

Parameterization and Calibration of Actuarial Models

Paul Kneuer

2008 Enterprise Risk Management Seminar

Tuesday, April 15, 11:45 a.m. - 1:00 p.m.

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ERM Symposium
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Parametizing Models: Volatility Measures

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Parametizing Models:

Volatility Measures

- The bad news: It's hard
- The good news: It's impossible

An integrated and dynamic view of the volatility of transactions would be a powerful insight:

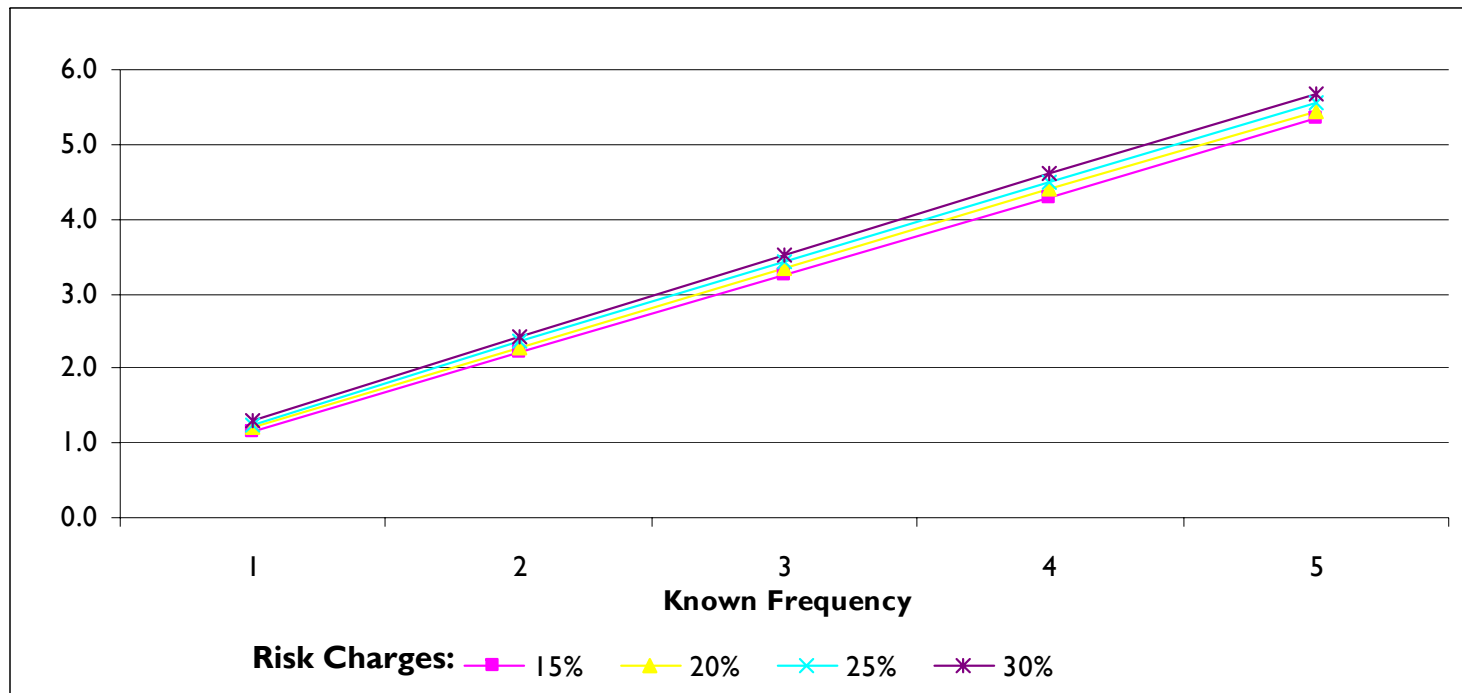
- Overall portfolio capital needs, real time
- Relative capital usage of transactions
- Marginal cost pricing

-
- ❑ Unfortunately, data is a poor source for the volatility of the next period. “Past performance is not a promise of future returns.”
 - ❑ In real world situations, what we don’t know that we don’t know can have more cost (and value) than what we do know.
 - ❑ **“It’s hard.”**

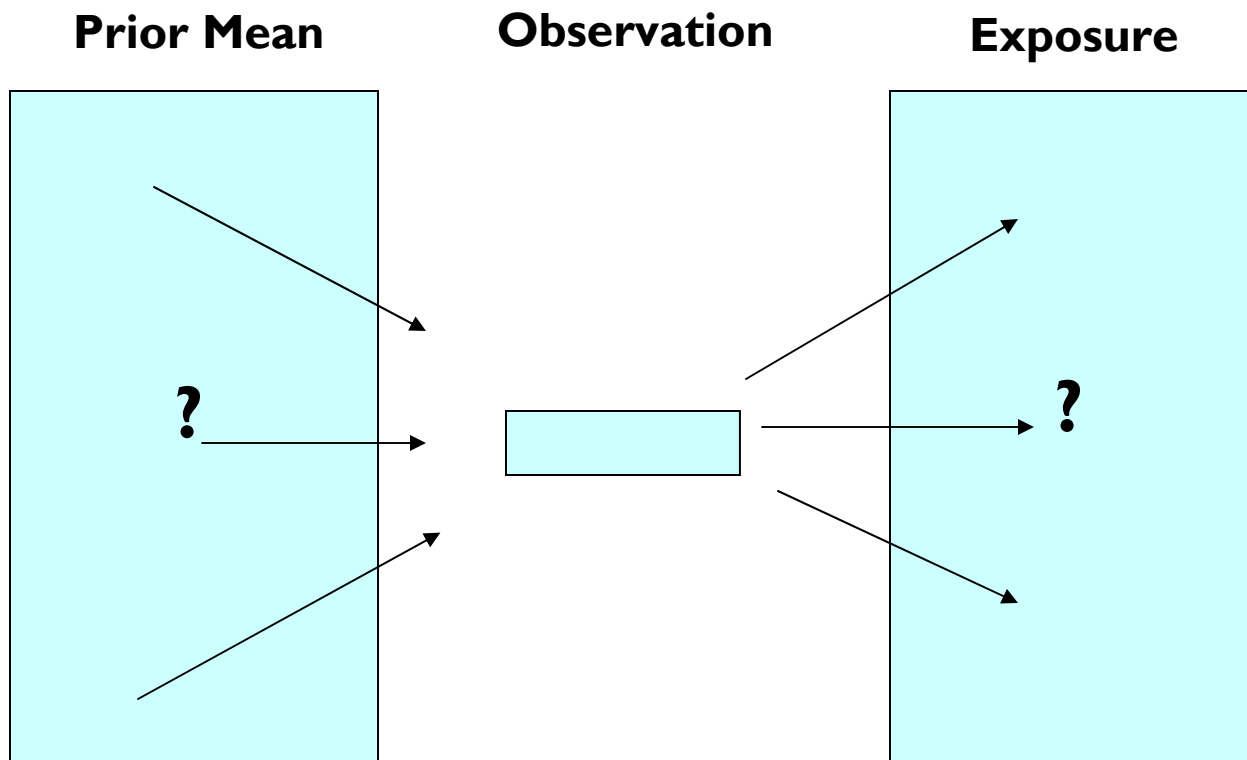
A simple model:

- Define a loss process so that it is a Poisson (counting) function: frequency, not severity.
- Look at an historical period and estimate the Poisson mean.
- Project the distribution of counts next year.
- Measure the value as $E(X) + R \times SD(X)$

Contract Value



Now, acknowledge that the data from last year is only sample from a distribution that we can never know.

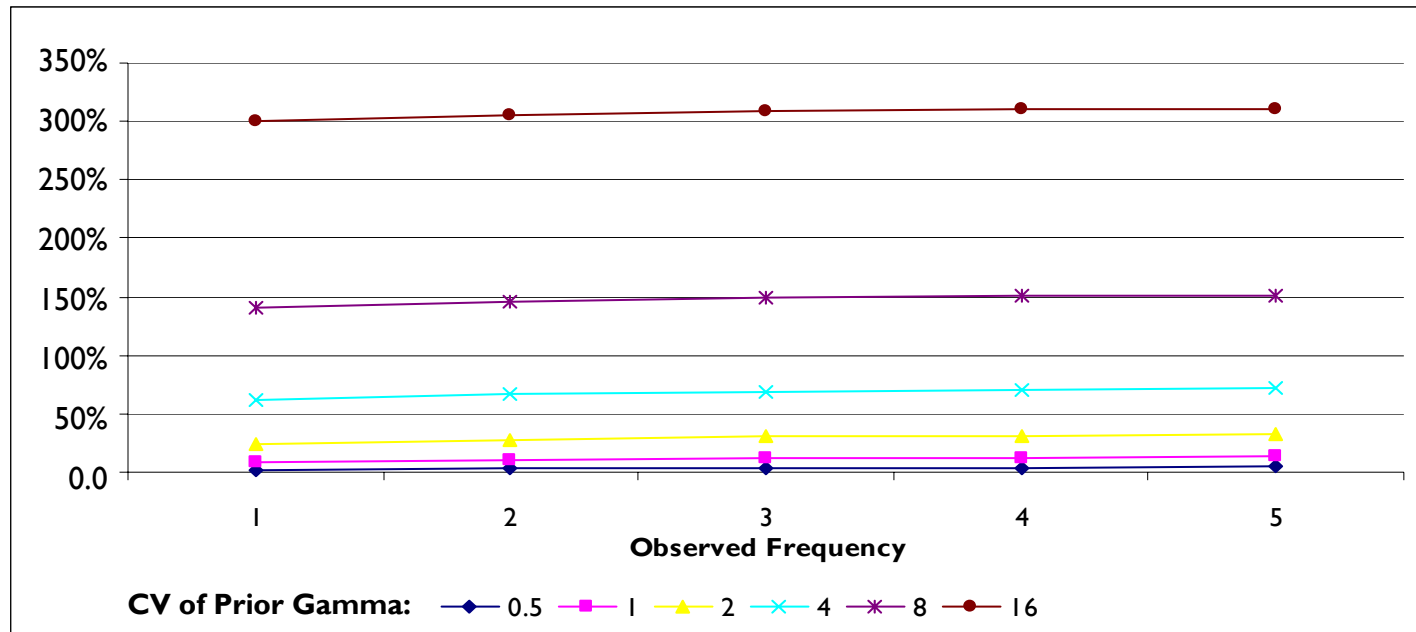


The Value of Next Year's Contract Reflects the Risk that We have a Poor Estimate of Last Year's Mean

