

# Apples and pears: the comparison of risk capital and required return in financial institutions\*

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February 2007

## Abstract

Risk capital is the contribution of an exposure to the default risk of a financial institution. We investigate its relationship with required shareholder returns, showing that the use of return on risk capital (RAROC) as a risk-adjusted performance measure is inconsistent with the standard theory of financial valuation and that using this one measure to represent at the same time both contribution to default risk and required shareholder returns can lead to substantial loss of shareholder value. We propose an alternative performance measure distinguishing these two aspects of risk and applicable to the efficient allocation of risk capital.[99 words]

Journal of Economic Literature number: G21

Keywords: Risk Management, Economic Capital, Performance Measurement, Financial Regulation.

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\*We are grateful for comments from audiences at Cass Business School, the University of Bergamo, Bocconi University and from two anonymous referees. Any remaining errors are our own responsibility.

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# 1 Introduction

Perhaps the greatest change in the management of financial institutions during recent years has been the widespread adoption of risk-adjusted performance measurement (RAPM).<sup>1</sup> The most widely used generic RAPM is return on ‘economic’ capital  $r_{RC}$  (often also referred to as RAROC after the first measure of this kind developed by Banker’s Trust in the 1980’s).<sup>2</sup> This is computed as:

$$r_{RC} \equiv \frac{\text{Expected Revenues} - \text{Costs} - \text{Expected Losses}}{\text{Risk Capital}} \quad (1)$$

where expected revenues are the anticipated receipts from the business activity or exposure, costs are the anticipated funding and operational costs and expected losses are the predicted level of losses (arising amongst other reasons from loan default, loss of capital value, and operational problems).

The denominator – what we call risk capital but which is perhaps more often referred to as ‘economic’ capital – is the contribution made by a particular business activity or exposure to the total capital required to protect the financial institution against risk of default, usually computed in a value-at-risk framework for some appropriate confidence threshold and time horizon e.g. 99.9% avoidance of default over a one-year horizon.  $r_{RC}$  is then used a measure of the ‘risk-return’ trade-off in many business applications – for comparing the contributions to shareholder value of different lines of busi-

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<sup>1</sup>Matten (2000, pp 146-166) describes several different performance measures used in the banking industry. The major consultancy companies now devote a great deal of effort to helping banks and other financial institutions develop these measures and consultancy company websites are a good source of current practitioner thinking. For a collection of recent views on the modelling and allocation of risk capital see Dav (ed) (2006).

<sup>2</sup>Smithson (2002), page 266, reports that 78% of the respondents to his 2002 Rutter Associates survey of credit portfolio management practices, used RAROC to evaluate the performance of their portfolio of credit assets.

ness, as an input into remuneration and bonuses, and for setting minimum prices for financial products. The process of assessing performance across an institution by computing risk capital and comparing returns on risk-capital is often referred to as ‘capital allocation.’

Why do we refer to the denominator of (1) as risk capital and not as economic capital, as it is more usually termed by practitioners? This choice has been made carefully and after much deliberation. The central point of our paper is that returns required by shareholders on an exposure cannot be determined from its contribution to risk capital. This in turn suggests that it is better to avoid the practitioner term ‘economic capital’, thus avoiding any suggestion that return on risk capital is a guide to the economic cost of risk.

The emergence of capital allocation has been a principal driver of the new Basel accord on bank capital adequacy (Basel II). One of the stated goal of the new accord has been to achieve a closer alignment of regulatory capital with economic capital.<sup>3</sup> Misalignment of economic and regulatory capital is thought to have distorted business decisions and encouraged the use of financial transactions such as securitisations to reduce regulatory capital without altering bank exposure to downside risks.<sup>4</sup> Furthermore a key principle of the new accord is the ‘use test’, the requirement that in order to qualify for the

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<sup>3</sup>The Basel committee writes (Basel Committee (1999), page 11) ”...during the 1990s the [1988 Basel] Accord became an accepted world standard, with well over 100 countries applying the Basel framework to their banking system. However, there also have been some less positive features. The regulatory capital requirement has been in conflict with increasingly sophisticated internal measures of economic capital....In addition the accord does not sufficiently recognise credit mitigation techniques such as collateral and guarantees. These are the principal reasons why the Basel committee decided to propose a more risk-sensitive framework in June, 1999.”

<sup>4</sup>See Jones (2000) for illustration of this practice of ”Regulatory Capital Arbitrage”.

reduced regulatory capital requirements available under the more advanced pillar 1 methods (the IRB method for credit risk and the AMA method for operational risk) and for regulators to agree the pillar 2 assessments of overall capital needs, the underlying systems must be applied by banks to their business decision making, not just used for regulatory compliance.

Despite the potential business benefits, and the strong incentives provided by regulators, there is in fact little agreement amongst practitioners on the implementation of RAROC and related measures.<sup>5</sup> Concerns arise, for example, about discrepancies between market prices and the ‘economic pricing’ of corporate loans from capital models. Bankers will often suggest that pricing based on capital allocation is simply too low for this particular market to bear. There is no consensus on the choice of a benchmark hurdle rate for RAROC. Different institutions choose very different confidence thresholds for allocated capital, some using 95%, some 99%, and yet other 99.97% or even higher. While capital allocation has been championed by many risk-professionals, others are more cautious and it has achieved only partial acceptance from other business professions such as accountants and credit professionals. Capital allocation is done in so many different ways by different institutions. Because of these and other difficulties many practitioners with substantial practical experience of implementing systems of capital allocation share our view that the term ‘economic capital’ is best avoided. Capital allocation is a long way from having proved itself to be an effective tool of enterprise wide risk management in financial institutions.

There is little research on capital allocation published in the finance journals (our literature review in Section 2 below indicates that these papers are largely limited to descriptions of professional practice investigating what

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<sup>5</sup>This paragraph is based on numerous discussions with practitioners, in bilateral meetings and at many seminars and conferences over the past five years.

financial risk managers do, rather than exploring the intellectual foundations of these procedures.) This is surprising if for no other reason than that the spread of capital allocation as a business tool has led to a substantial loss of influence and status for the finance profession. In those financial institutions where capital allocation is being applied as a central business tool, business decisions are being taken with no explicit reference to the basic concepts of finance established since the 1950s and 1960s, the concepts that are the foundation of financial economics as an intellectual discipline. As a consequence training and qualifications in financial economics are being devalued relative to those in natural sciences or statistics.

This paper aims to fill this gap in the finance literature, using standard tools of financial economics to explore the relationship between risk capital and the return required by shareholder for bearing risk. We obtain strong results. We show that RAROC is fundamentally inconsistent with the established framework of financial economics. Only in unrealistic special cases is it possible to apply a single hurdle rate for return on risk capital as a guide to whether an exposure creates shareholder value. The use of (1) confuses quite two separate concepts, downside tail risk and the market cost of bearing risk, and as a result its use may lead to substantial destruction of shareholder value. Capital allocation, implemented in this form, is a throw-back to pre-1958 thinking, ignoring the role of equity markets in establishing an equilibrium allocation of risk.

We draw practical conclusions. We argue that that the failure of capital allocation to establish itself as a effective means of enterprise wide risk management is precisely because its intellectual foundations are so weak. To correct this, we propose that (1) should be replaced by an alternative performance measure that distinguishes the market price of risk and of the

contribution of an exposure or business line to institution wide risk capital, and we suggest how to do this. We also argue that neither regulators nor bankers need be concerned about divergences between various measures of prudential risk (a useful umbrella term for referring to both risk capital and regulatory capital requirements) and the banks assessment of the actual business cost of accepting risk. The goal of ‘alignment’ of regulatory with economic capital is misplaced. Financial institutions would serve their shareholders better by devoting more substantial resources to the effective disclosure of their capital needs and much less to the design of risk models so as to minimise their regulatory capital requirements

Our discussion is presented as follows. Section 2 motivates our contribution, relative to both existing business practice and to the small academic literature on bank capital allocation. Section 3 presents a formal analysis of the relationship between risk capital and required returns. Section 3.1 introduces the assumptions of our model. Proposition 1 of Section 3.2 is our basic result. We use a simple arbitrage argument to derive necessary and sufficient conditions for the use of return on risk capital as a risk-performance measure. Proposition 2 provides a further more intuitive result in terms of skewness of return distributions. Section 3.3 extends the analysis to the case of a bank with limited shareholder liability and Section 3.4 to a bank with 100% deposit insurance (i.e. incorporating the assumptions of Merton (1973) and Merton (1974)). We find that these generalisations make little difference to our findings.

