Integrated Analyses and Assessment of Operational Risk: An Influence Diagrams Approach Based on Topological Data Model

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Abstract：

Considering the shortages of the general operational risk data model, a new operational risk data model -- operational risk topological data model is brought forward in this paper. It can not only contain the information about the connection between all risk causes during the risk event happening process, but also can support the operational risk measurement by influence diagrams approach. In the operational risk topological data model, a risk loss event is attributed to risk trigger cause which lead to the initial loss and successor causes which increase (sometimes maybe decrease) the initial loss. The record includes the initial loss of the trigger cause and the loss effect multiples of each successor cause.

Then the integrated analyses and assessment of operational risk is studied by an influence diagrams approach based on topological data model. Operational risk influence diagrams can not only assist the analysis of correlation between all the causes, but also can support the statistic analysis to operational risk data in the topological data model. The calculation of operational risk influence diagrams has three steps: calculating the company's operational risk loss severity distribution by the influence diagrams probabilistic inference; calculating the company's operational risk loss frequency distribution by the frequency of trigger causes; and combining them into an aggregated operational risk loss distribution. The approach to getting initial loss distribution of each trigger cause which is needed in the calculation of operational risk influence diagrams is studied at last.

The operational risk measurement by influence diagrams approach based on topological data model with objective data is a new operational risk assessment approach and effective risk analysis method which focus on the failure of control and process and using the information contained in the operational risk historical event more effectively. It also can be modified and improves agilely according to new information, improve the suitability of external data and support the operational risk management decision.

Keywords： Operational Risk; Measurement; Influence Diagrams; Topological; Data Model; Risk Cause; Correlation; Conditional Independence.

1. Introduction

The operational risk of a company is a very complicated system probably with dozens or hundreds of risk causes. Not only can every one of them lead to operational risk loss events, but

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also they may fit together in varied manners and result in more serious loss events. Furthermore, the complication of the operational risk system will increase with the growing of the company's scale and business. It is obviously so ideal that to assess such complicated risk system only with statistic approaches. Nowadays, many researches on operational risk measurement focus on developing statistic approaches according to the mathematic traits of the operational risk (e.g. small sample, heavy tail, etc.) to improve the measurement accuracy. However the risk measurement researches concerning the complication of operational risk system are few. Some scholars have introduced Bayesian network into operational risk assessment but most of those are for OPERATIONAL RISK management (e.g. Carol Alexander, 2003; Paolo Giudici, 2004). Influence diagrams are also mentioned in some papers but no complete assessment system is build yet.

Influence diagram brought forward by Professor Howard is an effective graph language to represent uncertainty decision problem by applying Bayesian conditional probability theory to Graph theory. Its topological structure and probability analysis can assist the integrated analysis of a company's operational risk and can support many kinds of risk measurement approaches. However it needs the support of the data model with consistent topological structures.

Otherwise, from our investigation, it is found that when those complex operational risk events resulting from more than one cause are recorded by the general record method and data structure which is the precondition of current operational risk measurement research, the gross event loss is easy to get, while the greatest difficulty is to separate the gross loss by each risk cause.

Therefore, a new operational risk data model-- operational risk topological data model is brought forward in this paper. It can not only reflect the connection between all risk causes during the risk event happening process, but also can support the operational risk measurement by influence diagrams approach. The integrated analyses and assessment of operational risk is studied by an influence diagrams approach based on topological data model.

2. The basis of operational risk measurement：Topological Data Structure

Data is the foundation of the operational risk measurement and management. If lack of data, it is impossible to quantify the operational risk objectively. To accumulate and use the loss data effectively, an operational risk data model should be built at first, which has to meet the following basic requirements:

1. Convenient to obtain the date.

The date model should be easy for comprehension and data collection practice.

2. Able to distill the most information contained in loss events.

Getting enough data for relevant research on operational risk is not as easy as other financial risks, especially the data of those events that are infrequent but lead to tremendous loss and important to operational risk management. So every loss event should be take as precious history information and risk sample of operational risk and the operational risk date model should be design to distill most information contained in the operational risk events.

3. Convenient to measure operational risk.

The data not only can help to recognize and control operational risk, but also the foundation of operational risk measurement. So operational risk data model should provide risk measurement with all information needed and be convenient to apply the risk measurement approaches.

Therefore, in this section, on the basis of the above requirements, the general operational risk data model is introduced firstly and its shortages are analyzed. Then a new operational risk data model -- operational risk topological data model is presented and its data record processes are also studies.

2.1 General operational risk data model

There are two kinds of popular operational risk database. One is the databases for recording operational risk Key Risk Index (KRI). KRIs are outstanding performance indexes or control indexes benefiting to risk tracking. By observing and recording KRIs, the analysis of the trend of KRIs can give an alarm to operational risk. KRI approach is one of the popular approaches of risk management, though this approach can not measure the risk. The other kind is operational risk loss database which can collect data for operational risk measurement. This kind of database records and accumulates information of loss event by uniform format. At present, the general data model of operational risk loss database has three dimensions of data -- Risk Event, Risk Cause and Event Effect. (See Figure 2.1)

Figure2.1 Risk event-cause-effect chain

Risk cause is an action or a series of actions which result in risk event. Risk causes are usually reflected in companies' risk categorization. Risk cause has been classified in detailed. Each operational risk event can be classified into a well-defined risk cause category by a certain series of rules. Event effect can be defined as the influence of risk event to the risk subject. In order to guarantee the objectivity of records, the international normal method of recording event effect is only counting the financial loss. The records of risk events mainly include the time of events happening or being detected which can support the risk frequency measurement and the description of the internal and external environment when events happened, event causes and disposal process, etc.

2.2 The mainly problems of general operational risk data model

The most important function of operational risk data model is to supply needed information for operational risk measurement. In the general operational risk data model, the risk frequency information can be got from risk events record; the loss records of the same risk category are looked as the samples of one random variable used to analyze statistic characteristic of the loss of this risk category. So the classification of risk events by their risk causes is the basis of risk measurement and determines the accuracy of risk measurement.

From our investigation, it is found that when those complex operational risk events resulting from more than one cause are recorded by the general operational risk data model, the gross event loss is easy to get, while the greatest difficulty is to separate the gross loss by each risk cause. In that situation, some companies have to roughly attribute the whole loss to the trigger cause (Such as Allianz) or one risk category. There is even an interesting experience in Global Loss Database Committee of British Banker Association that is the loss cause are determined by the way just like "law court".[1]

Actually after the original cause triggered the risk event, the subsequent causes can magnify the effect and loss of risk event (such as control failure). Many times of magnification may finally result in disasters. It is a trait of operational risk that can not be ignored.

The relation between risk causes is one of the focuses of operational risk measurement study at all times, which relates to the risk aggregation and the accuracy of operational risk measurement. Attributing the gross loss caused by many risk causes to one risk cause will lose much important and precious information about the correlation between different risk causes and each cause's effect to the loss. The general data model restricts the relativity study to operational risk causes.

2.3 Operational risk topological data model

2.3.1 Operational risk topological data model description

Considering the shortage of the general operational risk data model, a new operational risk loss event record method and data structure -- operational risk topological data model is brought forward in this section.

Topology was originally one of branches of geometrics focusing on the invariability of consecutive transmutation of geometric forms and becomes an important branch of mathematics studying on continuity. Topological structure is one kind of space data structure, which can help the correlative data to keep a consistent and compact space structure.

In order to dig out and keep more information of operational risk events, the operational risk topological data model not only record the loss, cause and happening time of the operational risk event, but also record the happening course of the operational risk event by topological structure.

