLPM is the **Probability-Weighted (expected) Uncovered Liquidity Shortfall (P-WULS).**

$$CLPM_1 = E\ \{(C_t) \text{ for all } C_t < 0 \ \mid\ D/E_t > D/E_{Target}\ \} \qquad \text{(M)}$$

The resulting risk measure has the attractive feature that it takes into account both the likelihood *and* size of liquidity shortfalls. With P-WULS, unlike PULS, we obtain that hedging to reduce the variability in cash flow never increases the risk estimate, which of course is a desirable property of a risk measure. What P-WULS does not capture, however, is that the consequences of uncovered shortfalls may be increasingly severe the larger they get. That is, the marginal cost of a one-unit shortfall may be increasing in the size of the shortfall. By simply weighting each shortfall with its probability of occurring, P-WULS does not recognize this possibility.

To appreciate the point made above, consider the following sequence of events. A firm's first response to insufficient liquidity might be to cut down the investment spending with the lowest expected return. It may then choose to cancel its dividend payment, sending clear distress signals to the investor community. For even larger uncovered shortfalls, the firm may have to scale back its strategic investments where the expected profitability is the highest. If a firm's interest payments are threatened, a variety of financial distress related costs might set in, as stakeholders demand a higher risk premium for doing business with the firm. That is, the first set of cash commitments that the firm would scale back on can be assumed to be less important than the next set. What the above example is meant to illustrate is that if the cost of uncovered shortfalls increases the larger they become, a conceptually appealing risk measure should apply a larger penalty to large shortfalls compared to smaller ones. One way to achieve this is by squaring the uncovered liquidity shortfalls. If a = 2, the CLPM is the **Uncovered Liquidity Shortfall (semi-) variance (ULS^2).**

$$CLPM_2 = E\ \{(C_t)^2, \text{ for all } C_t < 0 \ \mid\ D/E_t > D/E_{Target}\ \} \qquad \text{(N)}$$

The risk measures presented in this section each conveys some aspect of the firm's risk profile and should be considered complementary. As each send a slightly different signal as to the firm's risk profile, a simultaneous reading of many risk measures is the preferred approach. The next section discusses more in detail how the CLPM applies to corporate decision-making.

VI. CLPM as a steering tool: Understanding delta risk

Creating CLPM-based measures in practice means to specify dynamic simulation models of the firm's cash, debt, and equity for a selected time horizon (or some other financial ratio that is a proxy for its balance sheet constraints). These variables are stock measures that fairly well summarize the level of the firm's strategic risk taking[8] and its financial and hedging policies. Risk measures based on the variability and interaction between such stock variables will therefore be responsive to a large number of corporate policies. We therefore obtain an estimate of delta risk with respect not only to changes in market conditions and exposures, but also with respect to strategic risk taking and financial policy.

Using an CLPM -risk measure, delta risk can always be decomposed into the changes in one (or several) of the underlying factors affecting risk. While some are beyond the control of management, such as movements in the financial and macroeconomic risk factors influencing the firm's performance, there are quite a few policy variables, both in the financial and strategic area, that operate on ΔCLPM. The policy variables affecting delta risk can be illustrated as in Figure 1.

[8] As noted earlier, by strategic risk taking is here meant the levels of spending on investment and acquisitions