Section 4 illustrates with some computations of the required return on risk capital, under the assumption that the market price of risk can be computed using the CAPM model. We begin with two standard return distributions commonly applied to market risk, the arithmetic normal and log-

normal, finding that the required return on risk-capital is unaffected by portfolio variance in the case of the arithmetic normal, but that it is increasing in the case of the log normal. We also explain how the results of Crouhy et. al. (1999) can be interpreted as (near) special case of our own results. We then examine the required return on risk capital for credit portfolios using the Vasicek asymptotic default distribution, the distribution underlying the Basel II pillar 1 risk curves., showing that the required return on risk capital is very much lower than for our market risk distributions, due to the much greater left skew of credit returns. Section 5 summarises our contribution and draws conclusions for practitioners and policy makers, in relation to both practical risk management and to the operation of Pillar 1 (capital modelling) and Pillar 2 (capital assessments) in the new Basel accord.

## **2 Prudential and economic capital**

The purpose of this paper is to analyse the relationship between risk capital – the contribution of an exposure to default risk – and the returns required by shareholders compensate them for bearing the risk of an exposure. Before undertaking this tasks we first review what previous literature on bank performance measurement has to say about these two concepts.

The role of capital in protecting financial institution solvency is well understood (see for example Berger et. al. (1995).) The relationship between capital in financial institutions and business performance measurement – so called capital allocation – has been the subject of several descriptive studies, much of this summarised in Schroeck (2002). RAROC can be understand as a reformulation of standard models of risk pricing, with the ‘risk capital’ of an exposure representing the portfolio beta and hence related to CAPM pricing

(see e.g. Jokivuolle (2006), pages 462-463) but this relies on the assumption that all returns are normally distributed, it is not generally correct.

Since the late 1980s financial institutions have been designing systems to measure the risk in their different lines of business. As James et. al. (1996) point out these systems seek to achieve two goals. One is to help optimise capital structure i.e. find the proportion of equity to assets that minimizes the bank's cost of funding, addressing the tradeoffs between the benefits of leverage (the tax shield provided by tax-deductible interest payments and the discipline debt imposes on managers to operate as efficiently as possible) against the costs of financial distress arising from excessive leverage.

The second application of these systems of capital measurement is for enterprise risk management, i.e. comparing the risk exposure and performance of different business lines and products across the institution. i.e. their use in what is often referred to as the allocation of capital across the bank. Such measures are used for a wide range of purposes, including strategic decision making, as an input to employee compensation systems, and for product pricing.

Why do financial institutions use systems of capital allocation rather than the more familiar tools of financial project appraisal adopted by non-financial corporates? Merton and Perold (1993) emphasize that the costs of leverage for banks are different from those for industrial companies because bank customers are often also their largest liability holders and as a consequence, a high credit rating is generally essential for banks to maintain their business activities, e.g. as dealers or customers in OTC markets, to underwrite securities or to compete effectively in the corporate banking and deposit markets. In extreme cases, extensive use of debt financing can lead to default and a costly reorganization.

As stated in our introduction, it has been common to use equation (1) as such a performance measure. The underlying assumption here is that the denominator, *risk capital*, can be used to measure the cost to shareholders of carrying the risk exposure of each individual transaction, but as our analysis demonstrates this is not generally true. Still however, many institutions assume this approach is valid and use the risk measurement systems developed for bottom-up calculation of capital adequacy for business performance measurement (for documentation see e.g. Zaik et. al. (1996), Matten (2001) and Schroek (2002)). Similar thinking is evident in the evolution of the Basel accord (Basel Committee (1999a, 1999b, 2004) where the prudential risk that is the central concern of regulators and the measures of risk appropriate for internal business assessment are assumed to be the same.

How are these ‘bottom up’ calculations of risk capital conducted? The usual approach is to compute a broad measure of value at risk (VaR), covering market, credit, operational, and even sometimes even other risks, for some specified tail probability and time horizon. A common choice is a time horizon of one year and a loss threshold of 99.97% or 99.98% i.e. prudential capital should be enough to cover bank losses occurring in all but 2 or 3 out of 10,000 years.<sup>6</sup> A justification for this loss threshold is that it corresponds to the historical cross-sectional performance of AA rated corporate bonds.<sup>7</sup> Banks typically need to maintain a credit rating of AA in order to maintain their counterparty standing in wholesale and interbank markets. These choice of prudential capital parameters are therefore consist with this

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<sup>6</sup>For example according to Zaik and al. (1996) at Bank of America an annual default probability of 0,03% is targeted.

<sup>7</sup>Moody’s and Standard and Poor’s databases indicate that average annual default on US AA corporate bonds over the past fifty years has been around 0.02% or 0.03%.

business requirement.<sup>8</sup> There are substantial practical difficulties with these bottom-up calculations, especially for extreme confidence thresholds. Only in some cases, e.g. desks trading equities, derivatives, or government bonds, or in exposures such as corporate bonds or loans to quoted companies, is it possible to measure the risk of default on a mark to market basis. The greater proportion of bank activities, especially those associated with retail lending and smaller corporate credits, are illiquid. Risk measures must then be based on accruals accounting system, including arrears and recoveries.<sup>9</sup>

Performance measurement in financial institutions have adopted some of the features of the EVA<sup>®</sup> performance measure developed by Stern Stewart & Co's for application to fixed capital investment and other project decisions by non-financial companies. EVA<sup>®</sup> evaluates contribution to net present value using a required return calculated from the correlation with risk factors in standard asset pricing models such as the CAPM or the APT, although the required return is typically determined at the level of the firm, or subsidiary, rather than at the level of the individual investment project.

An key feature of EVA<sup>®</sup> is that allows the calculation of contribution to shareholder value added from each year that a project is active. This is important, for example, in order to set appropriate compensation incentives and to make sensible comparisons between different business lines.

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<sup>8</sup>Bottom-up capital modelling is not the only determinant of leverage. Practitioners also emphasise the role of rating agencies, who may insist on higher capital than suggested by the banks own capital management systems. Desired capital may also exceed prudential capital for other reasons e.g. in order to finance future asset growth or acquisitions, see for example Jokipii and Milne (2007) for discussion and empirical analysis.

<sup>9</sup>Practitioners have developed fairly sophisticated accounting based systems of risk measurement for bank loan portfolios. Many of these methods are set out in Ong(1999), the widely used account of the techniques for measuring bank prudential capital for credit exposures. See Milne (2007) for a discussion of some of practical implications for performance measurement.

The equivalent of EVA applied in financial institution performance is the use of a ‘value-added’ variation of the RAROC formula as an indicator of the amount of shareholder value created by a bank activity:

$$\text{value} = (r_{RC} - \hat{r}_{RC}) \times \text{Risk Capital} \quad (2)$$

where  $\hat{r}_{RC}$  is an appropriate hurdle rate for RAROC. An advantage of such a value added formulations, in comparison for example to (1) or an internal rate of return comparison of current costs and future earnings, is that they more directly incentivise creation of value, avoiding for example the rejection of large projects because that do not achieve a sufficient margin over the hurdle rate in favour of smaller projects with larger margins.

As noted by Zechner and Stoughton (2003), one reason for using performance measures for enterprise management is to enable the delegation of decision making within a large organisation when managers responsible for investment decisions have privileged information. They develop a a model, drawing on the literature on capital decisions in non-financial companies, showing that a RAROC performance measure can overcome the information asymmetries between divisional managers responsible for portfolio decisions and central bank management. They however assume that risks are normally distributed, which is a special assumption that allows a single measure of risk to be used for both required returns and risk capital.