In the operational risk topological data model, a risk loss event is attributed to risk trigger cause which lead to the initial loss and successor causes which increase (sometimes maybe decrease) the initial loss. The record includes the initial loss which equals to the loss suppose the successor causes do not affect the event and the loss scaling multiples caused by each successor cause.

Operational risk topological data model is composed of the graph and node data. The graph pictures how each risk cause leads to the loss event. The nodes of the graph represent the risk

[1] John Serwer. Build and Operate Operational Loss Data. Advances in Operational Risk: Firm-wide Issues for Financial Institutions, Risk Waters Group 2001

causes of the loss event. The directed arcs connected each node represent the action order of the risk causes. The node data record the effect of each cause on the event loss. When recording the effect, firstly the initial loss caused by the trigger cause is recorded at the first node, and then scaling multiples to the previous loss by each successor causes are recorded at the successor nodes. The scaling multiples are called as loss effect multiplier of successor cause to its previous causes.

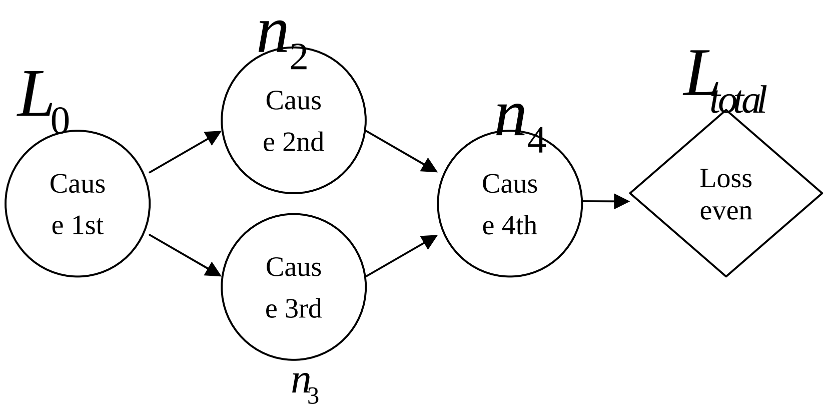


Figure 2.2 A demonstration of operational risk topological data model

Figure 2.2 is a demonstration of operational risk topological data model. L_0 is the initial loss. n_2, n_3 are the loss effect multipliers of the second cause and the third cause to the trigger cause. n_4 is the loss effect multiplier of the forth cause to the loss after the previous causes act. L_{total} is the total loss of the event.

The reason for adopting the multiplier instead of the addend to represent the loss relation between successor cause and previous cause is as followed:

Firstly, if using the addend to represent the effect of successor cause to the previous loss, the addend is correlated to the initial loss. For instance, in a familiar situation, if the original loss is L and the following operation or control detect the operational risk event and original loss is remedied so the effect of successor cause represented by the addend is –L and is obviously correlated to L. But when the effect is represented by multiplier, the effect multiplier of successor cause is 0 and is uncorrelated to the original loss. Multipliers imply the correlations between the loss effects of causes, so they can be considered as independent to previous loss, but they relate to the categories of the successor and the previous cause.

Secondly, successor causes are generally control or process. Multipliers can reflect the risk control function of successor causes and also can reflect successor causes efficiency and contribution of risk management.

Thirdly, in practice in order to measure work performance of each control point, there are some similar records and indexes in the financial companies already and most of them are relative indexes. All these existing data can help operational risk topological data model to accumulate data. And the relevant experts may be more confident in giving out the subjective estimation of multipliers with being familiar with those relative indexes.

Lastly, when subjective data are needed, experts are easy to offer them in multiple or proportion. It is hard for them to offer estimation of the absolute value.

Operational risk topological data model also need some free words record which assist to describe the loss event process, details, event handling result and internal/external environment, etc.. And the record also needs to include the risk event happening time and discoverable time.

A piece of record to the event caused by a trigger cause is a sample of operational risk frequency. Theoretically, there must be a initial trigger cause that can not be disassembled in every operational risk event, but a normative operational risk causes table is necessary to accumulate operational risk data and build data base. If the categorization of trigger causes is too rough, it obviously can not meet the need of operational risk measurement and management. On the contrary, if the categorization is too fine, it will enhance the cost of data accumulating and the difficulty of getting enough data for statistic and is not convenient to remember and use. So the operational risk causes should be categorized modestly.

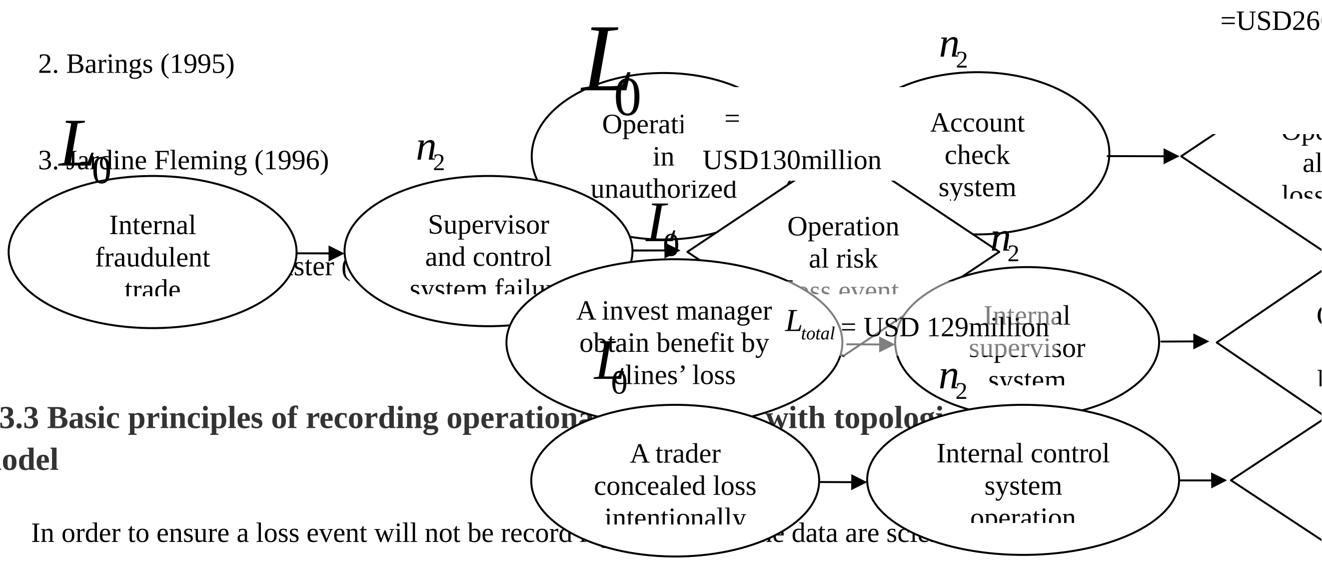
2.3.2 Operational risk topological data model illustration

Several famous operational risk loss events[1] in the international financial fields are recorded by the operational risk topological data model as followed for illustration. However it is a pity that the each cause's effect to the loss has not been found and the risk causes as the nodes can not be ranged into the normal causes with standard definitions for lack of information.

1. Sumitomo (1996)

2. Barings (1995)

3. Jardine Fleming (1996)



2.3.3 Basic principles of recording operationa... with topologi... model

In order to ensure a loss event will not be record... the data are sci... effective to the operational risk measurement, recording operational risk events with topological data model should be on the following basic principles.

1. Repeated failures are usually considered as separate, single events (high frequency events), even if they are made by a single individual and are due to a common misunderstanding, lack of training etc..
2. Multiple impacts from a single failure (e.g. many mis-priced transactions from a single incorrect price or reference data) are considered a single event.
3. Intentional activities and unintentional activities

The activities leading up operational risk events can be grouped into two types: intentional

[1] Jack L. King. Operational risk: Measurement and Modeling. John Wiley&Sons,Ltd.,2001
Hans U. Doerig. Operational Risk in Financial Services. Credit Suisse Group, April 2003

activities and unintentional activities. There is a special kind of intentional activities that is premeditation. In that situation, seemingly independent losses are connected by a common plan of action in fact, so they are considered as a single event whose operational risk cause is classified into internal fraud and external fraud.