The value of parameter risk is proportional to:

- C.V. of uncertainty around prior mean
- Square root of annual observed claims
- Correlation between this risk and overall market
- Market premium risk charges
- Square root of 1/experience period (in years)

**Relative Increase in Value
(Value of Parameter Risk/Expected Loss)**



Required Reading

“The Black Swan” by Nassim Taleb

- ❑ In many areas, the next data observations can be so discontinuous that it invalidates the form of distribution you would have chosen.
- ❑ For the population of ERM Symposium attendees, would you bet on the relative increase in the overall average size following the next arrival, based on:
 - ❑ Height?
 - ❑ Career experience?
 - ❑ Journal citations?
 - ❑ Net worth?

Black Swans

- ❑ Maximum swing to average from next observation:

<u>Observation</u>	<u>Swing to Average</u>
Aaron Gray (7' Center for Bulls)	< ¼ inch part of six feet, 0.3%
Charles Hewitt (FCAS, 1951)	< 1 week part of 20 years, 0.1%
Merton Miller (U. of C. Nobelist)	> 100 part of < 5, 20x
Bill Gates (Harvard Drop-out)	> \$100Mn part of < \$1Mn, 100x

- ❑ Fields of analysis exposed to black swans cannot be approximated with small, unbiased, normal errors.
- ❑ **“It’s impossible”**

Tools Invalidated by Black Swans: *Volatility Measures*

Sharpe ratio	Kreps pricing
CAPM	Mango Rhum ordering
APM	RBC
Black Scholes	BCAR
Duration immunization	DFA
VAR	Chain ladder confidence intervals
TVAR	Experience rating credibility

Alternatives to Transactional Volatility in ERM

- Scenario testing
- Maxi min (When am I the best off if the worst happens?)
- Contract aggregate management
- Natural hedging (Go short on what you are long of)

P&C Industry “Black Swans”

WHAT?	WHEN?	WHY A SURPRISE?
Katrina	2005	Levee failure in a windstorm
Hurricane frequency cycles	2000s, 1960s, 1930s	Short memories
Enron/Andersen, etc.	2003	Clash of D&O and E&O; Clash across firms
9/11 Attacks	2001	Foreign Terror in U.S.; Clash of Property, Liability, Life, WC and Aviation
Soft Casualty market	1998 – 2001	Cycles effect coverage, reserving and price monitoring, not just rates
Tobacco Liability settlements	Late 1990s	Government warning does not pre-empt manufacturers’ duties
Northridge earthquake	1994	Unmapped fault
Mold	1990s	Excluded physical damage collected as “water damage” or BI liability
LMX spiral	Early 1990s	Higher layers exposed when same amount counted again
Construction defects	1980s – 1990s	“Damage to own work” exclusion bypassed

P&C Industry “Black Swans” (cont’d)

WHAT?	WHEN?	WHY A SURPRISE?
Piper Alpha	1988	Multiple insureds and multiple limits at one rig
Widespread reinsurance uncollectibles	1980s	Not Cat driven
Repetitive stress injuries in WC	1980s	Neither accident nor illness
European windstorms	1987	Short memories
Superfund	Early 1980s	First party clean up costs covered as third party liability
Tenerife runway crash	1977	Collision causes clash of limits
Products coverage for asbestos	1970s (BI), 1990s (PD)	Workers not covered as WC; Clean up costs as liability
Pharmaceutical class actions	1960s	Expansion of “batch clause” concept

Value of a Contract (Assuming No Parameter Risk)

Known Frequency	Risk Charge Factor			
	0.15	0.20	0.25	0.30
1	1.150	1.200	1.250	1.300
2	2.212	2.283	2.354	2.424
3	3.260	3.346	3.433	3.520
4	4.300	4.400	4.500	4.600
5	5.335	5.447	5.559	5.671
10	10.474	10.632	10.791	10.949
50	51.061	51.414	51.768	52.121

Increase in Value from Parameter Risk

Observed Frequency	Increase in Value from Parameter Risk					
	Prior Gamma's Coefficient of Variation					
	0.50	1.00	2.00	4.00	8.00	16.00
1	0.024	0.083	0.247	0.625	1.412	3.006
2	0.032	0.104	0.283	0.671	1.465	3.062
3	0.037	0.115	0.301	0.693	1.489	3.087
4	0.041	0.124	0.312	0.706	1.503	3.102
5	0.045	0.130	0.320	0.716	1.513	3.112
10	0.055	0.147	0.342	0.739	1.538	3.137
50	0.076	0.174	0.373	0.772	1.572	3.172

Result is shown relative to expected losses

For Comments or Question

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Please leave business card if interested in formal paper of results shown today.

Parameterization and Calibration of Actuarial Models
Eric Sandberg

2008 Enterprise Risk Management Seminar
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Underwriting Risk Shocks for Economic Capital: Two Approaches for Determining the 99.5th Percentile Shock for Parameter Risk

March 2008

Methods for Determining UR Parameter Shocks

Two potential methods for determining the underwriting risk (UR) parameter shocks are discussed here:

Method A – Calculate the best-fit trend line and variance for historical claims experience (using the A/E in each calendar year for your data points.) Using the variance for the experience, calculate an additional year of experience equal to the 99.5% percentile A/E and calculate a new best-fit trend line with this extra data point. The difference between the original and new best-fit trend lines is the UR parameter shock.

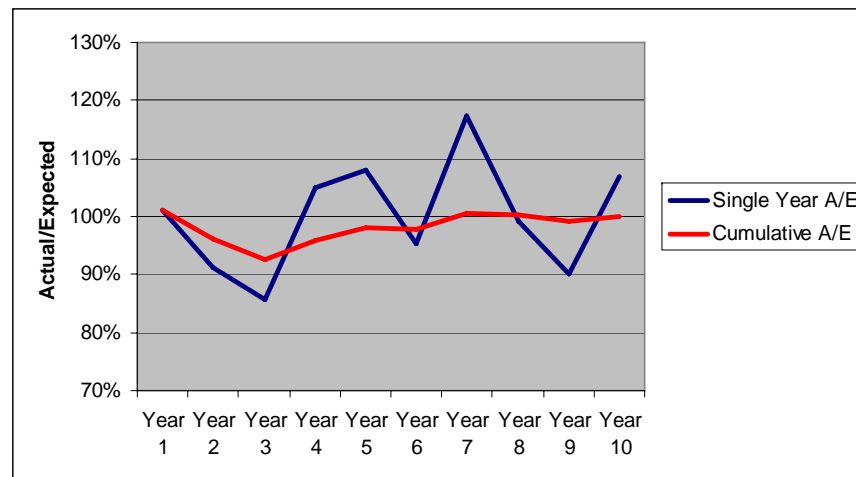
Method B – Use credibility/statistical techniques to determine the 99.5% confidence interval for the historical claims experience. The UR parameter shock is equal to the difference between the 99.5% confidence interval and the mean of the experience.