Why do financial institutions use a different approach to ‘capital allocation’ than that established for non-financial corporates e.g. the use of NPV rules? While the objective, delegated decision making, is the same, in the case of financial institutions substantial risk-exposure can arise with little or no upfront funding (e.g. in derivatives, in short term trading or the provision of committed lines of credit that are not immediately drawn down) and so it is inappropriate to measure performance relative to a required return on

funds supporting a position. Also, because risk exposures differ so greatly within a financial institution, the denominator of a performance measure must be risk-sensitive, it cannot be based simply on total funding. Since exposure specific risk is already measured for the purposes of capital management, the idea that this same measure of risk-capital can also be applied to performance measurement has obvious attractions.

A separate justification often made for the practice of comparing expected returns with the consumption of risk capital is that shareholders provide the risk capital of the bank, that this capital is in limited supply and hence must be efficiently rationed. Hence bank activities should be chosen according to the surplus value added relative to some target return on shareholder risk capital. Often this target return is further identified with the return on the accounting equity (ROE) of the bank.

This is an unacceptable argument. Accounting equity is a backwards looking reflecting historical performance, not a forward looking market measure of shareholder risk exposure. Moreover standard corporate finance tells us, even when equity is measured on a market basis, there equity capital is not in limited supply, it is freely available from shareholders provided that there is adequate compensation for risk, and hence there is no reason for the required return on equity to be the same from one institution to another, only that required returns should be appropriately adjusted for both asset risk and for leverage.

This paper explores these issues by analysing the relationship between RAROC and the standard financial measure of net present value. Section 3 obtains precise and very demanding conditions under which there is a constant zero net present value RAROC hurdle for performance measurement. Section 4 quantifies the gap between risk capital and required returns.

Changes the zero-NPV RAROC hurdle as portfolio characteristics are varied reveal very large potential losses of shareholder value when a single RAROC hurdle is mistakenly applied to business decision making.

The relationship between RAROC and NPV has been previously examined by Crouhy, et. al. (1999). They show (the final column of their Table 1, page 12) that for investments with log-normally distributed returns the RAROC hurdle (what they refer to as return on equity, i.e. the return required by the market) increases with the volatility of returns. They recommend an adjustment of RAROC which yields a constant hurdle rate for a log-normal return distribution. The analysis of our paper can be seen as a generalisation of their analysis, but we find that their adjustment to RAROC does not always yield a constant RAROC hurdle. We propose instead (in Section 5) a measure of risk for performance measurement that distinguishes the contribution to risk capital from required shareholder returns.

The academic literature discusses some other issues concerning bank performance measurement. One is the question of whether bank risk should be measured relative to the bank's own portfolio or relative to the market as a whole. Froot and Stein (1995) point out that, faced with an increasing cost of raising external funds banks will behave in a risk-averse fashion towards risks that are diversifiable at a market level. Specifically, a business unit's contribution to the earnings volatility of the bank will be an important factor in the capital allocation and capital structure decisions and also in the decision to hedge earnings risk. Capital structure, hedging and capital budgeting are therefore inextricably linked together.<sup>10</sup>

Froot and Stein (1998) further develop this point, demonstrating in a two period model that the hurdle rate for bank investments can be calculated

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<sup>10</sup>See also Stulz (1998)

from a two factor pricing model, namely the covariance of the return on the tradable component with the market  $R_m$  and the correlation on the non tradable component of the new risk ( $\mu_i^N$ ) with the non-tradable risks of the existing portfolio:

$$\mu_i = \gamma \text{cov}(\mu_i, R_m) + \lambda \text{cov}(\mu_i, R_P)$$

where  $\gamma$  is the market unit price of risk for the (market) priced factor  $R_m$  and  $\lambda$  is the unit cost for volatility of the banks portfolio.

The RAROC hurdle (1) can thus be seen as a one factor pricing model (see Wilson (1992) and James et. al. (1996)) that ignored entirely the systematic element of risk, the  $\gamma \text{cov}(\mu_i, R_m)$  in the Froot and Stein (1998) formulation. However such a simplification may be justified if the banks portfolio effectively mimics the market portfolio.

A different issue, discussed in Stoughton and Zechner (2003), is the optimal method of capital allocation when there are several business units. They show using standard portfolio calculations that the appropriate measure of economic capital for performance measure should depend on each unit's incremental contribution to total portfolio Value at Risk (the "IVaR"). This can be defined in such a way that sum of the IVaRs is equal to the institution's overall VaR. In other words capital allocation should take into account the diversification benefits that exist across the institution. Such portfolio diversification benefits are routinely taken into account by practitioners, when they conduct economic capital calculations.

These two issues – the choice of measuring risk on a market or bank portfolio basis and the existence of diversification benefits within a portfolio – are of practical importance, but they are already much discussed and fairly well understood by practitioners. The issue on which we focus, the relationship between risk capital and required shareholder returns has attracted little

previous attention from either academics or practitioners.

### **3 Risk capital and required returns: theoretical analysis**

This section analyses the conditions under which required shareholder returns are proportional to risk capital and hence when using  $r_{RC}$  with a constant hurdle rate is a valid performance measure. The theoretical framework we apply is standard and the results will be obvious to a reader familiar with standard asset pricing theory; at the same time other readers will find the analysis of this section rather far removed from the practical application with which we are concerned, that of performance measurement in financial institutions. Why then devote space to this theory? The reason is in order to demonstrate that the divergences between RAROC and shareholder value are not a consequence of the particular return distributions and valuation model used in Section 4. There is a fundamental theoretical inconsistency between the RAROC performance measure and standard analyses of shareholder value. Less patient or less technically orientated readers may prefer to accept this point and more directly to the quantification of this divergence in Section 4.

The first subsection sets out our assumptions. Remaining subsections present our results for three different cases, unlimited shareholder liability, limited shareholder liability, and 100% creditor protection (i.e. deposit insurance.) For concreteness the analysis is developed for the case of a bank, although it could equally well be applied to another financial intermediary such as an insurance company.

### 3.1 Notation and modelling assumptions

We consider a bank considering the choice of whether or not to invest in a loan asset or portfolio with a market value of  $\hat{A}^i(t)$  over the period  $t = 0$  until  $t = T = 1$ . This asset is one of a number of potential assets i.e.  $i \in (1, 2, \dots, I)$ . To avoid the need to discuss portfolio diversification, which will not anyway alter our results because these rest on risk-neutral valuation techniques, we make the further assumption that the bank invests only in a single asset. We therefore drop the superscript  $i$  except when explicitly considering the comparison between two assets. We use the ‘hat’ to distinguish market measures of assets ( $\hat{A}(0)$ ) and also net worth (capital) ( $\hat{E}(0)$ ) and return on risk capital ( $\hat{r}_{RC}$ ) from their corresponding accounting measures ( $A(0)$ ,  $E(0)$  and  $r_{RC}$ ).

If it proceeds, the bank finances this investment by issuing debt with a market value of  $D(t)$ . If shareholders are subject to unlimited liability i.e. this debt is risk-free then the bank market value balance sheet is:<sup>11</sup>

$$\hat{A}(t) = \hat{E}(t) + D(t), \quad \text{with } t \in [0, 1] \quad (3)$$

where  $\hat{E}(t)$  is the market value of equity at time  $t$ . The risk free rate of interest is  $r_f$  and we have  $D(1) = D(0)(1 + r_f)$

End period asset returns  $A(1)$ , net of all costs, are uncertain and continuously distributed. We will assume that all asset risks are tradeable in liquid markets and the returns on the asset are a (possibly non-linear) function of one aggregate priced market risk factor  $z$ .<sup>12</sup>  $A(1) = R_A + \alpha_1(z) +$

<sup>11</sup>This is the assumption of sub-section 3.2, relaxed in the following sub-sections.

<sup>12</sup>The assumption of a single pricing factor is inessential (we could instead work with an arbitrary number of  $n$  priced factors) but simplifies our exposition.

$\alpha_2\phi$  where  $\phi$  is the specific asset risk.  $z$  and  $\phi$  have the bivariate joint density function  $f(z)g(\phi)$  with  $\int_{-\infty}^{+\infty} zf(z) = \int_{-\infty}^{+\infty} \phi g(\phi) = 0$  and  $\int_{-\infty}^{+\infty} z^2 f(z) = \int_{-\infty}^{+\infty} \phi^2 g(\phi) = 1$  and with  $\phi$  uncorrelated with  $z$ .