If more than one unintentional activity lead to the loss at the same time, they are considered as coincidence and random phenomena. So they are recorded as different operational risk events separately.

4. Inartificial operational risk events are recorded on the same principle as events caused by unintentional activities.
5. Control/process type of cause and non-control/process type of cause

The control and process of a company generally had no bugs or were thought of having no bugs when they were designing. But after they were implemented for a period of time, some problems may be realized and even some loss events occurred. This is because the designers' knowledge is limited and the internal or external environment keeps changing. So even in the century-old financial companies, the control and process keep being improved and developed. Even if the control and processes have no bugs, when the trigger causes led to loss, the operational mistakes and offences of relevant control and processes will indulge the loss to the finial loss event. So the failures of control and processes are a significant type of operational risk cause.

Therefore, operational risk causes are classified into two types: control/process type of causes and non- control/process type of causes. Non- control/process type of causes mainly perform as trigger causes, while control/process type of causes perform as the successor causes which may magnify the initial loss caused by trigger cause when the relevant control and processes are failed or minify the initial loss when the relevant control and processes are effective.

The recording of operational risk events may have two types of topological structures: one is only having the trigger cause nodes and the other is having control/process type of causes as the successor of trigger cause nodes.

6. Every piece of operational risk record with topological data model must be a topological section of the company's operational risk influence diagram. In this way, it will be a effective sample for operational risk measurement by influence diagrams approach. Otherwise, it provides the information for the improvement of the operational risk influence diagrams.

7. Although the failures of control and processes seem as risk factors sometimes, the operational risk topological data model only concern about risk causes and their effect to the loss in stead of risk factors.

8. Using operational risk topological data model can help the company find the bugs of its operational risk control and operational processes. When a bug was found and the relevant control or process was improved, the loss data caused by this bug can be deleted out of the data base to reflect the improvement.

Operational risk is very complicated and variable, so there may be some special events which are not included above and which even didn't occur. Therefore, operational risk topological data model recording methods still needs modification and improvement in the practice.

2.3.4 Advantages of operational risk topological data model

Operational risk topological data model has the following advantages:

1. Digging more information out of operational risk historical data

Every operational risk historical event is precious because it contains the information useful to the operational risk management and measurement. Operational risk topological data model can assist to dig out more information because it not only records the loss, cause and occurrence time, and also use the topological structure to record the happening processes of operational risk events. These data supply samples for the statistic of correlation and conditional independence between operational risk causes.

2. Focusing on the failure of control and process

The failures of control or processes occurred in most of operational risk events that led huge loss as the four famous operational risk loss events illustrated in 2.4.2.. This is one of the traits of operational risk. The operational risk topological data model and the following influence diagrams measurement approach both focus on the failure of control and process. They can help the company detect the failures in the control and processes and calculate the loss caused by those failures. It is very useful to the operational risk management.

3. Representing a new effective risk analysis method and process

The topological new data model is not only a new method of operational risk's description and recording, but also represents a new risk analysis method and process. It helps to analyze the happening process of operational risk events, the interaction between every operational risk cause and each cause's effect to the loss. These reflect the correlations between each operational risk cause and simplify the summing of operational risk (see 4.1).

4. Having more dimensions

The depth and agility of using to the data resource are decided by the dimensions in some extent. In fact, the operational risk topological data model increased more plentiful dimensions on the basis of general operational risk data model.

3. Operational risk measurement: influence diagrams approach based on topological data model

3.1 Influence diagrams introduction

Influence diagrams are the directed acyclic graphs (DAGs) composed of nodes and arcs. The nodes represent the variables in the research problem, and the directed arcs represent the relations between the variables. Influence diagrams are a kind of visual graphs figuring the structure of the problem, the relations between the variables and especially the conditional independence of the variables and the information flows. Influence diagrams have two levels: the first one is diagram; the second one is the data structure of each note.

The general form of the relations between a group of variables is their joint probability. Suppose that there are n variables in the research problem which are $x_1, x_2, \cdots, x_n$, and their joint probability $Pr(x_1, x_2, \cdots, x_n)$ represents the information about the relations among the variables. According to the rule of probability train, the joint probability can be spread into $n!$ expansions which are products of different marginal probability and conditional probability. Equation 3.1 is one of the forms. Each spread equation is correspond to a probability assessment order which is represented by a group of directed arcs in the influence diagram.

$$Pr(x_1, x_2, \cdots, x_n) = Pr(x_1)Pr(x_2 | x_1) \cdots Pr(x_n | x_1, x_2, \cdots, x_{n-1}) \quad (3.1)$$

Depends on the components and the purpose, influence diagrams can be categorized into decision influence diagrams and probability influence diagrams. The former is the influence diagrams with decision nodes, while the latter only have chance nodes and determination nodes. In the probability influence diagrams,

(1) Chance nodes and determination nodes represent separately the random variables and the determination variables. Directed arcs indicate the possible probability dependence.

(2) The data of each node are the node's value and its conditional probability depended on the possible state of its predecessor.[1]

The operational risk influence diagram constructed in this paper is a probability influence diagram. The operational risk decision influence diagram can be constructed on the basis of the operational risk influence diagram which can assisted the operational risk management decision. (See 3.4)

3.2 Operational risk measurement with influence diagrams approach

To use operational risk influence diagrams approach measure operational risk should be based on the following hypotheses：

1. The company is willing to manage operational risk.
2. There are rational and effective systems supporting the accumulation of operational risk date with the topological data model.
3. Once an operational risk event was triggered, it is discovered by effective processes or otherwise its loss was realized at last. (See 4.1.2)

On the basis of operational risk topological data model, the construction of influence diagram is to identify all the possible trigger causes as the decision nodes in the operational risk influence diagrams, take their successor cause as the chance nodes and the 'operational risk event' as the final node. Therefore, each path from a trigger cause node through successor cause nodes to the final node or from a trigger cause node directly to the finial node is the abstract to happening processes of a type of operational risk events, and the whole operational risk influence diagram is a network combined of the paths abstracting all type of possible operational risk events in the company. Every operational risk event recorded with the topological data model is a sample of a certain path in the operational risk influence diagram of the company.

A company's original operational risk influence diagram can be constructed by logical analysis and deep interview with experts. Then risk managers can reform the existing operational risk data from the general data model to the topological data model, and modify the original operational risk influence diagram by the reformed data.

One of the important traits of operational risk influence diagrams is the dynamic development which fits operational risk's traits. The operational risk events are diversified and vary with the change of management decisions and the internal environments of the company. Consequently, the operational risk influence diagram has to be modified constantly. The operational risk topological data model can provide the information to the modification and support the operational risk measurement with the new influence diagram.

The objective data are always more convictive than the subjective data which are though more agile and more predictive. When the operational risk measurement is used to allocate the economic capital or calculate capital adequacy ratio, the company has the motivation to influence the internal experts to offer the fake estimation for saving the capital. So the objective data is the

[1] Zhan Yuanrui, influence diagrams theory and application, Tianjin University Publication, 1995.11

must of operational risk measurement especially for supervision. Influence diagrams approach based on topological data model can dig out and make the use of the information contained in the history data more completely. If the data are enough, the operational risk measurement with influence diagrams approach can be based on objective data entirely. Certainly, the influence diagrams approach also suits for the subjective data, and the topological data model can assist experts' estimation.