Simplified Example Assumptions

An example to illustrate the methodology and results of these two methods is included in the following slides. The assumptions used for this simplified example are:

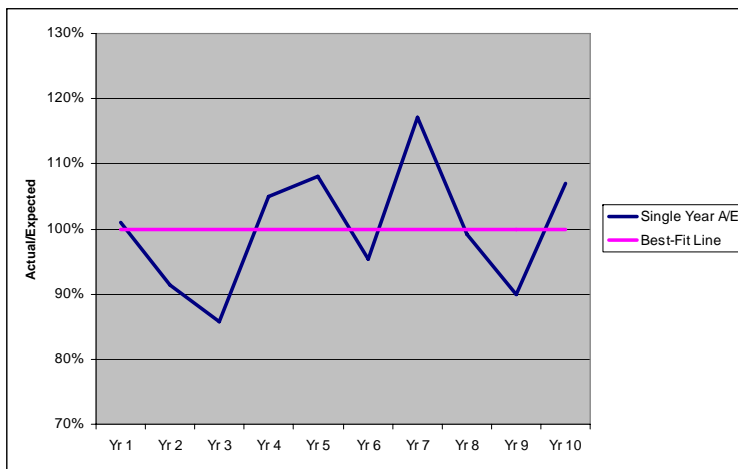
- Secular improvement is 0 (i.e., the trend is assumed to be 0)
- The number of deaths in a single year follows a binomial distribution
 - this examples uses A/E based on lives vs. amounts
- 10 years of historical claims experience is available (summary below)

	Single Year		Cumulative	
	A/E	Claims	A/E	Claims
Year 1	101%	101	101%	101
Year 2	91%	91	96%	192
Year 3	86%	86	93%	278
Year 4	105%	105	96%	383
Year 5	108%	108	98%	491
Year 6	95%	95	98%	586
Year 7	117%	117	101%	704
Year 8	99%	99	100%	803
Year 9	90%	90	99%	893
Year 10	107%	107	100%	1,000
Total	100%	1,000		



Simplified Example – Method A

Step 1 – Calculate best-fit line and mean & variance for historical A/E's



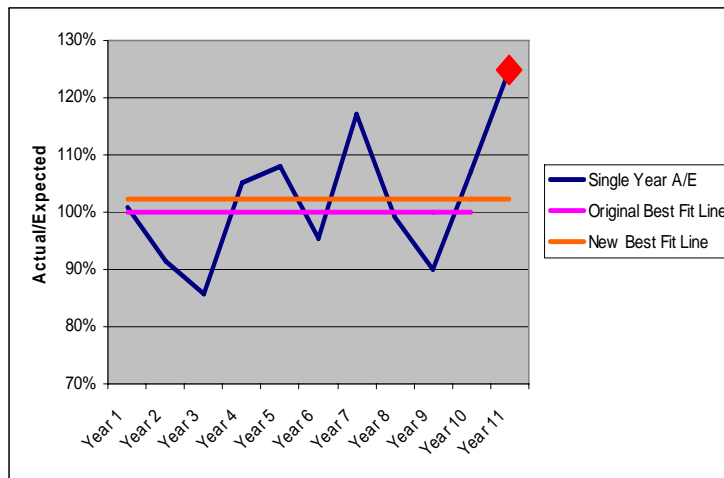
	Single Year A/E
Year 1	101%
Year 2	91%
Year 3	86%
Year 4	105%
Year 5	108%
Year 6	95%
Year 7	117%
Year 8	99%
Year 9	90%
Year 10	107%
Mean	100%
Variance	0.9%
Std Dev	9.6%

Step 2 – Calculate an additional year of experience equal to 99.5th percentile A/E

- 99.5th percentile A/E = $\mu + 2.58 \cdot \text{StdDev} = 100\% + 2.58 \cdot 9.6\% = 124.8\%$

Simplified Example – Method A

Step 3 – Add the additional data point (i.e. the 99.5th percentile A/E) and calculate the new best-fit line



Original Best Fit Line = 100.0% A/E

New Best Fit Line = 102.2% A/E

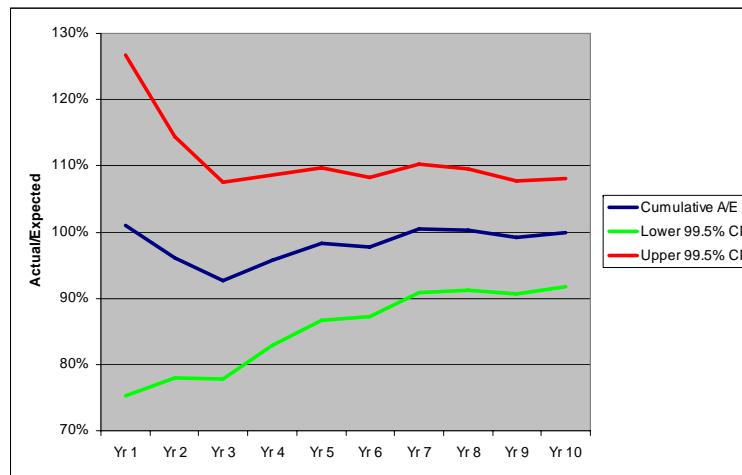
Result – The 99.5th percentile UR parameter shock is 2.2%

- Calculated as the difference between the Original and New best fit lines

Simplified Example – Method B

Step 1 – Calculate a 99.5% confidence interval around the aggregate A/E

- Note that the confidence interval shrinks as more years of experience are used



	Single Year A/E	Cumulative A/E	99.5% CI for Cum A/E
Year 1	101%	101%	+/- 25.7%
Year 2	91%	96%	+/- 18.2%
Year 3	86%	93%	+/- 14.9%
Year 4	105%	96%	+/- 12.9%
Year 5	108%	98%	+/- 11.5%
Year 6	95%	98%	+/- 10.5%
Year 7	117%	101%	+/- 9.7%
Year 8	99%	100%	+/- 9.1%
Year 9	90%	99%	+/- 8.6%
Year 10	107%	100%	+/- 8.1%

Result – The 99.5th percentile UR parameter shock is 8.1%

- Based on the 99.5% confidence interval for the experience A/E

Comments on Method A

Strengths

- Intuitive - magnitude of shock decreases as more data points are added
- Consistent with how an acquiring party might interpret our experience after a shock event

Weaknesses

- Doesn't take into account the credibility for each data point
 - All A/E's given the same "weight" regardless of number of claims
- Can produce very strange results when there are only a few data points
- Using too many data points can be an issue since it may dampen the size of the shock too much
 - AEGON limits to using 10 years of data

Comments on Method B

Strengths

- Intuitive - magnitude of shock decreases as more data points are added
- Takes into account the credibility for each data point and the experience set as a whole
- Can be used with 1 year of experience data or 20+

Weaknesses

- May not be consistent with how an acquiring party might interpret our experience after a shock event

Parameterisation and Calibration of Actuarial Models

AJ Czernuszewicz PhD FIA
PD England PhD

2008 Enterprise Risk Management Seminar
Tuesday April 15, 11:45 am - 1:00 pm



Parameterisation and Calibration of Actuarial Models

A.J. Czernuszewicz and P.D. England

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Abstract. In the underwriting risk component of standard simulation based capital models, parameters are usually obtained by fitting distributions to data, using maximum likelihood techniques. The parameters are usually then treated as fixed and known. In this presentation, we consider extending the methodology using Markov chain Monte Carlo techniques, commonly used in Bayesian statistics, to obtain distributions of parameters. Furthermore, the underlying data are often not known with certainty, but are themselves partially estimated, and it is uncommon to allow for this data uncertainty. We present a way to incorporate data uncertainty into the parameter estimation process. Model uncertainty is also considered simply by observing the effect on the results of using different distributions.

The methodology is illustrated with an example that fits a distribution to a history of loss ratios, and using the parameters obtained to forecast the aggregate claims in a new underwriting year. Stochastic reserving techniques are used to incorporate data uncertainty in the loss ratios. The price of a stop loss reinsurance applied to the aggregate distribution is then estimated assuming the parameters and data are known, and compared to the price that would be estimated if parameter uncertainty were considered, and also if parameter and data uncertainty were considered. We demonstrate that capital requirements and reinsurance prices are likely to be underestimated when parameter uncertainty and data uncertainty are not considered.

Keywords. Bayesian, Dynamic Financial Analysis, Internal Capital Models, Markov chain Monte Carlo, Parameter Uncertainty, Pricing, Stochastic Reserving.

Parameterisation and Calibration of Actuarial Models

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Agenda

- Description of a standard approach for non-life insurance risk
 - Including a simple example
- Taking account of parameter uncertainty
 - Extending the example
- An approach to including data uncertainty
 - Extending the example further!