$R_A$  is the expected absolute return on the bank's asset while  $r_A = R_A/\hat{A}(0) - 1$  is the expected rate of return. The variance of absolute returns is given by:

$$V_A = \int \int (A(1) - R_A)^2 f(z)g(\phi) dz d\phi \quad (4)$$

$$= \int \int (\alpha_1^2 z^2 + 2\alpha_1\alpha_2 z\phi + \alpha_2^2 \phi^2) f(z)g(\phi) dz d\phi \quad (5)$$

$$= \alpha_1^2 + \alpha_2^2 \quad (6)$$

and  $\sigma_A = \sqrt{V_A}/\hat{A}(0)$  is the standard deviation of rates of return.

Markets for risk are complete so the market value of the bank asset can be expressed as a linear function of the aggregate (priced) risk factor:

$$\hat{A}(0) = (1 + r_f)^{-1} \left( R_A + \int \alpha_1(z)q(z)dz \right) < (1 + r_f)^{-1} R_A \quad (7)$$

where  $q(z)$  is the risk-neutral linear pricing function. Since investors are risk averse  $\int \alpha_1(z)q(z)dz < 0$  and this valuation is less than the discounted expected return.

$w = \alpha_1(z) + \alpha_2\phi$ , the random component of asset returns, is a continuous random variable with density function given by:

$$h(w) = \int_{-\infty}^{\infty} f(\alpha_1^{-1}(w - \alpha_2\phi))g(\phi)d\phi \quad (8)$$

with corresponding cumulative density  $H(w) = \int_{-\infty}^w h(w)dw$ .

The bank is insolvent if  $A(1) = R_A + w < D(1)$ . The bank chooses its capital structure (the amount of debt  $D(1)$ ) so that the probability of insolvency is maintained at a target level  $p^*$ . This requires that

$$D(1) = R_A + H^{-1}(p^*) \quad (9)$$

and – in the case of unlimited shareholder liability where the banks debt is risk-free – we have  $D(0) = (1 + r_f)^{-1}D(1)$  and the risk capital of the bank, measured at market values, is then given by:

$$\hat{E}(0) = \hat{A}(0) - (1 + r_f)^{-1} (R_A + H^{-1}(p^*)) \quad (10)$$

while the return on risk capital for this zero-NPV investment is:

$$\hat{r}_{RC} = \frac{R_A - (1 + r_f)D(0)}{E(0)} - 1 \quad (11)$$

where the superscript denotes an evaluation of return on risk measured at market value.

Note that all expected costs (the costs of debt, anticipated loan losses etc.) are deducted from the numerator while all adjustment for risk comes through the denominator, higher risk opportunities are associated with higher risk capital  $\hat{E}(0)$ .

The following argument establishes that  $\hat{r}_{RC}$  from equation (11) – the return on risk capital evaluated when the loan asset is measured at market values – is an appropriate hurdle rate for the corresponding performance measurement computed when the loan asset and risk capital are based on accounting conventions at acquisition value rather than market value  $r_{RC}$ .

Suppose that the bank asset requires total funding of  $L(0)$  (in the case of a lending operation  $L(0)$  corresponds to the book acquisition value the loan portfolio, but the funding requirement can equal zero, or even be negative if the bank is undertaking business such as the writing of options). The funding provided by equity holders is  $L(0) - D(0)$  and the investment creates value if  $\hat{A}(0) - D(0) > L(0) - D(0)$  i.e. if  $\hat{A}(0) > L(0)$  i.e. the relevant comparison is very simple – an exposure creates value if its market value exceeds its acquisition value. A marginal project, one that neither adds to nor subtracts from shareholder value, is one where  $\hat{A}(0) = L(0)$ .

It is then possible to compute return on risk capital calculated on a book acquisition basis, i.e. using the cost of acquisition of the bank asset being analysed. We obtain:  $E(0) = L(0) - (1 + r_f)^{-1} (R_A + H^{-1}(p^*))$  where the absence of a ‘hat’ on the equity value indicates that this is book rather than market value and return on book risk capital.

$r_{RC}$  can then be calculated from equation (11) but with  $\hat{E}(0)$  substituting for  $E(0)$  – with our assumptions this is the measure of return on risk capital that corresponds to equation (1) in our introduction. It is apparent from comparing these expressions that  $r_{RC}$  can be used as a performance measure using equation  $\hat{r}_{RC}$  from (11) as the appropriate hurdle rate. This is because the investment creates value (i.e.  $\hat{A}(0) > L(0)$ ) if and only if  $r_{RC} > \hat{r}_{RC}$ . Our task is therefore to establish, under what conditions is  $\hat{r}_{RC}$  constant, so that a single institution wide hurdle rate can be applied to  $r_{RC}$  in order to determine if an exposure creates value, i.e. when are risk capital and required returns aligned?

### 3.2 The case of unlimited shareholder liability

The following proposition summarises necessary and sufficient conditions for applying a single zero NPV hurdle rate for return on risk capital ie for the equivalence of risk capital and required returns.

**Proposition 1** *Consider a bank with unlimited shareholder liability. The rate of return on risk capital for a marginal bank asset takes the same value  $r^*$  for any marginal investment opportunity (one where  $A^i(0) = L^i(0)$ ) drawn from a set of potential opportunities indexed by  $i \in (1, 2, \dots, I)$  **if and only if** the ratio  $\theta^i = \int \alpha_1^i(z)q(z)dz/H^{-1}(w^i)$  for asset  $i$  is a constant for all  $i$ .*

**Proof**

Substituting for its component terms, the expression for the return on risk capital for a marginal (zero NPV) investment i.e. equation (11) can be rewritten:

$$\hat{r}_{RC} = \frac{-(1+r_f)H^{-1}(p^*)}{\left(\int \alpha_1^i(q(z))dz - H^{-1}(p^*)\right)} - 1 = \frac{\theta^i + r_f}{1 - \theta^i}$$

The left hand side of this equation remains constant for all investment opportunities if  $\theta^i$  is a constant. Similarly if  $\hat{r}_{RC}$  is constant so is  $\theta^i$ . Both necessity and sufficiency are established.

QED  $\square$

The following sufficient condition, while less general than the previous proposition, provides a more practical intuition:

**Proposition 2** *A sufficient condition for Proposition 1 to apply is that the distribution of asset returns  $w^i$  for any given  $i$  can be expressed as a mean-preserving spread of a single underlying asset return distribution  $w^0$ .*

Proof. Consider a mean-preserving spread of the underlying asset return distribution, for convenience indexed by the parameter  $i$ , so that  $w^i = iw^0 = i\alpha_1(z) + i\alpha_2\phi$ .

The denominator of the first term in the RAROC performance measure  $r_{RC}$ , the risk capital required to protect against  $w^i$  can then be rewritten as:

$$\begin{aligned} RC^1 &= \hat{A}(0) - D(0) \\ &= (1+r_f)^{-1} \left( R_A + \int k\alpha_1(q(z))dz \right) - (1+r_f)^{-1} (R_A + iH^{-1}(p^*)) \\ &= (1+r_f)^{-1} \left( \int i\alpha_1(q(z))dz - H^{-1}(p^*) \right) \\ &= iRC^0 \end{aligned}$$

i.e. risk capital increases in proportion to  $i$ .

The numerator of the first term of  $r_{RC}$  can be rewritten as:

$$R_A - (1 + r_f)D(0) = R_A - (R_A + iH^{-1}(p^*)) = iH^{-1}(p^*) \quad (12)$$

and this also increases in proportion to  $i$ . Hence, the return on risk capital  $\hat{r}_{RC}$  for a marginal investment opportunity is unaffected by a mean-preserving spread in asset returns.

QED  $\square$

It should be apparent that the sufficient conditions for applying a RAROC hurdle given in Proposition 1 and 2 are very strong. They imply for example that the degree of skewness of all bank investments is the same, in order that tail risk (i.e. risk capital) can be a sensible measure of the cost of risk to shareholders.