At the begging of the operational risk measurement study, people once doubted the measurability of operational risk because of the data absence. With the continuous efforts and practice of some famous financial institutions in the accumulation of operational risk data, the operational risk measurement and management based on the objective data has been accepted widely. So although there is no datum of the conditional distribution of the operational risk sub-category now, the operational risk measurement using objective data with influence diagrams approach based on topological data model will be a effective operational risk assessment approach in stead of an air castle if this new data model is accepted abroad and the operational risk data are recorded by it.

3.3 Operational risk influence diagrams construction

As mentioned above, influence diagrams have two levels -- diagram of topological network and data structure of each note. The circulatory processes of influence diagrams construction are demonstrated in Figure 3.1.

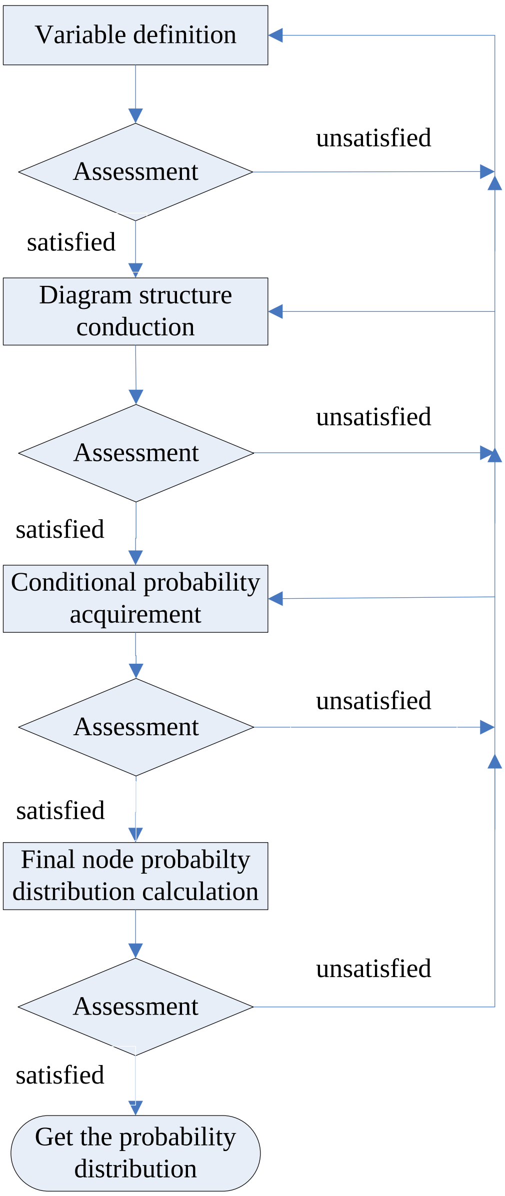


Figure3.1influence diagrams construction processes

The first step of influence diagrams construction is variables definition. The variables of operational risk influence diagrams are all the operational risk causes in the company. The identification and definition of operational risk causes are the foundation of operational risk management and operational risk data accumulation, so the decision of variables should be circumspective and should consider of the following aspects:

- ✓ Traits of operational risk in the company;
- ✓ Businesses and operation processes of the company;
- ✓ The available external loss data;
- ✓ Convenient to obtain operational risk data;
- ✓ Able to offer necessary information for operational risk management.

The key of operational risk influence diagrams construction is to identify the conditional independence between the nodes, which can simplify the topological structure of the operational risk influence diagrams and reduce the necessary amount of data for operational risk measurement. According to the investigation, the logical analysis and the basic principles of recording operational risk events with topological data model (see2.3.3), we bring forward the following basic guidelines which the influence diagrams construction needs:

1. The loss event caused by more than one operational risk cause because of premeditation should be considered as a single event whose risk cause should be categorized into internal fraud or external fraud. So the possible correlations between some operational risk causes arise from premeditation need not to be considered.

2. The situations in which the same subject adopts the unauthorized or deregulation operation to hide or remedy the operational risk event happened already and lead to a more severe loss event will not occur if the operational risk incentive mechanism and the audit approached are effective.[1] It is hypothesized that the operational risk incentive mechanism and the audit approached are effective in the company. Therefore, more conditional independence can be established.

3. There is no evidence found for the correlations between the marginal distributions of trigger causes by logical analysis and interviews to practical experts until now. So it is hypothesized that the marginal distributions of trigger causes are independence. That means there is no directed arc between trigger cause nodes. When the objective data are enough to the statistic analysis of correlations, the hypothesis can be tested.

4. The operational risk causes are classified in to two types (see 2.3.3), so the nodes in the operational risk influence diagrams are correspondingly classified into two types: trigger cause nodes and control/process type of cause nodes. To simplify the diagrams, the operational risk influence diagrams with control/process type of cause nodes and without control/process type of cause nodes can be constructed separately, and they can be combined directly into one complete operational risk influence diagram if needs.

5. By the analysis of the processes, it can be aware that which category of operational risk events can be detected in which process, and these connections are represented by directed arcs.

6. The complication of operational risk influence diagrams will affect the analysis of the problem, the data accumulation, the efficiency of calculation and the measurement cost, so it is a very important management decision problem.

We constructed the operational risk influence diagrams of the insurance company for illustration (see Figure 3.2 and Figure 3.3). The general operations in insurance companies are classified into six basic progresses which are underwriter process, claim verification process, maintenance process, capital application process, finance process and slip management process. These processes can be disassembled finer according to the special practice of each insurance company.

[1] Zhang Qinghong, Zhao Lei. A Study of Incentive Mechanism for Operational Mistakes Control in Financial Services Organizations. Asia-Pacific Risk and Insurance Association 10th Annual Conference, Tokyo, Japan, 2006.8

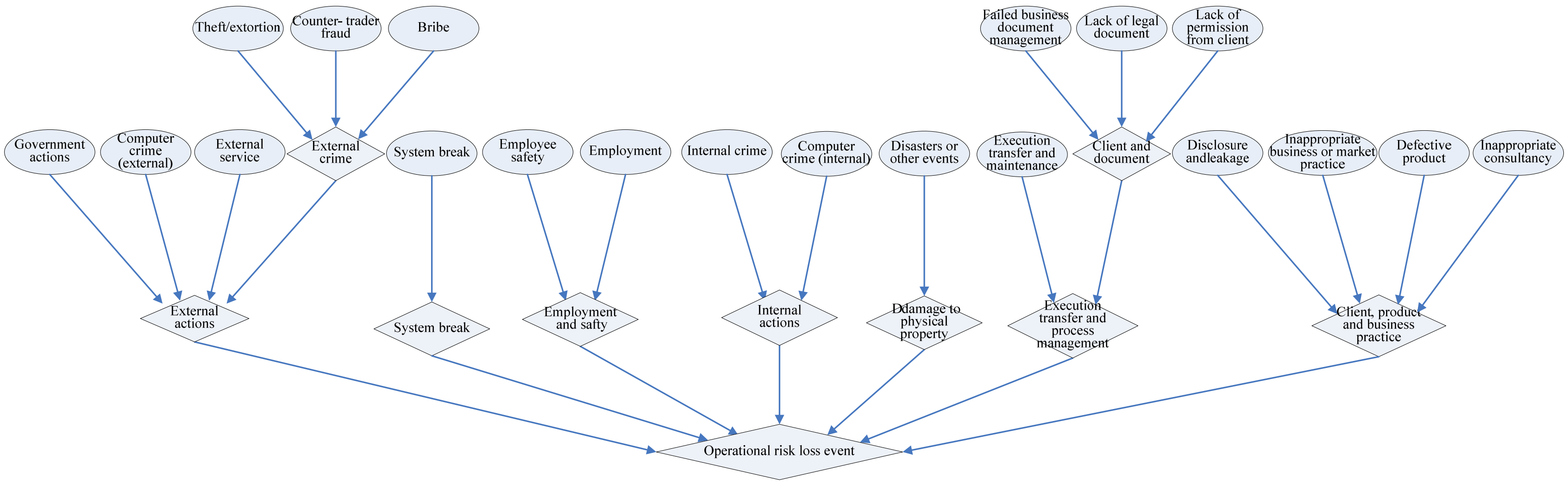


Figure3.2 operational risk influence diagram without control/process type of successors