Non-Life Insurance Risk A Standard Approach

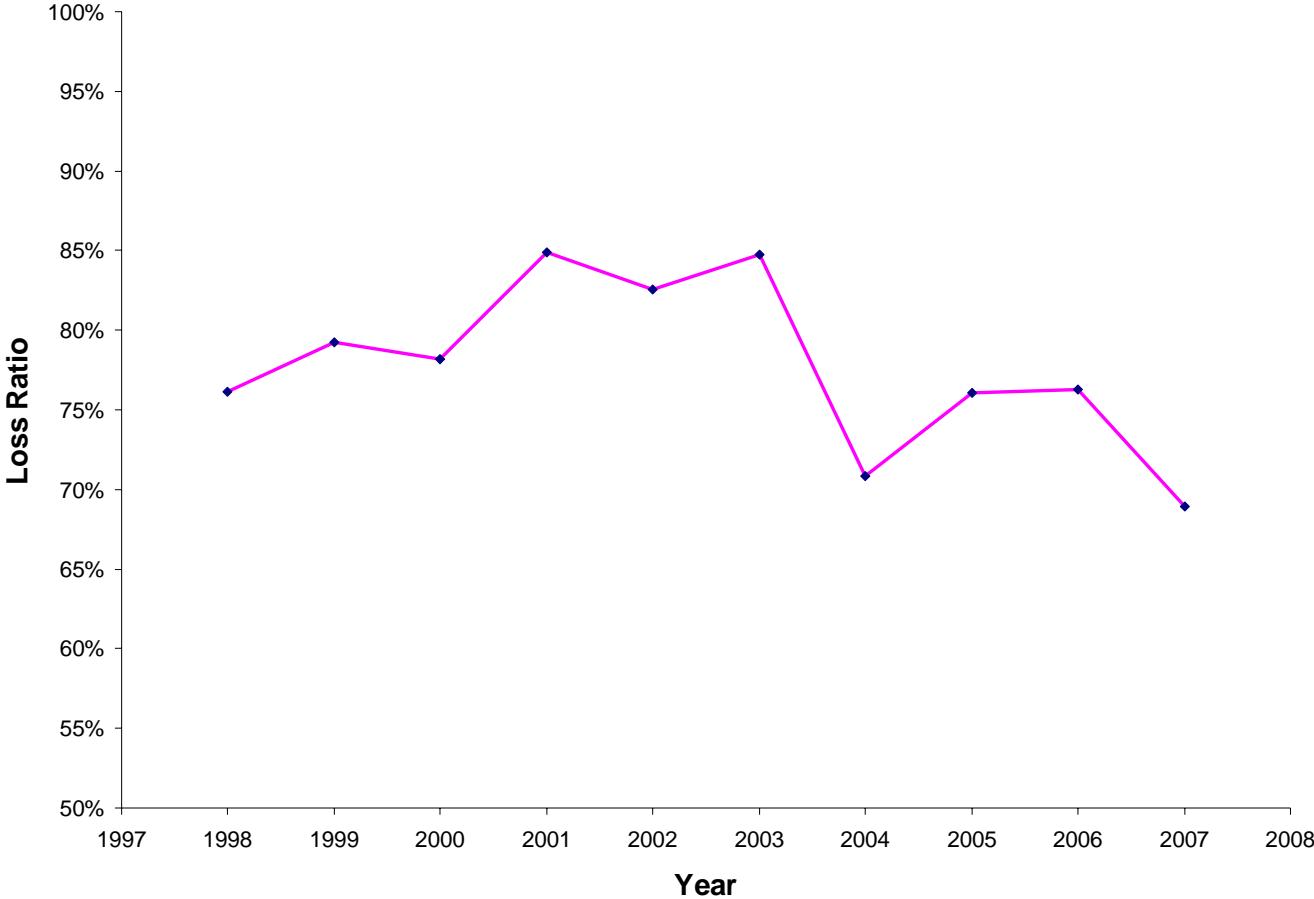
- Prior year liabilities (Reserve Risk by Line of Business)
 - Distributions of gross outstanding liabilities obtained using bootstrapping (or Bayesian approaches)
 - Net down using a simple methodology
 - Note: Parameter uncertainty *is* taken into account
- New business (Underwriting Risk by Line of Business)
 - Exposure/Premium given by Business Plan assumptions
 - Claims split between Attritional, Large and Catastrophe
 - Attritional claims simulated in aggregate
 - Large claims simulated individually
 - Catastrophe claims simulated by event
 - Net down using the actual reinsurance programme
 - Note: Parameter uncertainty *is not* usually taken into account



New Business A Simplified Example

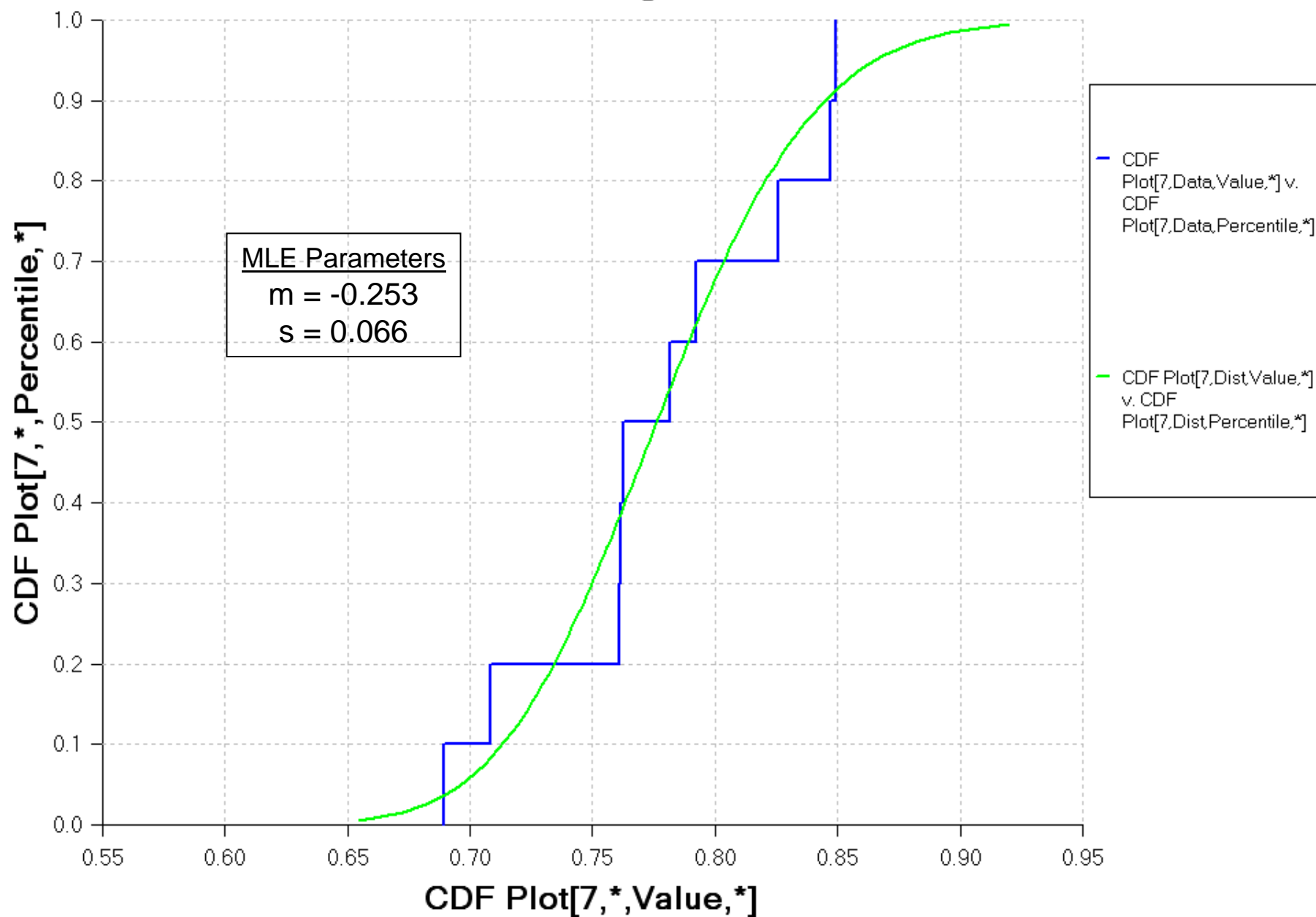
- Do not split between Attritional, Large and Catastrophe
 - Just simulate all claims in aggregate
- Given a 10 year history of premium and claims information:
 - Inflation and rate adjust the premiums
 - Inflation adjust the claims
 - Create a history of “loss ratios”
 - Fit a Log-normal distribution using maximum likelihood
- Given a Gross Premium estimate:
 - Simulate a forecast loss ratio from a Log-normal distribution, using the parameters obtained above
 - Multiply by the premium estimate to give an aggregate claims distribution

Loss Ratios (Rate and Inflation adjusted) by Year



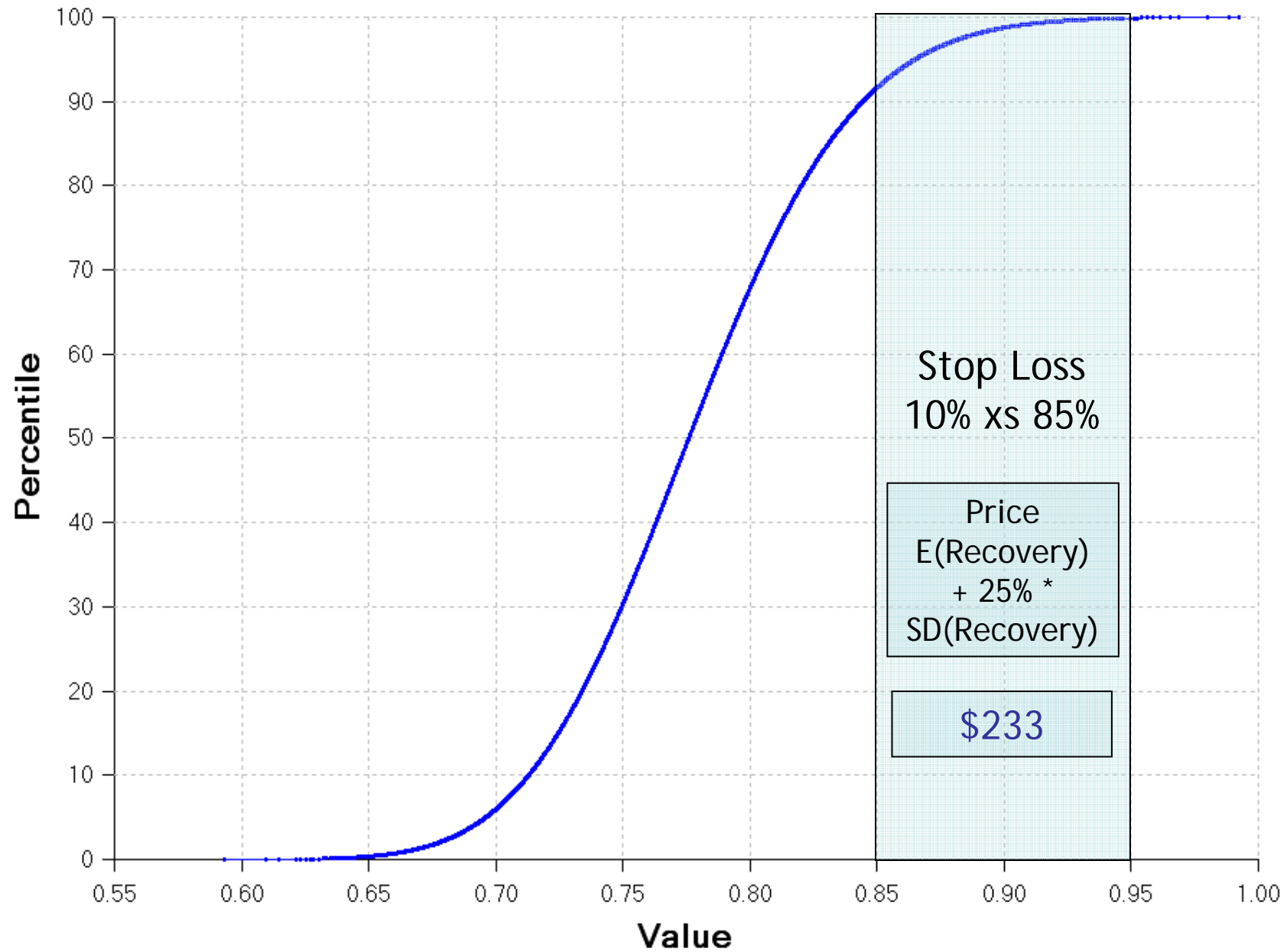
<i>Year</i>	<i>Expected Loss Ratio</i>
1998	76%
1999	79%
2000	78%
2001	85%
2002	83%
2003	85%
2004	71%
2005	76%
2006	76%
2007	69%
2008	?