In practice banks must make choices for investment opportunities that differ greatly in their degree of skewness and there is therefore potential for substantial bias in business decision making from applying a single RAROC hurdle.

### 3.3 Limited liability with risky deposits

Unlimited liability is a simple and transparent special case. But our results also apply to the case of limited liability with risky financial institution debt.

**Proposition 3** *In the case of shareholder limited liability, provided that debt holders are fully liable for any losses not borne by shareholders and the bank is able to pre-commit its asset choice at the time it issues debt, then Proposition 1 continues to hold.*

### Proof

End-period debt  $D(1)$  is determined as before by  $p^*$  through equation (9). The credit riskiness of debt now reduces the current market value by an amount  $V_{\text{put}}$ , the present market value of the put option on banks assets written by deposit holders:

$$D(0) = (1 + r_f)^{-1}D(1) - V_{\text{put}} = (1 + r_c)^{-1}D(0) \quad (13)$$

Here  $r_c$  is the implied interest rate on credit risky debt.

The present value of equity (the required risk capital) then becomes:

$$\hat{E}(0) = \hat{A}(0) - D(0) + V_{\text{put}} \quad (14)$$

while expressing the expected absolute return on equity is given as:

$$R_E = R_A - (1 + r_c)D(0) + (1 + r_f)V_{\text{put}} \quad (15)$$

and we have as before  $\hat{r}_{EC} = R_E/\hat{E}(0)$

Substituting (13) into the right hand side of (14) and (15), all terms in  $V_{\text{put}}$  cancel. Neither the expected return nor the amount of required risk capital is affected by the presence of risky debt. It follows immediately that the return on risk capital for each project  $\hat{r}_{EC}$  is also unaffected by limited liability in this case with risky debt and so Proposition 1 continues to apply.

QED  $\square$

This result reflects the completeness of markets. Debt holders must be compensated for bearing default risk and since this compensation is paid by equity holders, the outcome is that neither the market value of equity nor the expected return on equity is affected by limited liability. The pre-commitment assumption is needed because otherwise the bank could increase the value of  $V_{\text{put}}$  after raising debt finance, hence transferring wealth from debt to equity holders (the agency cost of debt).

### 3.4 Limited liability with 100% protection for creditors

Creditor protection, most obviously the provision of deposit protection through either an explicit insurance scheme or an implicit safety net for failed institutions, increases both the return to equity holders and the absolute return to equity holders. In this case (focussing on the case of a purely deposit financed bank with 100% deposit insurance), while we have not been able to prove Proposition 1, a version of Proposition 2 still applies:

**Proposition 4** *In the case of shareholder limited liability with 100% insured bank debt, then under the assumption of Proposition 2 (that all risk can be described as a mean-preserving spread of a single underlying distribution) the zero-NPV RAROC hurdle is a monotonically decreasing function of the spread of returns  $i$ , falling between the unlimited liability threshold  $r^*$  and the risk-free rate with limiting values  $\lim_{i \downarrow 0} \hat{r}_{RC} = r^*$  and  $\lim_{i \uparrow \infty} \hat{r}_{RC} = r_f$*

#### Proof

Let  $r^* = -H^{-1}(p^*) / \left( \hat{A}(0) - (1 + r_f)^{-1} (R_A + H^{-1}(p^*)) \right) - 1 > r_f$ , be the return on risk capital for the zero-NPV project under unlimited liability. With 100% deposit insurance we have:

$$\hat{r}_{RC} = \frac{-H^{-1}(p^*) + (1 + r_f)V_{\text{put}}^i}{\hat{A}(0) - (1 + r_f)^{-1} (R_A + H^{-1}(p^*)) + V_{\text{put}}^i} - 1 \quad (16)$$

$$= \frac{1 - (1 + r_f)V_{\text{put}}^i/H^{-1}(p^*)}{(1 + r^*)^{-1} - V_{\text{put}}^i/H^{-1}(p^*)} - 1 \quad (17)$$

$$= (1 + r_f) \frac{(1 + r_f)^{-1} - V_{\text{put}}^i/H^{-1}(p^*)}{(1 + r^*)^{-1} - V_{\text{put}}^i/H^{-1}(p^*)} - 1 \quad (18)$$

This establishes that when  $i = 0$  i.e. the volatility of return equals zero and so  $V_{\text{put}}^i = 0$ ,  $\hat{r}_{RC} = r^*$ .  $V_{\text{put}}^i$  is an increasing function of  $i$  and hence also  $\hat{r}_{RC}$

is a decreasing function of  $i$ . Finally since  $\lim_{i \uparrow \infty} V_{\text{put}}^i = \infty$ ,  $\lim_{i \uparrow \infty} \hat{r}_{RC} = r_f$

QED  $\square$

100% deposit insurance does make a difference to our results. But it should be realised that for a safe bank, one say holding capital to maintain its annual default probability to a target of 0.1% or less, the value of the the deposit insurance put option  $V_{\text{put}}^i$  held by equity holders is very small and will make little difference to the RAROC hurdle.

## 4 Illustrations of zero-NPV RAROC hurdles

The previous section has shown that the use of RAROC as a performance measure is consistent with standard asset pricing theory only under very restrictive conditions. But the more important issue is whether these departures are large enough to be of practical significance. This section will show that the departures can be very large indeed. It quantifiesthe impact on the zero-NPV hurdle for return on risk capital of assuming different return distributions or altering exposure characteristics such as volatility of returns or probability of default.<sup>13</sup> In particular it demonstrates that, because of the greater left-hand skew of credit portfolio returns, the appropriate RAROC hurdle for credit risks can be as little as one half of the appropriate hurdle for market risks.

Two cases are considered in detail. Section 4.1 compares two standard cases appropriate to the analysis of market risks, those of arithmetic and lognormal returns. This will show how previous work of Crouhy et. al. (1999) is related to our own, their results emerging as a special case. Section 4.2 analyses the determinants of the hurdle RAROC in a standard credit risk

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<sup>13</sup>Mathematica coding for all the Figures reported in the section is available from the authors.

model, the asymptotic portfolio loss model of Vasicek underlying the Basel II pillar 1 risk curves and widely used in contexts such as CDO tranche pricing.

The figures of the zero-NPV RAROC hurdle for return on risk capital reported throughout this section are all computed using (equation (11)). For any given return distribution  $A(1)$  and confidence threshold  $p$  for avoiding default, we compute the current market value  $A(0)$  of the prospective investment and thus the market value of the initial equity  $E(0) = A(0) - D(0)$  that must be provided by shareholders to reduce the default probability to  $p$ , yielding the required return on this risk capital.

The figures all assume quadratic investor utility, so that the pricing function  $q(z)$  used to compute  $A(0)$  is the capital asset pricing model, in which the expected rate of return on the market value of the asset is given by:

$$r_A - r_f = \beta_{A,M} (r_M - r_f) = s_A \rho_A \Phi_M \quad (19)$$

and  $\beta_{A,M}$  is the beta of the return on asset  $A$  with the market  $M$  and  $r_M(t)$  is the market return at time  $t$ . This assumption, while convenient, is not especially restrictive. We could instead have adopted one of many other asset pricing models. While the quantitative differences between required returns and risk capital would then differ from those we report here, the general conclusions would be unaffected. The calculations make use of the right-hand expression in (19), the reformulation more closely related to the Sharpe ratio, where  $\rho_A$  is the correlation of the asset return with the return on the single factor driving market returns, where  $s_A$  is the standard deviation of asset returns, and  $\Phi_M$  is the market price of risk.<sup>14</sup>

We assume that all the portfolios (equity or credit) are fully diversified i.e. that  $\rho_A = 1$  – an appropriate assumption when risk capital is measured

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<sup>14</sup>obtained using  $\beta_{A,M} = \rho_A / (s_A s_M)$  and  $(r_M - r_f) / s_M = \Phi_M$

by contribution to the default risk of a very large financial institution where the factors driving its returns may be assumed identical with those for the economy as a whole. We thus avoid discussion of a related issue, the application of RAROC hurdles in smaller financial institutions whose portfolio returns are driven by specific risks or a different combinations of risk factors than the market portfolio. We further assume that  $\Phi_M = 1$ , but this is only a scaling factor, assuming a larger value would raise all RAROC hurdles proportionately and not affect the differences in these hurdles which we report.