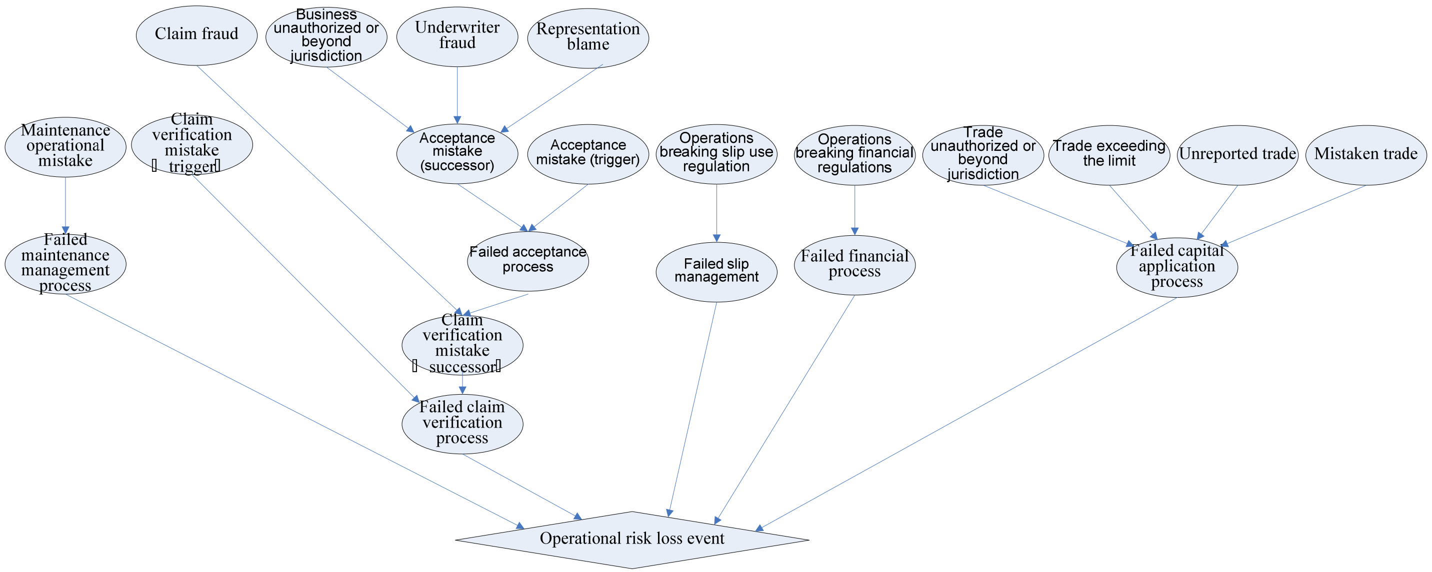


Figure3.3 operational risk influence diagram with control/process type of successors

3.4 Advantages of influence diagrams approach

Influence diagrams approach has the following advantages compared with the operational risk measurement approaches based on general operational risk loss data.

1. The diagram structure can reflect the correlations and conditional independence between the operational risk causes qualitatively and quantitatively. So it is convenient for analysis and calculation.

2. It can support the modification and improvement according to new information. The topological character of influence diagrams permits adding or deleting variables without influence to the other part of the network and the topological transformation.

3. Influence diagrams calculation can entirely use objective data accumulated by topological data model, and also can use subjective data or both kind of data agilely together. So the choice of data is more flexible and can take the advantages of both kinds of data.

4. It can improve the suitability of external data. Since operational risk influence diagrams reflect the action progress of operational risk causes and they support the research to the part, the sections unrelated to the company's individuality can be found to use the corresponding external data.

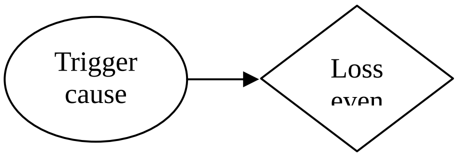
5. It supports the operational risk management decision. Operational risk influence diagrams are probability diagrams. They can be reconstructed into decision diagrams by adding the decision nodes. The risk resulted from each cause especially the control/process type of cause can be calculated with the influence diagrams approach. So the risk decrease result from the control of each operational risk cause (e.g. to improve the effectiveness of a certain progress) can be got. With risk decrease and the control cost known, the operational risk management decision influence diagrams can calculate out the optimal decision.

4. Operational risk influence diagrams calculation based on topological data model

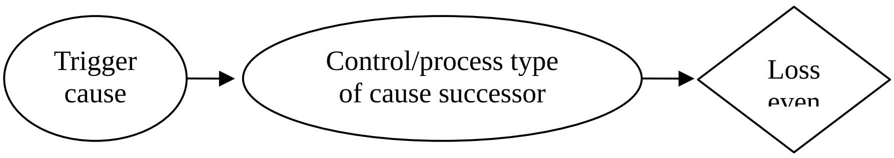
4.1 Operational risk influence diagrams calculation

Operational risk influence diagrams can not only assist the analysis of correlation between all the causes, but also can support the statistic analysis of operational risk data based on the topological data model. The calculation of operational risk influence diagrams has three steps: firstly, calculating the company's operational risk loss severity distribution by the influence diagrams probabilistic inference with the data of initial loss and effect multipliers; secondly, calculating the company's operational risk loss frequency distribution by the frequency of trigger causes in the influence diagrams; lastly, combining them into an aggregated operational risk loss distribution.

To get the operational risk loss severity distribution needs to calculate loss distribution of each path separately and add them together. Each path in the operational risk influence diagram pictures a kind of operational risk events caused by the same series of risk causes. The paths can be grouped into tow types: one represents the events which arise only from the trigger cause, as the situation in Figure4.1.This type of paths exist in the operational risk influence diagram without control/process type of successors (Figure3.2). The other type of paths exists in the operational risk influence diagram with control/process type of successors (Figure3.3). Its character is the existence of more than one risk cause successor after the trigger cause node. The simplest path of this type is showed as the situation in Figure4.1, which only has one control/process type of successor.



Situation



Situation

Figure4.1 operational risk influence diagrams basic paths

In Situation , the happening process of the operational risk event which affected by one of the control/process type of operational risk causes also has two situations:

Situation A: If the control/process type of operational risk cause j (hereinafter referred to as "Process j ") has no failure and be performed correctly, the loss event touched off by Trigger cause i can be discovered. This situation is recorded as G_{ij}, its probability is $P(G_{ij}) = q_{ij}$.

Situation B: Even if Process j has no failure and be performed correctly, the loss event touched off by Trigger cause i can not be discovered. This situation is recorded as $\overline{G}_{ij}$, its probability is $P(\bar{G}_{ij}) = 1 - q_{ij}$ 。

q_{ij} only related to the operational risk cause types of Trigger cause i and Process j .

In Situation , the loss distribution of the path is just the loss distribution of the event touched off by Trigger cause i. So the following research will focus on the calculation of the path's loss distribution in Situation .

4.1.1 Loss severity distribution calculation

The two situations of Situation will be calculated separately and then added together to get the path's loss distribution in Situation .