CDF Plot - LogNormal



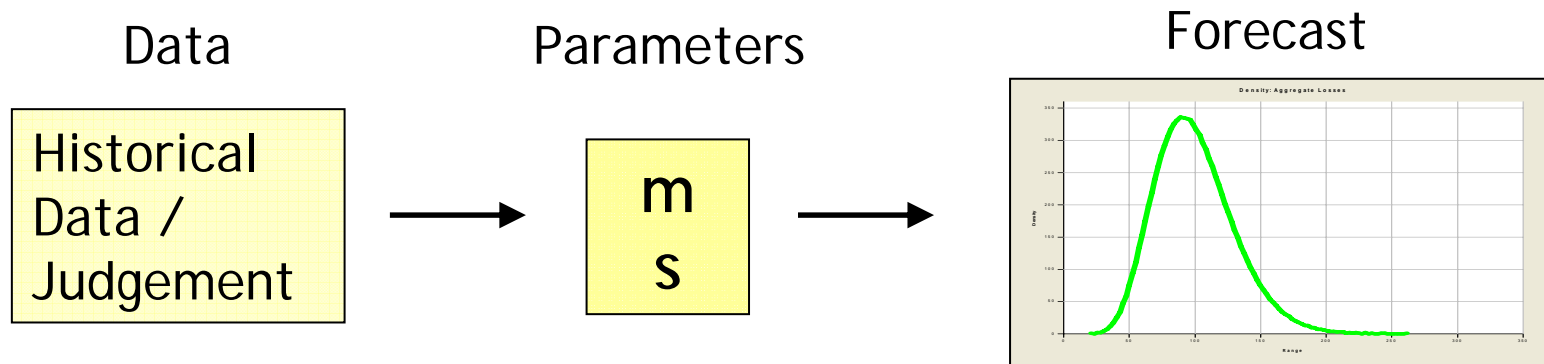


Gross Loss Ratios (LogNormal)



The Story So Far

- The data and parameters are considered fixed and known



- But the parameters were obtained from a sample of data, so we should take account of the precision of the parameters

Quantifying Parameter Uncertainty

- “Classical Statistics”
 - Asymptotic distribution of ML estimates
- Bootstrapping
 - Resample with replacement from the original sample
 - Re-fit model to each sample
 - Gives a (joint) distribution of parameters
- Bayesian Statistics
 - Use Bayes’ Theorem to determine posterior distribution of the parameters, given the data and prior beliefs

Parameter Uncertainty: Likelihood-based

- Covariance matrix of parameter estimates is inverse of Fisher Information Matrix

$$Cov(\theta) = [I(\theta)]^{-1} \quad I(\theta_{rs}) = -nE\left(\frac{\partial^2 l(\theta)}{\partial \theta_r \partial \theta_s}\right)$$

- Standard error of parameters is square root of diagonal elements

	Estimate	Std Err
m	-0.253	0.021
s	0.066	0.015

- But how good is an assumption of multivariate normality?

Forecasting with Parameter Uncertainty: A Bayesian Approach



- Select a distribution for the data
- Select a prior distribution of the parameters
- Form posterior distribution by revising the prior in light of the data sample
- Simulate parameters from the posterior distribution using MCMC methods
 - Gives (joint) distribution of parameters
 - Note: If non-informative Uniform priors are used, the posterior log-likelihood is essentially the log-likelihood of the data (plus a constant)
- Simulate the forecast conditional on the (simulated) parameters

Gibbs Sampling

Posterior likelihood:

$$f(\theta | X) \propto L(\theta | X) \pi(\theta)$$

Gibbs Sampling:

$$\theta_1^{(1)} \sim f(\theta_1 | \theta_2^{(0)}, \dots, \theta_k^{(0)})$$

$$\theta_2^{(1)} \sim f(\theta_2 | \theta_1^{(1)}, \theta_3^{(0)}, \dots, \theta_k^{(0)})$$

⋮

$$\theta_j^{(1)} \sim f(\theta_j | \theta_1^{(1)}, \dots, \theta_{j-1}^{(1)}, \theta_{j+1}^{(0)}, \dots, \theta_k^{(0)})$$

⋮

$$\theta_k^{(1)} \sim f(\theta_k | \theta_1^{(1)}, \dots, \theta_{k-1}^{(1)})$$

A generic sampling algorithm (e.g. ARS/ARMS) is used where the conditional distributions cannot be recognised

Gibbs Sampling



Initial Parameters	$\theta_1^{(0)}$	$\theta_2^{(0)}$.	.	$\theta_k^{(0)}$
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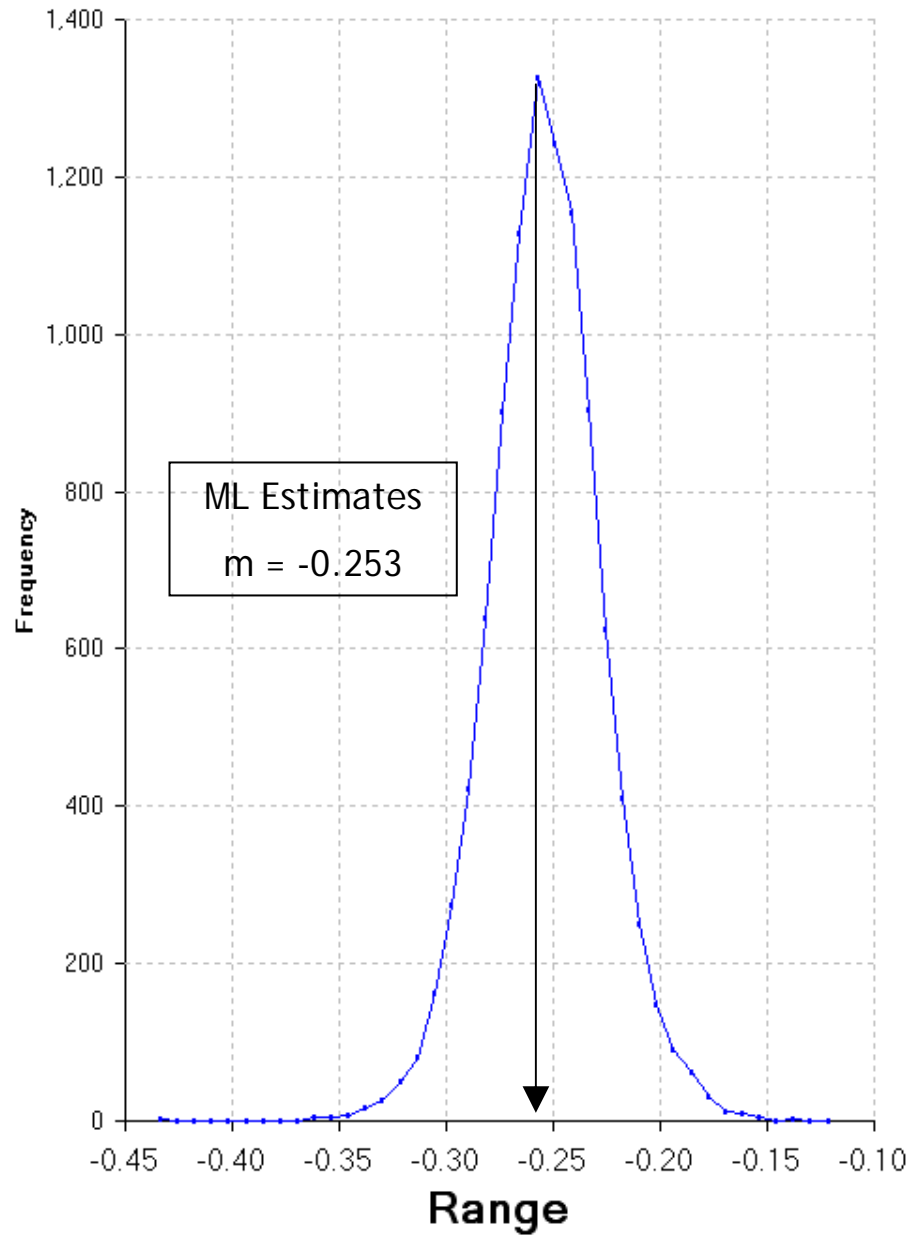
Iteration