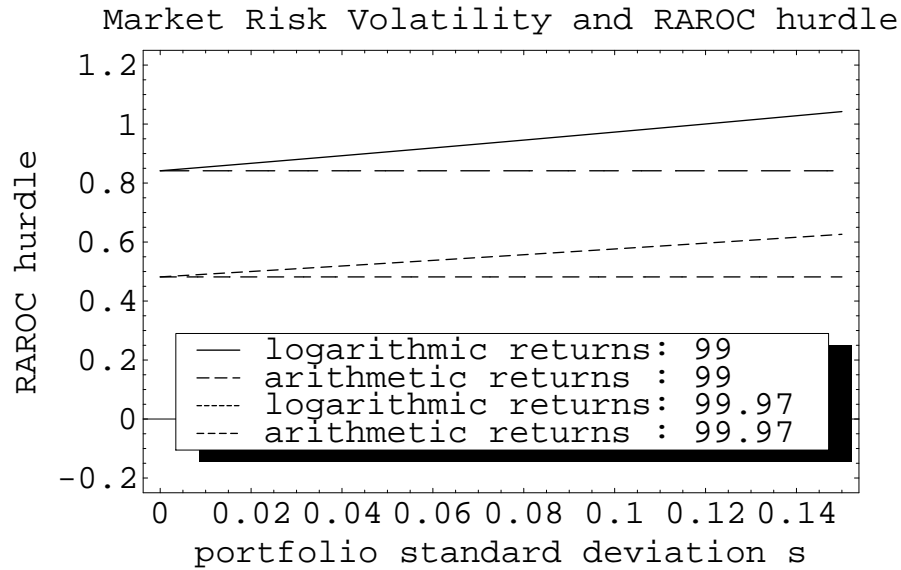
#### 4.1 Arithmetic versus lognormal returns.

This subsection presents calculations of the RAROC hurdle for a marginal investment opportunity (the  $\hat{r}_{RC}$  evaluated on a market value basis as in equation ) while varying the standard deviation of returns on a market investment portfolio.

The results are shown in Figure 1 . The horizontal lines (where the RAROC hurdle constant, i.e. the case when RAROC *is* a valid performance measure) is derived assuming an arithmetic normal distribution. This is as predicted by proposition 2, in this case an increase in the standard deviation of returns is an mean-preserving spread in the return distribution, and hence  $\hat{r}_{RC}$  is not affected.

The lines that slope upwards are for the log-normal distribution of returns, previously analyzed by Crouhy et. al. (1999). This distribution or returns has a right hand skew. An increase in the standard deviation of returns results in a less than proportionate increase in downside tail risk. The denominator of the expression for return on economic capital rises less than proportionately to the increase in asset returns (the numerator). Hence

Figure 1: RAROC hurdles for market risks<sup>†</sup>



<sup>†</sup> Computed assuming  $\rho = 1$ ,  $\Phi = 1$ . In the case of the log-normal using exact discrete time expected returns and standard deviations.

the RAROC hurdle rises as the standard deviation of returns  $\sigma$  increases.

We could instead have computed the RAROC hurdles for an undiversified portfolio, for example  $\rho = 0.2$  a degree of correlation with the market that might be appropriate for a single equity held by an undiversified bank. In this case the risk capital requirement would be relatively large and the required return on risk capital would be correspondingly smaller. This explains the quantitative difference between our figure and the lower hurdle rates reported in Table 1 of Crouhy et. al. (1999).<sup>15</sup>

<sup>15</sup>Our calculations differs slightly from theirs, for two other more technical reasons. First we do not include the put option arising from deposit insurance. However the probability of default is low and the put option makes only a small difference to the results. Secondly we use a slightly different approach, using an exact

The case of log-normal returns is usually appropriate for market investments, e.g. the applications of RAROC in investment management. The figure illustrates the impact of increasing the volatility of returns from 0% to 14%, as a result of which the required return on risk capital increases from 48% to about 63% at the 99.97% confidence threshold for the log-normal distribution, whereas for the arithmetic normal it remains constant at 48%. At a 99% confidence threshold the required returns are about twice as high as at 99.97%, increasing in the case of the log normal from 84% to 105%.

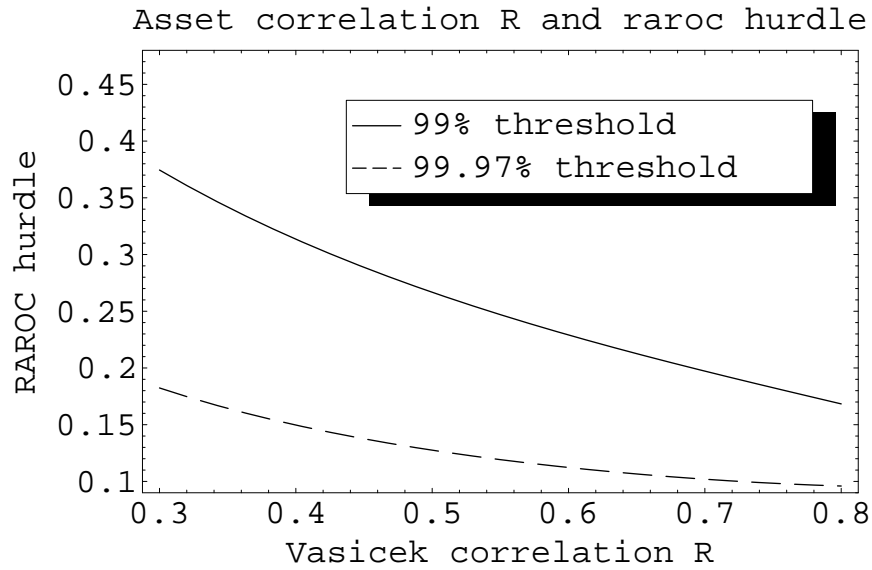
## 4.2 An asymptotic credit portfolio distribution

Figures 2 and 3 illustrate the RAROC hurdle for the standard credit portfolio model proposed by Vasicek (1987), an asymptotic model of the distribution of returns on a portfolio of defaultable claims and the model underlying the IRB risk-curves in pillar 1 of the Basel II accord. The two parameters varied here are the probability of default  $PD$  and the correlation  $R$  with the single aggregate risk factor triggering defaults. In its application in the new Basel accord  $R$  is set by the regulator for different categories of exposure while  $PD$  is determined by the bank's own estimation of the one-year ahead default for the appropriate internal rating.<sup>16</sup>

The Vasicek model of defaultable losses reproduces many basic features of credit risk that cannot be captured by either arithmetic or log-normal return distributions. The return distribution is leftward skewed, bounded rather than approximate conversion between continuous time returns and standard deviations (used for the log-normal distribution) and discrete time returns and standard deviations.

<sup>16</sup>We can ignore LGD since, assuming independence from default, higher  $LGD$  increases both risk capital and required returns while leaving the RAROC hurdle unaffected. We similarly ignore  $EAD$  since under the same assumption of independence this has no impact on the RAROC hurdle.