Situation A:

In Situation A, there still two possible situations. One is that the loss event touched off by Trigger cause i is discovered by the Process j because of the correct performance of Process j . It is called Process j effective. The other is that loss event touched off by Trigger cause i is not discovered by the Process j because of the failure performance of Process j . It is called Process j ineffective.

When Trigger cause i touched off an operational risk event, if Process j is effective, the effect of Process j to the loss caused by Trigger cause i is described by a minification which is a random variable and its probability density function is $f_y(n_{ij} \mid yes)$; while if Process j is ineffective, the effect of Process j to the loss caused by Trigger cause i is described by a magnification which is a random variable and its probability density function is $f_n(N_{ij} \mid no)\ (N_{ij} \geq 1)$. The minification and the magnification are called jointly the effect multiplier of Process j to Trigger cause i .

Therefore, every control/process type of risk cause has two random variables which only related to the operational risk cause types of itself and its predecessor and can be got from statistics of operational risk data collected in topological data model although it needs to accumulate historical data during a period of time.

To simplify, hypothesize the effect multiplier is a random variable C_{ij} which obey binomial distribution with the means of $f_y(n_{ij} \mid yes)$ and $f_n(N_{ij} \mid no)$ as the two points.

Let $E[f_y(n_{ij} \mid yes)] = n_{ij0}$, $E[f_n(N_{ij} \mid no)] = n_{ij1}$.

So, in the condition of an operational risk event touched off by Trigger cause i , Process j is effective on the probability p_j and its effect multiplier is $C_{ij} = n_{ij0}$, while Process j is ineffective on the probability $1 - p_j$ and its effect multiplier is $C_{ij} = n_{ij1}$. p_j only related to the operational risk cause type of Process j .

C_{ij}	n_{ij0}	n_{ij1}

P_r	p_j	$1-p_j$

If an operational risk loss event has occurred, let the distribution of the initial loss caused by Trigger cause i be $f_{L_i}(l)$ ($f_{L_i}(l)$ only related to Trigger cause i) and let the final loss of the event in Situation A be X_{ij} whose distribution is $F_{X_{ij}}(x)$. Then

$$
\begin{aligned}
F_{X_{ij}}(x) &= P(X \le x) \\
&= P(X \le x \mid C_{ij} = n_{ij0}) \cdot P(C_{ij} = n_{ij0}) + P(X \le x \mid C_{ij} = n_{ij1}) \cdot P(C_{ij} = n_{ij1}) \\
&= P(L_i \le \frac{x}{n_{ij0}} \mid C_{ij} = n_{ij0}) \cdot p_j + P(L_i \le \frac{x}{n_{ij0}} \mid C_{ij} = n_{ij1}) \cdot (1-p_j) \\
&= P(L \le \frac{x}{n_{ij0}} \mid C_{ij} = n_{ij0}) \cdot p_j + P(L \le \frac{x}{n_{ij0}} \mid C_{ij} = n_{ij1}) \cdot (1-p_j)
\end{aligned} \tag{4.1}
$$

The effect multiplier C_{ij} is independent of the initial loss L , so

$$
\begin{aligned}
F_{X_{ij}}(x) &= P(L_i \le \frac{x}{n_{ij0}}) \cdot p_j + P(L_i \le \frac{x}{n_{ij0}}) \cdot (1-p_j) \\
&= F_{L_i}(\frac{x}{n_{ij0}}) \cdot p_j + F_{L_i}(\frac{x}{n_{ij1}}) \cdot (1-p_j)
\end{aligned} \tag{4.2}
$$

The probability density function of the finial loss X_{ij} in Situation A is

$$
f_{X_{ij}}(x) = f_{L_i}(\frac{x}{n_{ij0}}) \cdot \frac{p_j}{n_{ij0}} + f_{L_i}(\frac{x}{n_{ij1}}) \cdot \frac{1-p_j}{n_{ij1}} \tag{4.3}
$$

Situation B:

Since the operational risk event triggered can not be discovered in Situation B, the probability density function of the finial loss X_{ij} in Situation B is

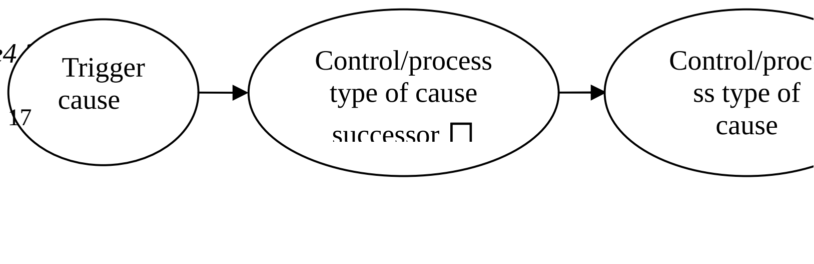
$$
g_{X_{ij}}(x) = f_{L_i}(l) \tag{4.4}
$$

Now the loss severity distribution in Situation A and Situation B were got. So the loss severity distribution in Situation (Figure4.1) is

$$
\begin{aligned}
h_{X_{ij}}(x) &= [\, f_{L_i}(\frac{x}{n_{ij0}}) \cdot \frac{p_j}{n_{ij0}} + f_{L_i}(\frac{x}{n_{ij1}}) \cdot \frac{1-p_j}{n_{ij1}}] \cdot q_{ij} + f_{L_i}(x) \cdot (1-q_{ij}) \\
&= f_{L_i}(\frac{x}{n_{ij0}}) \cdot \frac{p_j q_{ij}}{n_{ij0}} + f_{L_i}(\frac{x}{n_{ij1}}) \cdot \frac{(1-p_j) q_{ij}}{n_{ij1}} \cdot + f_{L_i}(x) \cdot (1-q_{ij})
\end{aligned} \tag{4.5}
$$

If a path has two successors after the trigger cause node, it can be considered as a path adding a new control/process type of cause node after Process j in Situation . We can take the loss severity distribution of the first part as the initial distribution of the new node, and calculate the finial distribution as above. The following calculation is the demonstration of this kind of paths as Situation (Figure4.2). The finial distribution of a path with more than two successors can be calculated as the same way assisted by computer programs.

Figure4



Let the finial loss distribution after Process j is $f_{L_j}(l)$.

$$f_{L_j}(l) = h_{X_j}(x) = f_{L_i}(\frac{x}{n_{ij0}}) \cdot \frac{p_j q_{ij}}{n_{ij0}} + f_{L_i}(\frac{x}{n_{ij1}}) \cdot \frac{(1-p_j)q_{ij}}{n_{ij1}} \cdot + f_{L_i}(x) \cdot (1-q_{ij}) \qquad 4.6$$

Let the effect multiplier of Process k to its predecessor is a random variable C_{jk} which obey binomial distribution.