1	$\theta_1^{(1)}$	$\theta_2^{(1)}$.	.	$\theta_k^{(1)}$
2	$\theta_1^{(2)}$	$\theta_2^{(2)}$.	.	$\theta_k^{(2)}$
3	$\theta_1^{(3)}$				
.					
.					
10,000					

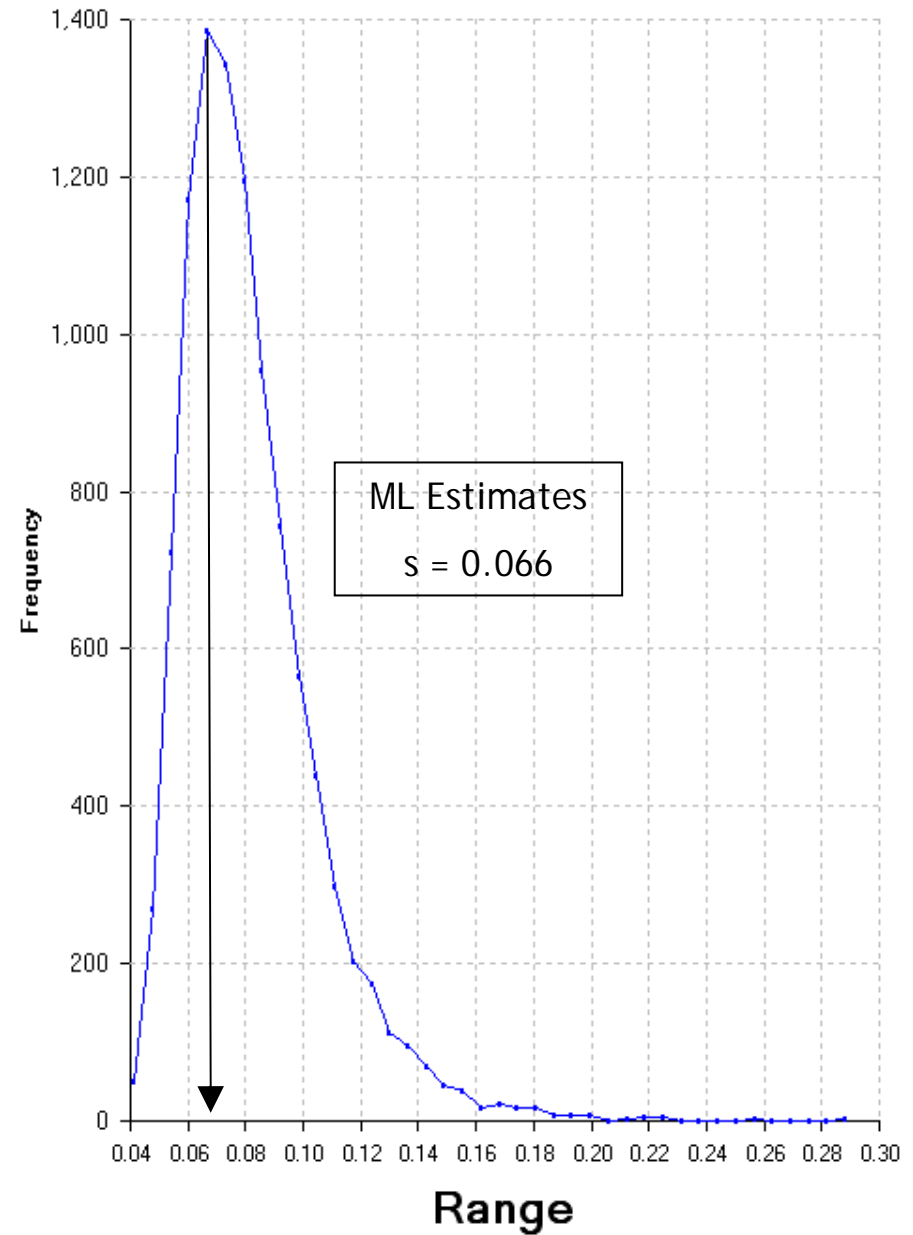
Where the joint log-density cannot be factorised, it is treated sequentially as a function of 1 parameter, conditional on the most recent value of all other parameters.

Where the log-density is not from a standard distribution, a generic sampling routine is used, such as Adaptive Rejection Metropolis Sampling (ARMS)

Distribution of m

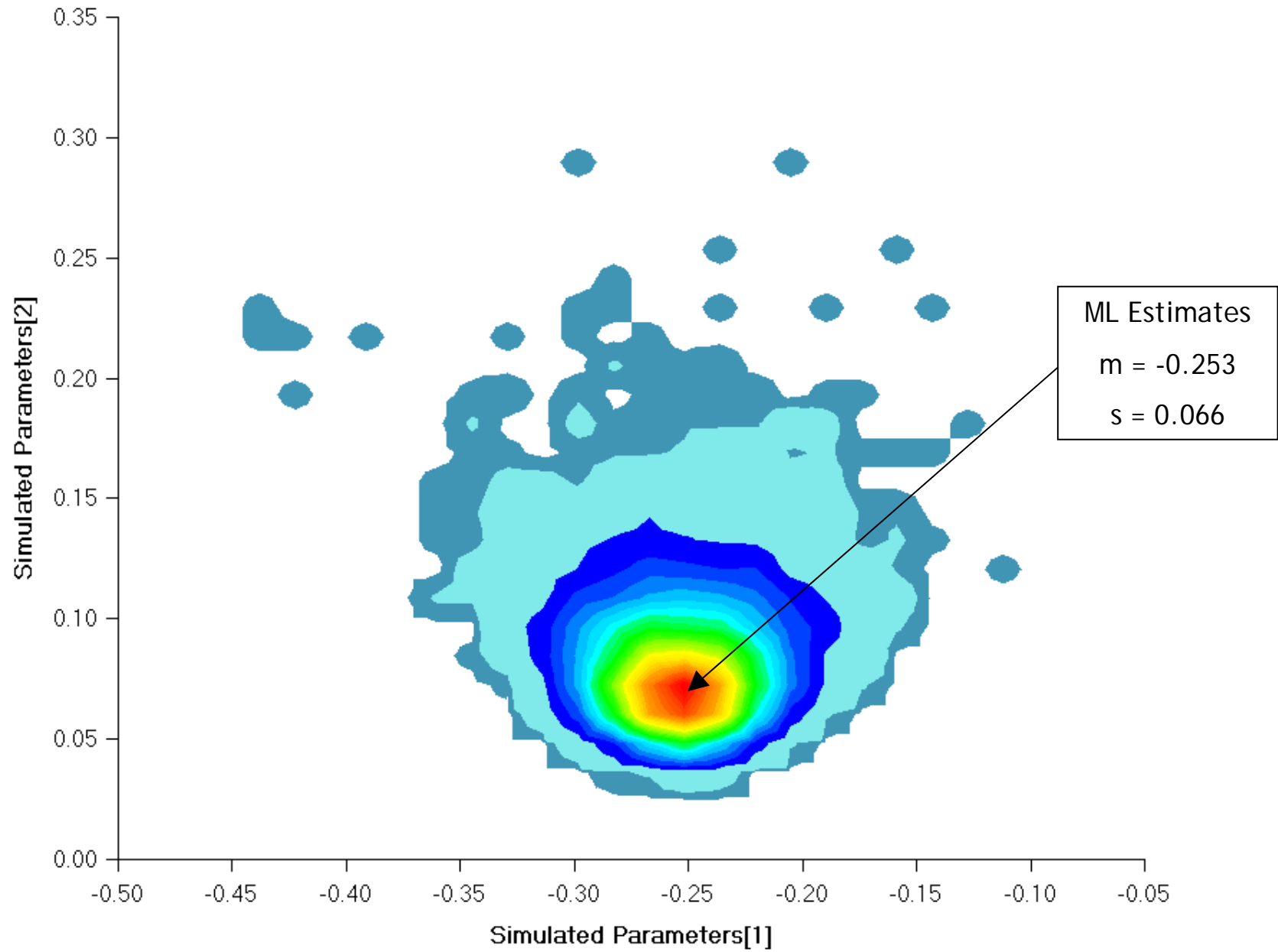


Distribution of s





Value Scatter



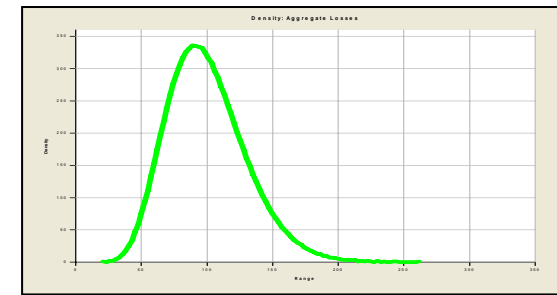
Incorporating Parameter Uncertainty

Excluding parameter uncertainty

Historical
Data /
Judgement

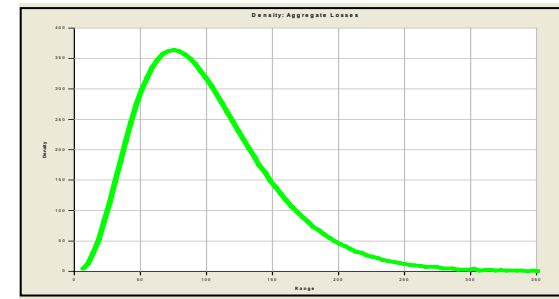
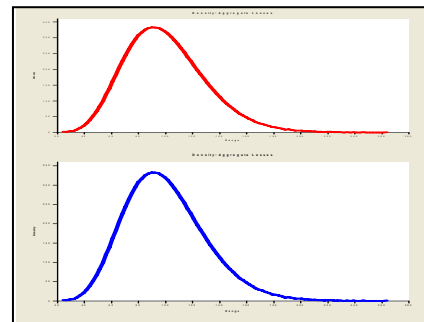