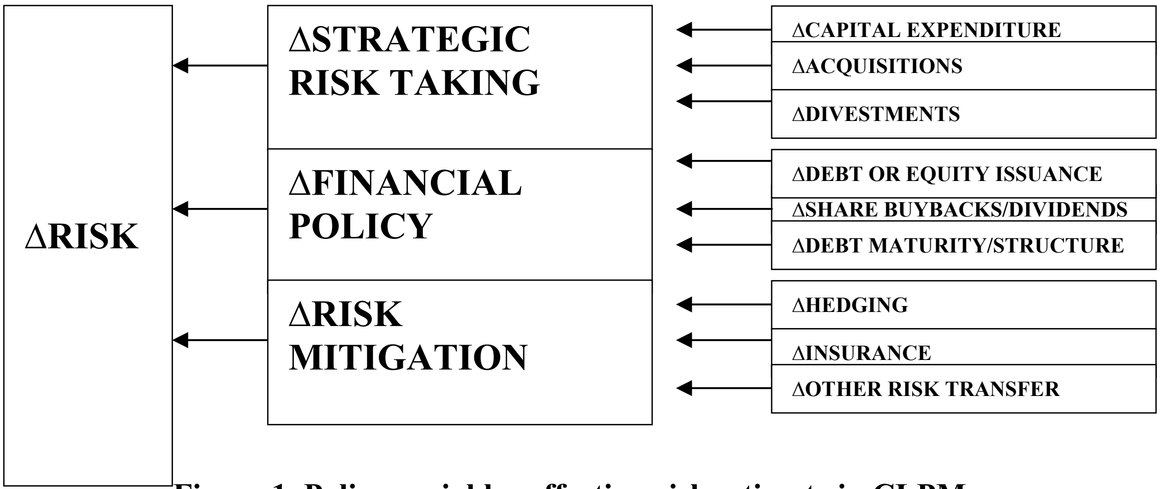

Figure 1. Policy variables affecting risk estimate in CLPM

In the CLPM two major risk drivers are sufficiently large cash- or debt-financed acquisitions or share buyback-programs/ dividends. The former will tend to stretch the firm's balance sheet and may make the firm more vulnerable to deteriorating markets, especially if the asset(s) acquired yields little or no immediate cash flows. The latter will also tend to increase risk since it uses up cash and at the same time weakens the equity base.

To see how the framework can be applied, consider a firm that entertains the prospect of making a cash-financed acquisition of an asset that yields little or no immediate cash flows. In the acquisition scenario, the forecasted level of cash will decrease by approximately the transaction amount. The increase in planned acquisition spending represents an increase in the level of strategic risk taking and will consequently increase $P(C_t < 0)$. What happens to CPLM-based risk measures, for example PULS, will depend on the current health of its balance sheet. If the firm has a weak balance sheet, then PULS will increase to reflect the firm will not be able to cover all funding needs that could arise given the forecasted level and volatility of cash. To counter-balance the increase in risk from the acquisition the firm might hedge its exposure to price risk, thereby seeing risk return to more appropriate levels.[9]

[9] An important point is that actions that increase risk must be evaluated against their contribution to upside potential. If no consideration is given to the benefits of each alternative course of action then the obvious preference would be the not-so-efficient strategy of always minimizing risk. For example, a share buyback will tend to increase the return on the firm's remaining equity, so the change in

While any measure of corporate risk will ultimately be very uncertain, *changes* in risk could be more informative and robust. That is, delta risk may be more useful as a management tool than the risk estimate itself. While a lot of assumption goes into creating a risk measure, changes in such a measure should to a larger extent reflect genuine changes in the firm or its environment (at least as long as the base assumptions remain unchanged). Successive measurements of ΔCLPM signal the direction of risk: is risk on the increase or is it decreasing? Monitoring trends in CLPM allows management to respond at an early stage to a changing risk-picture. Furthermore, knowing the *source* of the direction in ΔCLPM will indicate the appropriate response. Having in place a risk measure like CLPM thus offers a richer understanding of a firm's total risk and changes in that risk profile. Knowing the impact on risk from various pro-active policies increases the chances that risk considerations can play an active part in forming the firm's strategies on both the strategic and financial level.

VII. Concluding Remarks

This paper has outlined the Conditional Lower Partial Moments-framework (CLPM) for measurement of corporate level risk. A rich palette of risk measures is obtained by analyzing the firm's operating cash flow, demand for liquidity and balance sheet constraints in a unified framework. By taking optimal execution of strategic initiatives as the main objective, the need to subject risk management objectives to the shareholder value criterion is recognized.

A key managerial benefit of CLPM is that in the CLPM *delta* risk thus takes on added meaning since it will not just be a function of changing exposures or market conditions. This occurs because when risk measures are based on stock variables like the cash balance and items in the firm's balance sheet, they will be responsive to a large number of business- and financing policies. Successive measurements of CLPM-based risk measures indicate the development in the overall risk profile of the firm, and the

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the expected return-on-equity could proxy for the upside. In other instances, a Net Present Value-figure will be more appropriate if a risk-increasing investment or acquisition is made that increases cash flows beyond the time horizon of the risk analysis. Another way a proxy for upside can be created is by looking at the upper partial moments in the distribution for outgoing cash.

changes in this risk profile can, for the identification of the appropriate management response, be decomposed to identify the factors driving these changes.

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