C_{jk}	n_{jk0}	n_{jk1}
P_r	p_k	$1-p_k$

So the finial loss distribution of Situation is

$$h_{X_k}(x) = f_{L_j}(\frac{x}{n_{jk0}}) \cdot \frac{p_k q_{jk}}{n_{jk0}} + f_{L_j}(\frac{x}{n_{jk1}}) \cdot \frac{(1-p_k)q_{jk}}{n_{jk1}} \cdot + f_{L_j}(x) \cdot (1-q_{jk}) \qquad 4.7$$

Substitute equation 4.6 into equation 4.7:

$$\begin{aligned} h_{X_k}(x) = {} & f_{L_i}(\frac{x}{n_{ij0}n_{jk0}}) \cdot \frac{p_j q_{ij}}{n_{ij0}} \frac{p_k q_{jk}}{n_{jk0}} + f_{L_i}(\frac{x}{n_{ij1}n_{jk0}}) \cdot \frac{(1-p_j)q_{ij}}{n_{ij1}} \frac{p_k q_{jk}}{n_{jk0}} \\ & + f_{L_i}(\frac{x}{n_{ij0}n_{jk1}}) \cdot \frac{p_j q_{ij}}{n_{ij0}} \frac{(1-p_k)q_{jk}}{n_{jk1}} + f_{L_i}(\frac{x}{n_{ij1}n_{jk1}}) \cdot \frac{(1-p_j)q_{ij}}{n_{ij1}} \cdot \frac{(1-p_k)q_{jk}}{n_{jk1}} \\ & + f_{L_i}(\frac{x}{n_{jk0}}) \cdot (1-q_{ij}) \frac{p_k q_{jk}}{n_{jk0}} + f_{L_i}(\frac{x}{n_{jk1}}) \cdot (1-q_{ij}) \frac{(1-p_k)q_{jk}}{n_{jk1}} \qquad 4.8 \\ & + f_{L_i}(\frac{x}{n_{ij0}}) \cdot \frac{p_j q_{ij}}{n_{ij0}} (1-q_{jk}) + f_{L_i}(\frac{x}{n_{ij1}}) \cdot \frac{(1-p_j)q_{ij}}{n_{ij1}} \cdot (1-q_{jk}) \\ & + f_{L_i}(x) \cdot (1-q_{ij})(1-q_{jk}) \end{aligned}$$

The basic principles of recording operational risk events with topological data model (see 2.3.3) and the operational risk influence diagram construction (see3.3) can ensure the independence of loss severity distributions of every path in the operational risk influence diagrams. Thus, the company's whole operational risk loss severity distribution can be calculated by Convolution formula or Monte Carlo method after the distributions of every path in the operational risk influence diagram.

4.1.2 Loss Frequency distribution calculation

Based on the 3rd hypothesis in 3.2, all the operational risk events can be detected by the company. So the trigger frequency of each trigger cause can be obtained using the data recorded with the topological data model and the total operational risk events frequency equals to the summation of all the trigger frequency of every trigger cause in the operational risk influence diagram. Furthermore, the basic principles of recording operational risk events with topological data model (see 2.3.3) and the operational risk influence diagram construction (see3.3) can ensure the independence of the trigger frequency of each trigger cause. Thus, the company's whole operational risk loss frequency can be calculated by adding up the frequency of every trigger cause, and then the loss frequency distribution can be acquired.

4.1.3 Operational risk loss distribution calculation

Having calculated separately both the severity and frequency processes, we now need to combine them into one aggregated loss distribution (see Figure 4.3).

If the annual amount of operational risk events is n which is a scattering distribution function $b(n)$ and the annual loss in the condition of a certain n is x which is a continuous distribution function $g(x|n)$, the annual loss is a composite probability distribution function $f(x)$.

$$f(x) = \sum_{n=0}^{\infty} b(n)g(x|n) \qquad 4.9$$

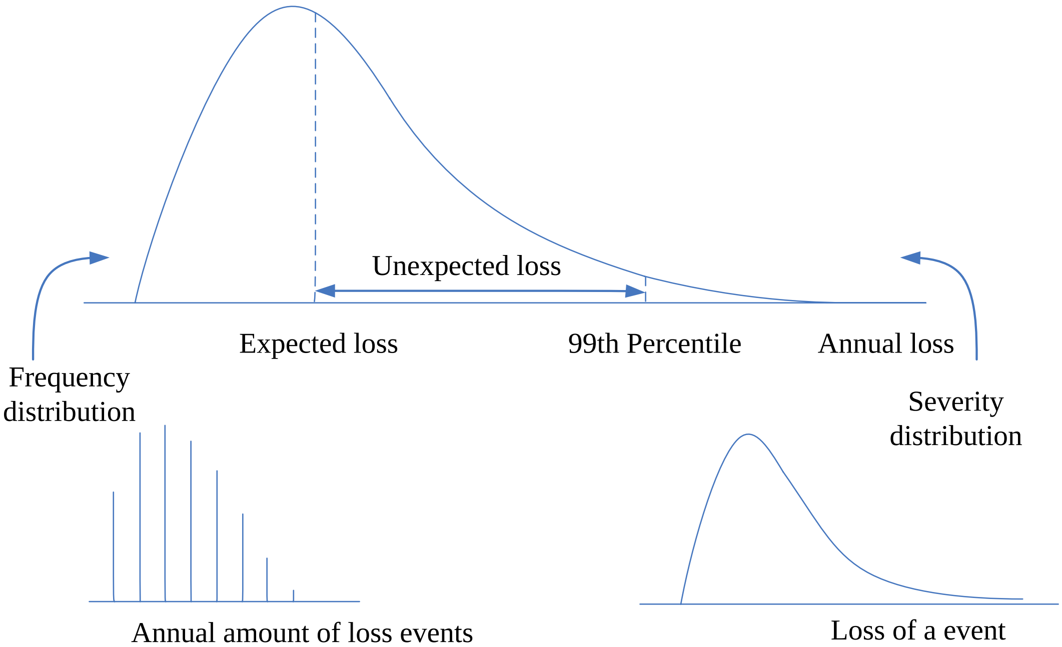


Figure4.3 Combination of the severity and frequency distribution

Generally, we hypothesize the loss severity distribution is independent of the loss frequency distribution. Then the aggregated operational risk loss distribution can be calculate by Monte Carlo method.

When there are obvious evidences which can prove the correlations between the loss severity distribution and the loss frequency distribution, the aggregation can be implemented by the improved simulate method which can deal with some special correlations.[1]

4.2 Trigger cause initial loss distribution acquirement

The calculation of operational risk influence diagrams needs to kwon the distribution $f_{L_i}(l)$ of the initial loss L_i caused by Trigger cause i. There are two approaches to get $f_{L_i}(l)$: using objective data and using subjective data.

The method to record the initial loss caused by trigger causes in the operational risk

[1] Carol Alexander, Statistical Model of Operational Risk Loss, Operational Risk, Pearson Education Limited, 2003

topological loss data model is same as which to record the loss in general operational risk data model. So the statistical approaches of the objective data can take advantage of the existing outcomes of operational risk measurement.

The models suitable for the initial loss distribution includes some parametric distributions which are Normal distribution, Lognormal distribution, Exponential distribution, Weibull distribution, Gamma distribution, etc., and some non-parametric distributions which are histogram and disjunction distribution, or simple random sampling with inputted data.[1]

If there are not enough objective data for statistics, the subjective data will be needed. Operational risk managers can invite relevant experts in the company to estimate the trigger cause initial loss objective distribution. In fact, the distribution format should be decided at first, and then the experts can estimate the parameters of the distribution in stead of the whole distribution. Commonly used distributions suitable for operational risk in subject estimation are Triangular distribution (Figure 4.4), Transformed triangular distribution (Figure 4.5), BetaPERT distribution (Figure 4.6), General distribution (Figure 4.7), etc.