m
s



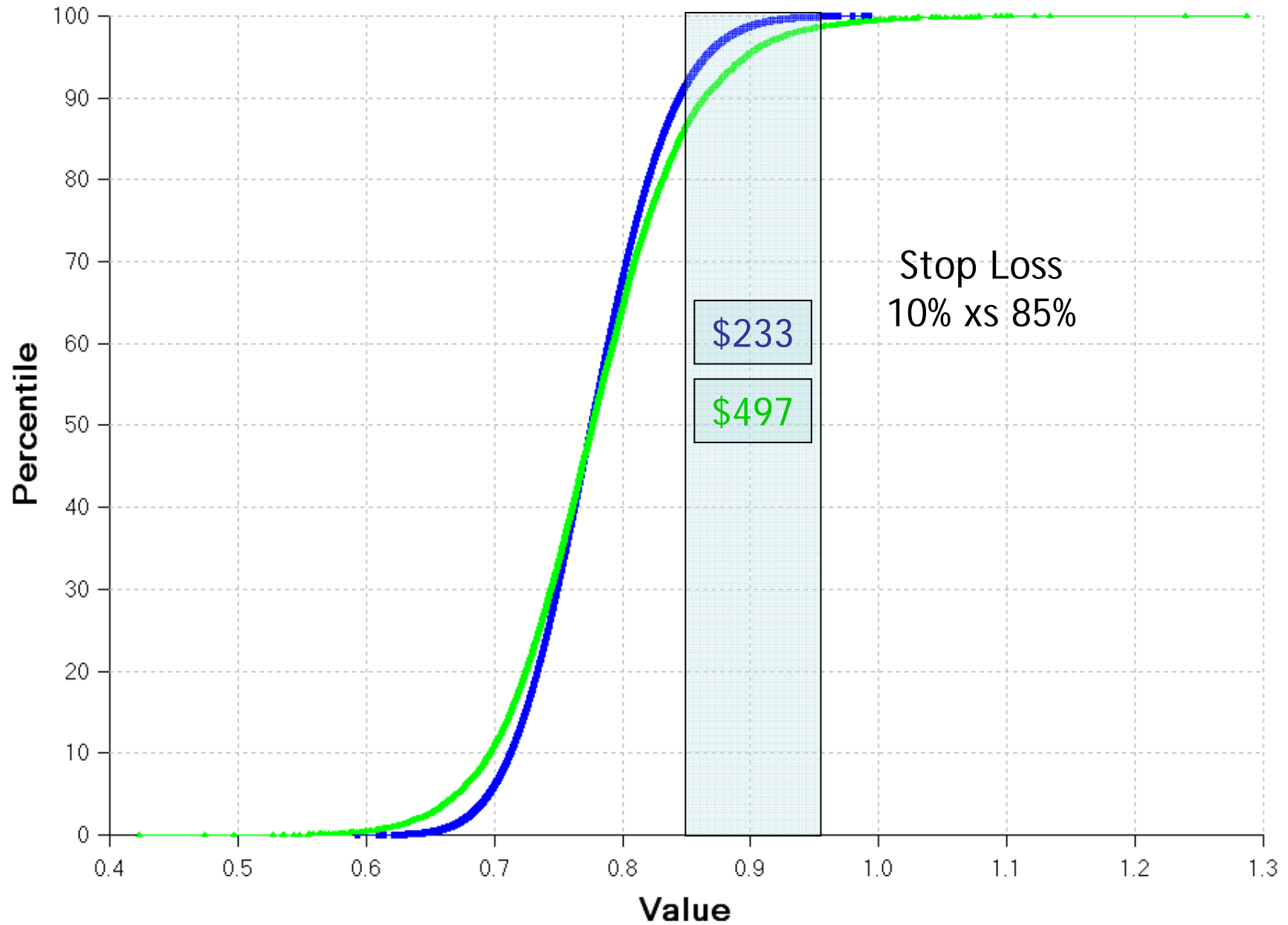
Including parameter uncertainty

Historical
Data /
Judgement





Gross Loss Ratios (Lognormal with Parameter Uncertainty)



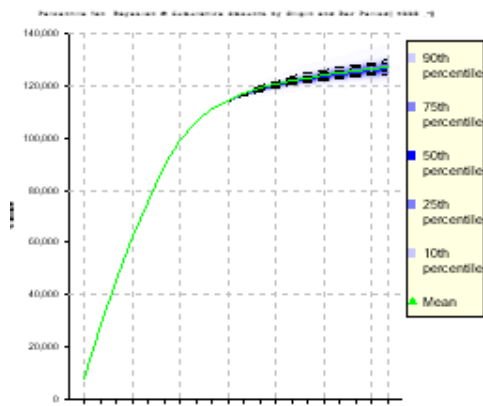
Year	1	2	3	4	5	6	7	8	9	10
1998	7,729	28,846	45,003	62,748	75,460	89,604	98,994	106,163	110,978	114,505
1999	9,111	33,697	51,559	66,717	82,816	90,475	95,801	103,245	105,940	
2000	6,421	27,266	43,315	66,449	79,884	88,105	91,245	94,959		
2001	6,055	26,210	43,261	58,119	68,994	75,261	81,590			
2002	4,475	17,318	33,698	45,968	60,533	68,071				
2003	4,678	16,836	28,422	42,352	56,817					
2004	3,208	10,285	20,994	30,525						
2005	2,370	10,043	18,953							
2006	2,593	10,484								
2007	2,118									

Inflation adjusted cumulative claim amounts

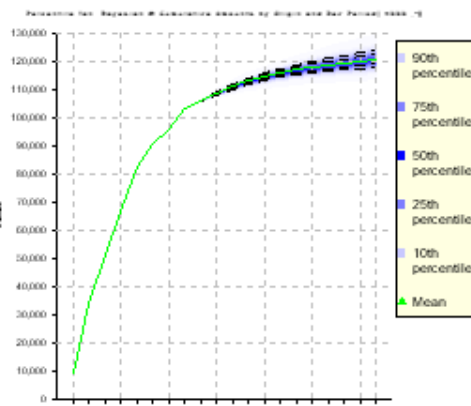
Year	Rate and Inflation Adjusted Premium	Expected Ultimate Claims	Expected Loss Ratio
1998	166,673	126,878	76%
1999	152,536	120,857	79%
2000	144,144	112,653	78%
2001	120,365	102,198	85%
2002	111,673	92,231	83%
2003	102,386	86,747	85%
2004	79,816	56,547	71%
2005	64,631	49,172	76%
2006	59,654	45,497	76%
2007	51,742	35,666	69%
2008	50,000	?	?

Data Uncertainty:

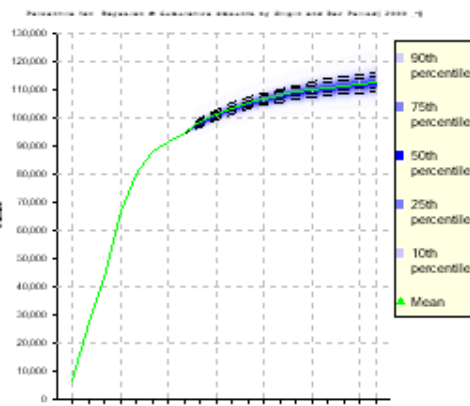
The expected loss ratios are derived from a forecast of the ultimate claims, which is itself uncertain