Figure 2: RAROC hurdles and credit correlation<sup>†</sup>



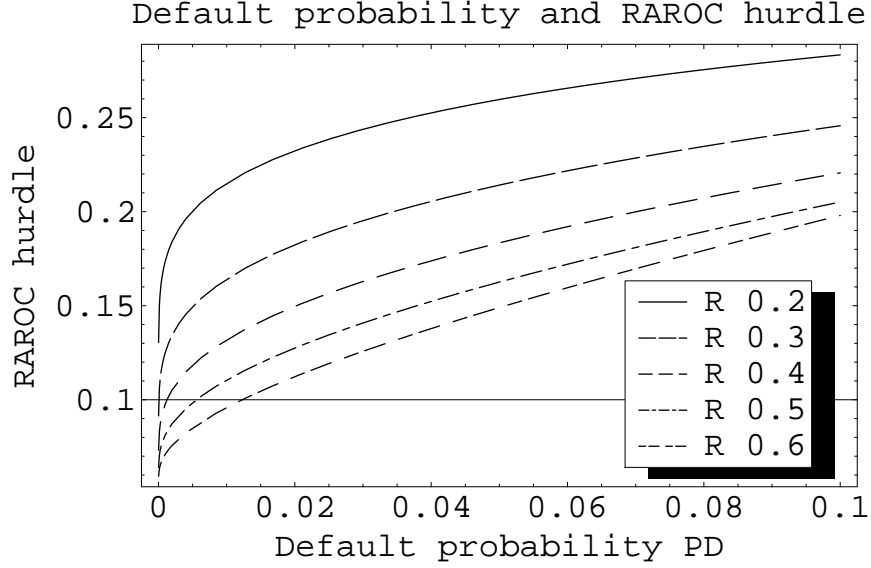
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<sup>†</sup> Computed assuming  $\rho = 1$ ,  $\Phi = 1$ ,  $PD = 0.02$ .

above at the par value and bounded below at zero. In this model, all but very high risk portfolios with a large proportion of credits bordering on default, the standard deviation of annual returns on a credit risky portfolio is relatively small. With the range of parameter values we have explored the standard deviation of annual portfolio returns falls in the range 0% to 2%, much less than the range illustrated in Figure 1 appropriate for market investments such as equities.

In the Vasicek model the credit value at risk at a confidence threshold

Figure 3: RAROC hurdles PD<sup>†</sup>



<sup>†</sup> Computed assuming  $\rho = 1$ ,  $\Phi = 1$ .

of  $p$  on a diversified credit portfolio is given by :<sup>17</sup>

$$A(1)[p] = LGDN\left(\frac{N^{-1}(PD) + RN^{-1}(p)}{\sqrt{1 - R^2}}\right) \quad (20)$$

where  $N()$  is the standard cumulative normal density function,  $R$  is the correlation of the asset values of an individual obligor with the aggregate risk factor driving defaults and  $PD$  represents the quality (expected default rate) of the portfolio.<sup>18</sup> Note that while the default distribution depends on

<sup>17</sup>This same equation used, with the further introduction of a relationship between  $PD$  and  $R$  for corporate risk, underlies the IRB calculations of Pillar 1 of the new Basel accord

<sup>18</sup>We have adopted a slightly different convention than the New Basel accord, our  $R$  is the correlation between the individual obligor factor and the aggregate risk factor, this is the square root of The Basel ‘correlation’  $R$  in the IRB risk curves, the much lower correlation between two different individual obligor risk

the extent of correlation between individual asset values and the aggregate risk factor, the portfolio itself is fully diversified with all specific risk to individual obligors fully diversified away.<sup>19</sup>

Figure 2 shows the impact of the asset correlation  $R$  on the RAROC hurdle rate, assuming a default probability of  $PD=2\%$ . The RAROC hurdle is very much lower than in Figure 1, between 18% and 10% for a 99.97% confidence threshold, compared with thresholds of over 40%. The difference in threshold is equally marked at the 99% confidence threshold.

Figure 3 shows the impact of the other main parameter of the Vasicek model – the probability of default  $PD$  – on the hurdle rate for return on risk capital. This Figure assumes a confidence threshold of 99.97%. As  $PD$  rises from over zero to 10%, the hurdle rate rises, but never by enough for the RAROC hurdle to approach that of the market exposures shown in Figure 1.

The differences in threshold, comparing Figure 1 with Figures 2 and 3 are very large indeed. They indicate that using a single RAROC hurdle rate for both market and credit exposures can lead to very substantial loss of shareholder value, with the appropriate RAROC hurdle nearly three as large for market risk exposures as for credit risk exposures. This implies that if a hurdle rate appropriate for market exposures is used then loan assets will factors. For translation to the Basel formulae  $R$  in our equation must thus be replaced by  $\sqrt{R}$  and the figures for  $R$  shown in Figures 1 and 2 must be squared before they can be interpreted in terms of Basel accord formula. Thus our  $R = 0.3$  corresponds to a Basel correlation of 0.09, while our  $R = 0.8$  corresponds to a Basel correlation of 0.64

<sup>19</sup>We assume that the same aggregate risk factor drives both defaults and market returns, when  $R$  is close to zero the aggregate risk factor has a small but close to linear impact on portfolio returns so the correlation between portfolio and market returns  $\phi$  is close to one, but as  $R$  increases the relationship between the aggregate factor and portfolio returns becomes increasingly non-linear and  $\phi$  declines, falling to around  $\phi = 0.6$  for  $R = 0.8$ .

be rejected with market values which are at least twice as great as their cost of acquisition (because the required expected return under a RAROC rule is more than twice as high as what shareholders actually require.)

Why are these required RAROC hurdles for credit and market risks so hugely different? This is because of pronounced differences in the shape of the loss distributions. The credit portfolio return distribution computed using the Vasicek model (as in Figure 2 and 3) have a very pronounced left skew. This is in contrast to the arithmetic and log-normal distributions used for Figure 1. This substantial difference in the skewness of returns means that the amount of shareholder equity i.e. the risk capital, required to protect a credit portfolio from default is around five times larger as multiple of portfolio return volatility than is required to protect investment in an equity portfolio. A credit portfolio thus absorbs a much larger amount of risk capital than an equity portfolio, relative to the return required to compensate shareholders for accepting the portfolio risk (which under the CAPM assumption underlying these figures depends only on the volatility of returns and their correlation with market returns.)

Deeper analysis of the difference in required RAROC thresholds for market and credit risk, going beyond that contained in these figures, is warranted. The calculations here assume that the compensation required by shareholders for bearing risk can be computed from the CAPM model. Alternative pricing models, with a relatively higher cost required by shareholders in order to compensate for downside risk, would lead to higher required returns on left skewed investment portfolios and thus would reduce the gap between the RAROC thresholds appropriate for credit and market portfolios.

Calibration of these figures to the skewness in returns on actual banking data would also affect gap between the RAROC thresholds. To mention

two possibilities: taking account of the additional liquidity costs of closing a losing position in a stressed market might introduce a more substantial left-hand tail in the return distribution for market risk than imposed by either the arithmetic or normal distributions (lowering the RAROC hurdle for market risk); the left-hand tails of credit risk may be greater than imposed by the Vasicek portfolio distribution (lowering the RAROC hurdle for credit risk).

These caveats suggest caution about the magnitude of the quantitative difference between the thresholds shown in Figures 1 and 2, but they do not dispose of the fundamental theoretical issue. It is simply not possible to summarise risk using a single metric which captures at the same time both the exposure to extreme negative returns and the return required by shareholders for absorbing risk. There is no single RAROC hurdle consistent with the maximisation of shareholder value and using RAROC with a single institution wide hurdle rate as a performance measure leads to substantial loss of shareholder value.

## 5 Conclusions and further work

This paper has examined the relationship between risk capital (the contribution of an exposure to default risk for a financial institution) and required shareholder returns. If required returns were proportional to risk capital then RAROC (equation (1)) could be used as a performance measure of the creation of shareholder value.

Our central result is Proposition 2 of Section 3.2. There we show that return on risk capital can only be used as a valid measure of contribution to shareholder value, if all bank asset returns distributions belong to a sin-

gle ‘family’, each a mean preserving spread (or contraction) of each other. Otherwise, from Proposition 1, a constant RAROC hurdle will only emerge if changes in the shape of the distribution by coincidence offset by changes in the degree of correlation with aggregate priced risk factors.

Section 4 illustrates the practical significance of these propositions, by computing the hurdle return on risk capital for some standard return distributions. Section 4.2 reproduces the finding of Crouhy et. al. (1999) that in the case of the log-normal distribution this hurdle rate rises with volatility. We contrast this with the case of the arithmetic normal where the hurdle rate remains constant.

Neither the log-normal nor the arithmetic normal is an adequate description of the returns on a credit portfolio. Section 4.3 shows that for credit portfolios, which are characterised by much more pronounced left-skews than market portfolios, the required return on risk capital is very much reduced, to around half the level appropriate for market risk exposures in the case of the asymptotic Vasicek portfolio model.