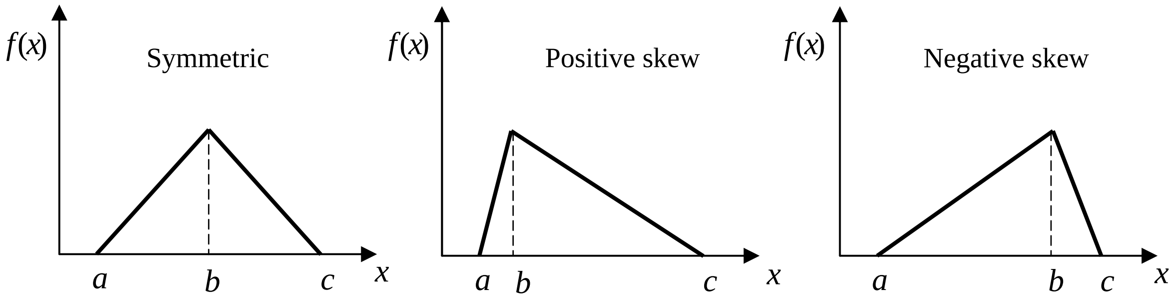


Figure4.4 Triangular distribution

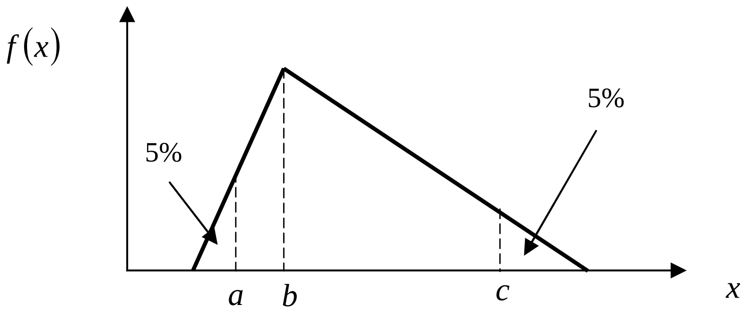


Figure 4.5 Transformed triangular distribution $Trigen(a,b,c,5\%,95\%)$

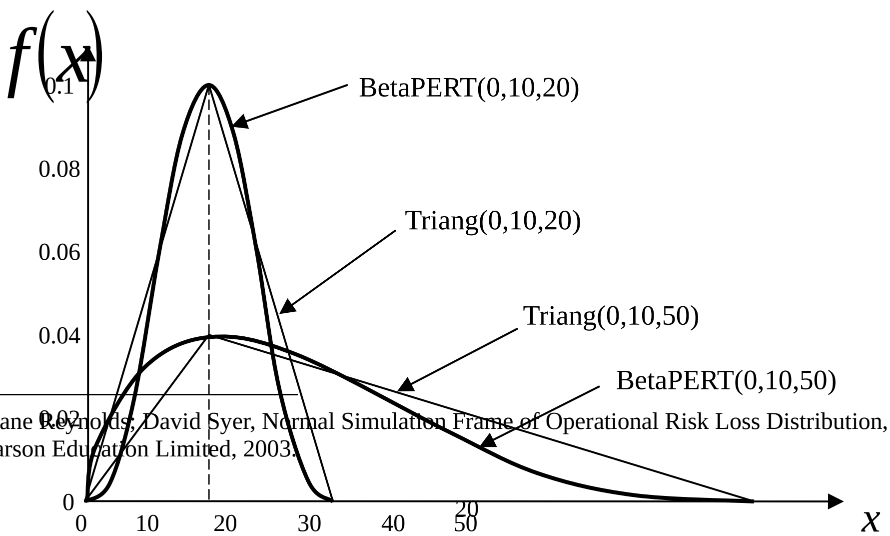


Figure4.6 Comparison of BetaPERT distribution and triangular distribution with the same parameter

[1] Diane Reynolds, David Syer, Normal Simulation Frame of Operational Risk Loss Distribution, Operational Risk, Pearson Education Limited, 2003.

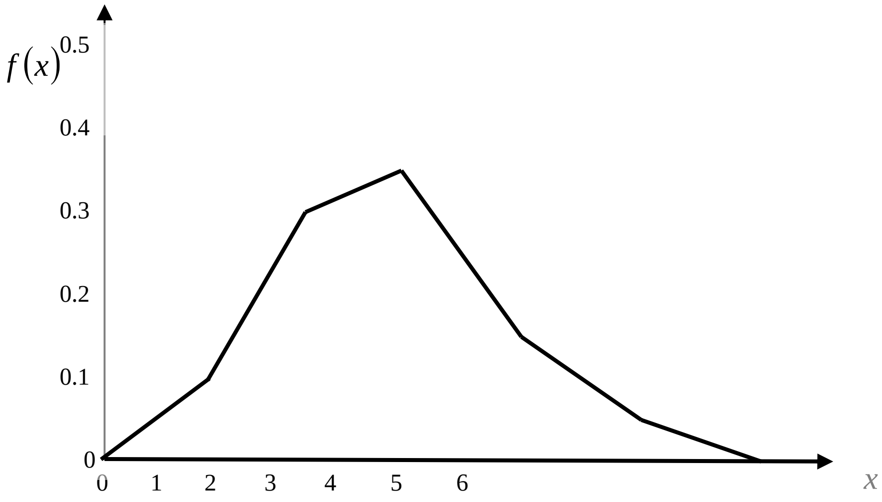


Figure4.7 Probability density function of $General(0, 6, \{1, 2, 3, 4, 5\}, \{2, 8, 9, 4, 1\})$

The BetaPERT distribution is recommended to descript the subjective estimation of the initial loss distribution of operational risk trigger cause. Because:

1. The shape of BETAPERT distribution varies as the changes of its parameters. It suits the diversity of the operational risk severity distribution. It can characterize the asymmetry and tail heaviness of the operational risk severity distribution.

2. The BETAPERT distribution has a complete distribution function in stead of subsection function, so it is convenient for the formulas deduced above.

3．There are only three parameters in BETAPERT distribution, so it is easier for expert to estimate.

To help the estimate, the operational risk managers can use the Verbal and Numerical Probability Scale (Figure4.8) which appears to be an aid for researchers and domain experts in estimation.

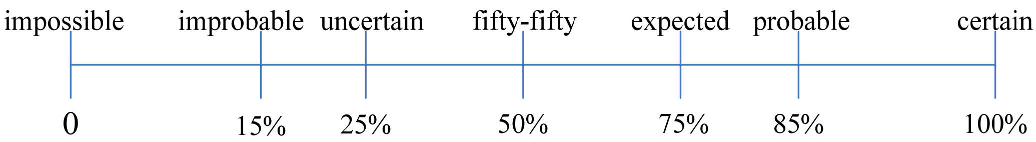


Figure4.8 Verbal and Numerical Probability Scale

The linear opinion pool and logarithmic opinion pool usually are used to combine the experts' opinions. The experts' weights of the estimations of parameters in each trigger cause loss distribution should be decided by the experts' familiarity to the corresponding businesses or operations.

5. Conclusion

Operational risk topological data model can not only contain the information about the connection between all risk causes during the risk event happening process, but also can support the operational risk measurement by influence diagrams approach. In the operational risk topological data model, a risk loss event is attributed to risk trigger cause which lead to the initial loss and successor causes which affect the initial loss. Operational risk topological data model is composed of the graph and node data. The graph pictures how each risk cause leads to the loss event. The nodes data includes the initial loss and the loss effect multiplier of successor cause to its previous causes.

Operational risk topological data model having more dimensions and representing a new effective risk analysis method and process focus on the failure of control and process and can dig more information out of operational risk historical data.

Influence diagrams approach based on topological data model is a new effective method of integrated analyses and assessment of operational risk. It simplified the analysis of correlation between all the causes and the statistic analysis of operational risk data. The three steps of calculating operational risk influence diagrams is that calculating the company's operational risk loss severity distribution by the influence diagrams probabilistic inference; calculating the company's operational risk loss frequency distribution by the frequency of trigger causes; and combining them into an aggregated operational risk loss distribution.

Influence diagrams approach not only has advantages in operational risk measurement, but also can be modified and improves agilely according to new information, improve the suitability of external data and support the operational risk management decision.

The Operational risk topological data model and the influence diagrams approach of operational risk measurement are entirely new concepts. They still need modification and improvement in the practice. The operational risk measurement by influence diagrams approach based on topological data model with objective data will be an effective operational risk assessment approach in stead of an air castle only if this new data model is widely accepted and used to accumulating operational risk data.

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