This analysis leaves several issues for further work:

- The quantitative impact of alternative pricing models than the CAPM, especially pricing which imposes a relatively higher penalty on downside outcomes. Such a framework could lead to higher required returns on risk capital for credit portfolios than we obtain here in Section 4.
- Our analysis assumes that all risks can be priced against market risk factors. A major issues, clearly important for general insurance companies but also for a number of bank exposures, is how to measure performance for risks not priced on financial markets.
- The incorporation of an impact on shareholder value of financial dis-

tress, the very reason why financial institutions are concerned about risk capital in the first place. It would be possible to derive a performance measure from a two factor pricing model which incorporated these costs of default as well as costs to shareholders of bearing market risk. We leave this extension for further work.

Still, our results are enough to show that it is inadvisable to apply on an institution wide basis, a single hurdle rate for return on risk capital. These weak intellectual foundations help explain why (as documented in our introduction) RAROC based performance measures have faced such difficulty when extended outside their original application to market risk management and why their uptake in financial institutions has been so uneven, limited often to risk specialists.

How then should capital allocation to be conducted, in order to overcome the weaknesses of RAROC? One possible approach would of course be to return to basic concepts of financial economics and evaluate the net present value of every exposure, but this approach does not seem to find favour with management of financial institutions.

There are alternatives. One is to impose exposure specific hurdle rates for  $r_{RC}$ , for example a simple approach based on our analysis in Section 4 suggests that this hurdle might be much lower for credit portfolios than for market portfolios. Finer distinctions could be introduced. Alternatively, in order to keep a single hurdle rate, it would be possible to change the denominator - the amount of risk-capital - by just the amount required to offset the differences in required return on risk capital

We recommend a different approach, using a single hurdle rate but achieving this through a risk adjustment of the numerator rather than the

denominator, according to:

$$r_{RC}^* = \frac{\text{Expected Revenues} - \text{Expected Losses} - \text{Market Cost of Risk} - \text{Other Costs}}{\text{Risk Capital}} \quad (21)$$

where the new term – the market cost of risk – can be calculated using a standard asset pricing framework. It is the difference between the expected value of portfolio returns and their certainty equivalent value. It can also be interpreted as the cost of hedging or insuring risk on the market.

For example, assuming the notation used in this paper and the CAPM pricing model, then this hedging cost is given by:

$$\begin{aligned} & (\text{Expected Revenues} - \text{Expected Losses}) \left( \frac{1}{1 + r_f} - \frac{1}{1 + r_f + s_A \rho_A \Phi_M} \right) = \\ & (\text{Expected Revenues} - \text{Expected Losses}) \frac{s_A \rho_A \Phi_M}{(1 + r_f)(1 + r_f + s_A \rho_A \Phi_M)} \end{aligned} \quad (22)$$

where  $s_A \rho_A \Phi_M$  is the exposure specific cost of risk derived from the CAPM. The use of  $s_A \rho_A \Phi_M$  is only one simple example, the risk-adjustment could instead be based on one of many other models for pricing risk, including those that allow for much greater aversion to downside tail risks than the CAPM.

Having adopted this expression (21) the only question that then remains is the choice of hurdle rate for  $r_{RC}$ . The natural choice is in fact zero, i.e. the financial institution should acquire any asset for which  $r_{RC} > 0$  and should use  $r_{RC} \times$  risk capital as the corresponding value added measure to be used for assessing remuneration and bonuses. This zero hurdle rate is appropriate for an institution that it is unconstrained by its use of risk capital, which may well for example be true of a high margin but slow growing retail bank.

It is rare for risk-managers and the board of a financial institution to have such little call on their balance sheet as to be able to ignore risk capital constraints entirely, they have to also the efficient use of their balance sheet as well as contribution to shareholder value. An advantage of our proposed performance measure is that it offers a simple way to structure this task. Now the hurdle rate  $\hat{r}_{RC}$  for applying (21) should be adjusted above zero to the point at which the accepted investments (those for which  $r_{RC}$  exceeds the hurdle rate) exactly utilise all the risk capital available on the balance sheet. This then maximises the creation of shareholder value subject to the prudential constraint that risk-capital absorbed by individual exposures does not exceed the total risk capital on the balance sheet. In this case  $r_{RC} \times$  risk capital can still be used as a measure of shareholder value added (not  $(r_{RC} - \hat{r}_{RC}) \times$  risk capital, because the hurdle rate is a shadow price not a real resource cost.) The magnitude of  $\hat{r}_{RC}$  then provides an indication of the shortage of risk capital and can be used to make a case for retaining additional earnings or raising additional capital.

Our work also suggests a re-assessment of the resources devoted to estimating risk-capital, relative to those devoted to estimation of the required market return for bearing risk. For most financial institution exposures, estimation of extreme tail risk cannot be undertaken with any precision, because time series of data are so short and the difficulties of marking returns to market. Moreover shareholders do not require that risk capital be measured accurately, only that the financial institution has clear procedures for determining its risk capital requirements. therefore relatively small amount of risk-modelling resource should be devoted to risk-capital estimation. Shareholders however are particularly concerned with the market cost of risk, a task which can be achieved even when data is limited (because correlations with

market factors are relatively easy to estimate) so a relatively large amount of risk-modelling resource should be devoted to this task. Distinguishing these two measures thus suggests also that internal risk modelling resource should be re-allocated towards the estimation of the market costs of risk.

Our work suggests some related messages for bank regulators. The central point of our paper is because of the substantial differences in the skewness of portfolio return distributions, the contribution to default risk of an individual exposure differs from the cost to shareholders of absorbing that risk. As we have also noted, the contribution of individual exposures to default risk – its capital consumption – is only a business concern to a bank if it has insufficient capital to support all its exposures.

We point out that the identical logic applies to regulatory capital. The contribution of an individual exposure to its regulatory capital requirement is only a business concern to a financial institution if it has – or is danger of having – an insufficient capital to meet the overall regulatory requirement. Most banks have a very substantial buffers of capital over and above their regulatory capital requirements. Moreover, even if they are not comfortable with these buffers of capital are, healthy financial institutions do not find it too difficult to raise additional capital should they so need it, since it is the market cost of risk not capital per se that is costly to shareholders. Hence financial institutions should not be concerned with the level of capital that regulators require to back a particular exposure. Just as there is no reason for shareholders to require returns based on consumption of risk capital, it does not matter if regulatory capital and the financial institution’s own measure of risk capital do not correspond.<sup>20</sup>

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<sup>20</sup>There is therefore a problem of comparing not just apples and pears, but apples, pears, and oranges, with three measures of risk (required return, financial institutions assessment of prudential risk i.e. risk capital, and the regulators as-

While the goal of ‘aligning’ economic and regulatory capital is misplaced, it is desirable that the calculation of regulatory capital requirements is conducted in parallel with the bank’s own calculations of risk capital and using, where possible, inputs from bank systems. The encouragement given by the new accord for banks to improve their systems and procedures for estimating their risk capital needs is therefore welcome.

But this leaves a concern about the ‘use test’ to be applied under the new Basel accord and the forthcoming Solvency II regulations for insurers in Europe. This use test is the requirement that systems used to compute the pillar 1 capital requirements and the pillar 2 capital assessments (or ICAP), should also be used for internal decision making, not just created for the purpose of regulatory compliance.

The use test itself is reasonable, but an implication of our analysis is that it should only be applied to the data systems used for calculation of risk capital needs. Regulators should not normally expect to see financial institutions take much direct account of either risk capital or regulatory capital in business performance measurement and business decision making, and this should be done *only* when the bank is under pressing risk or regulatory capital constraints.

Such an approach will in fact make the job of the financial regulator much easier, reducing potential tensions with financial institutions. Regulators naturally should be cautious when setting regulatory capital requirements, for example because where there is little historical data and thus great uncertainty about the magnitude of tail risks. As a result it is part of the job of regulators to press banks to increase their capitalisation. This is much easier to impose once it is recognised that increased regulatory capital does 

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assessment of prudential risk i.e. regulatory capital), and no justification for making direct comparisons between any of these three.

not usually have any direct impact on the business. Measures of downside risk and the cost to the business of holding these risks, are indeed apples and pears, and should not be confused.

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