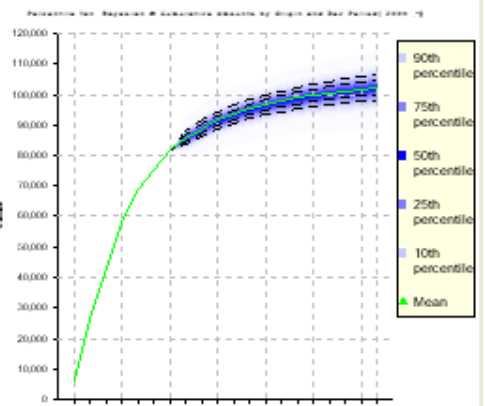
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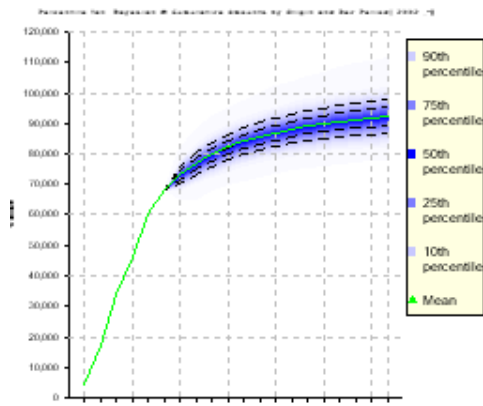
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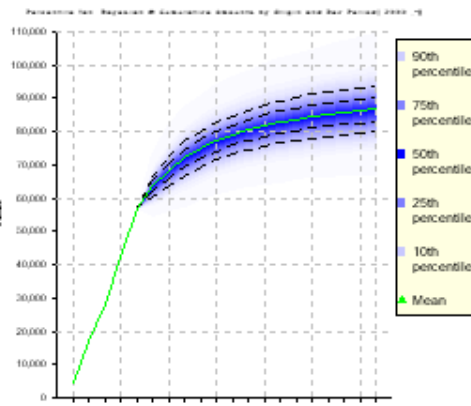
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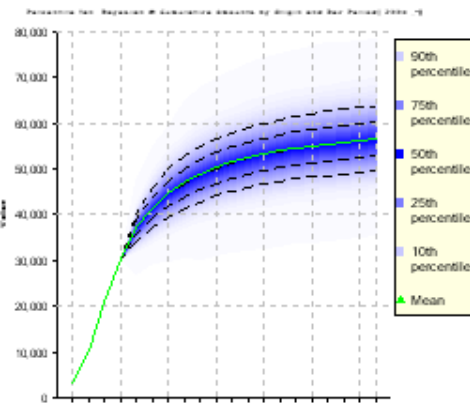
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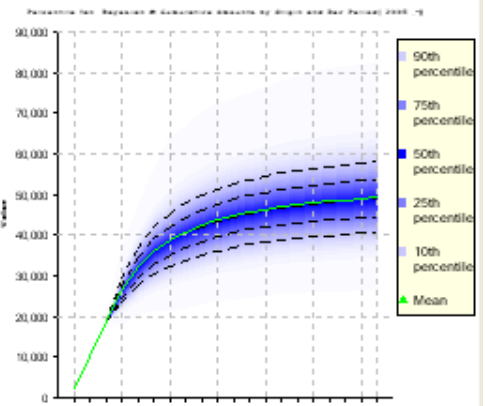
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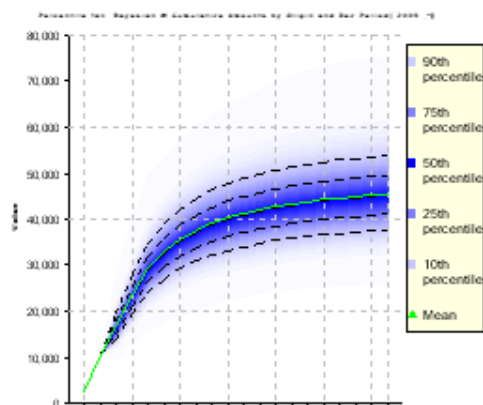
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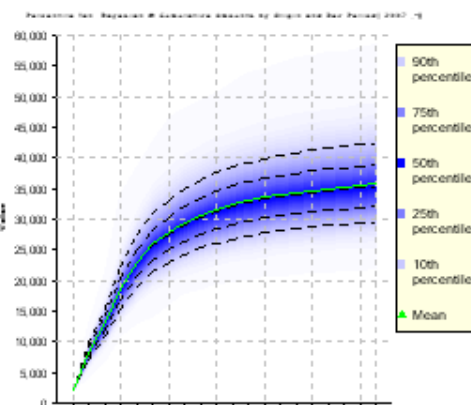
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Forecast the cumulative claims by origin year

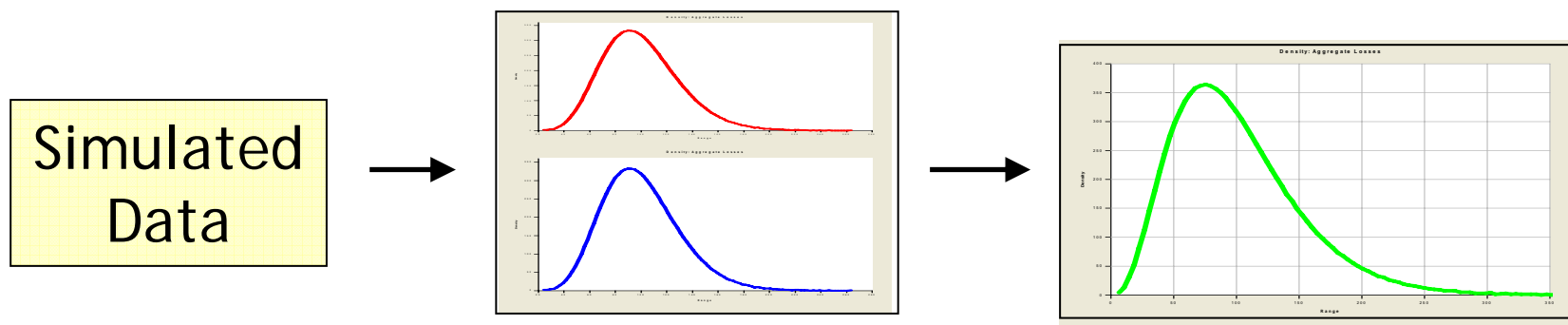
Bayesian version of Mack's model, with curve fitting for tail estimation

England PD & Verrall RJ (2006) Annals of Actuarial Science, Vol 1 Part II

Incorporating Parameter and Data Uncertainty: A Pragmatic Approach



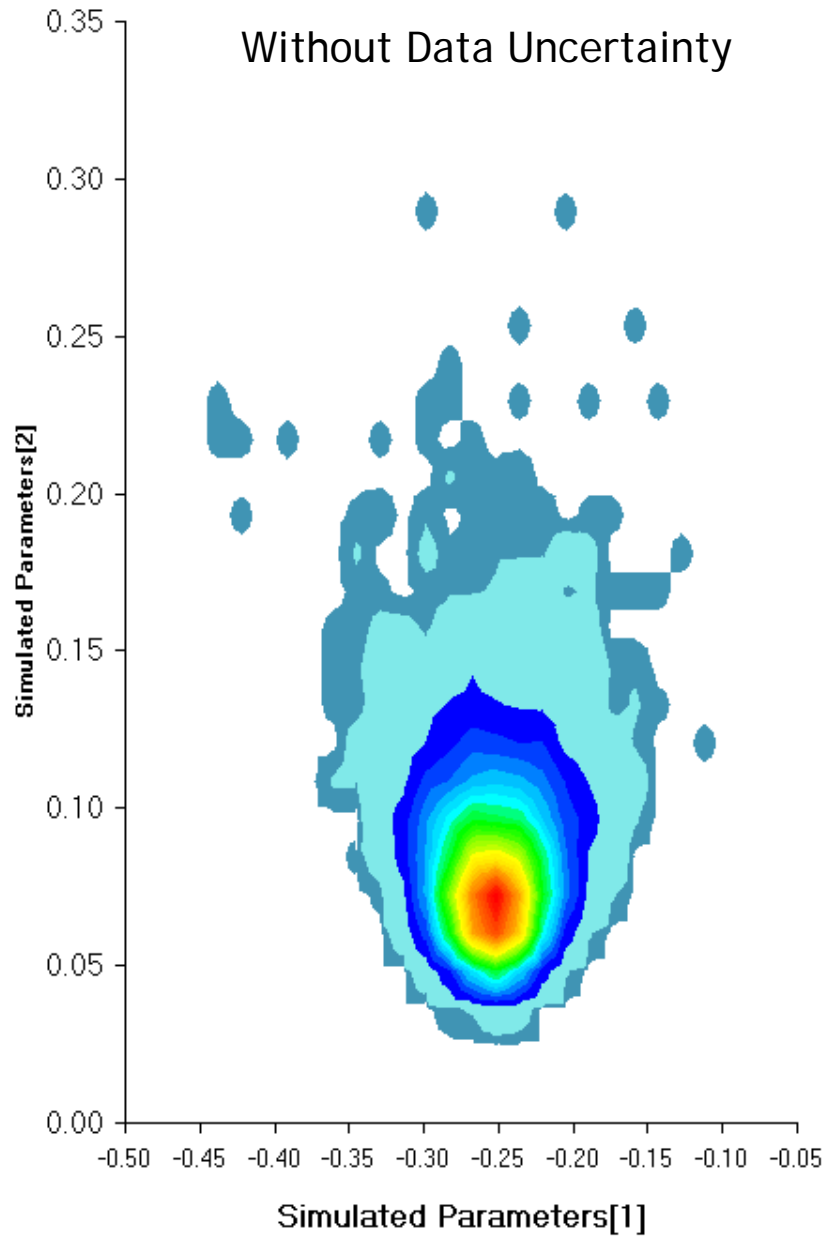
- Instead of considering the data as fixed and known, use simulated data instead
- Then obtain a distribution of parameters, conditional on the simulated data
- Then forecast, conditional on the parameters





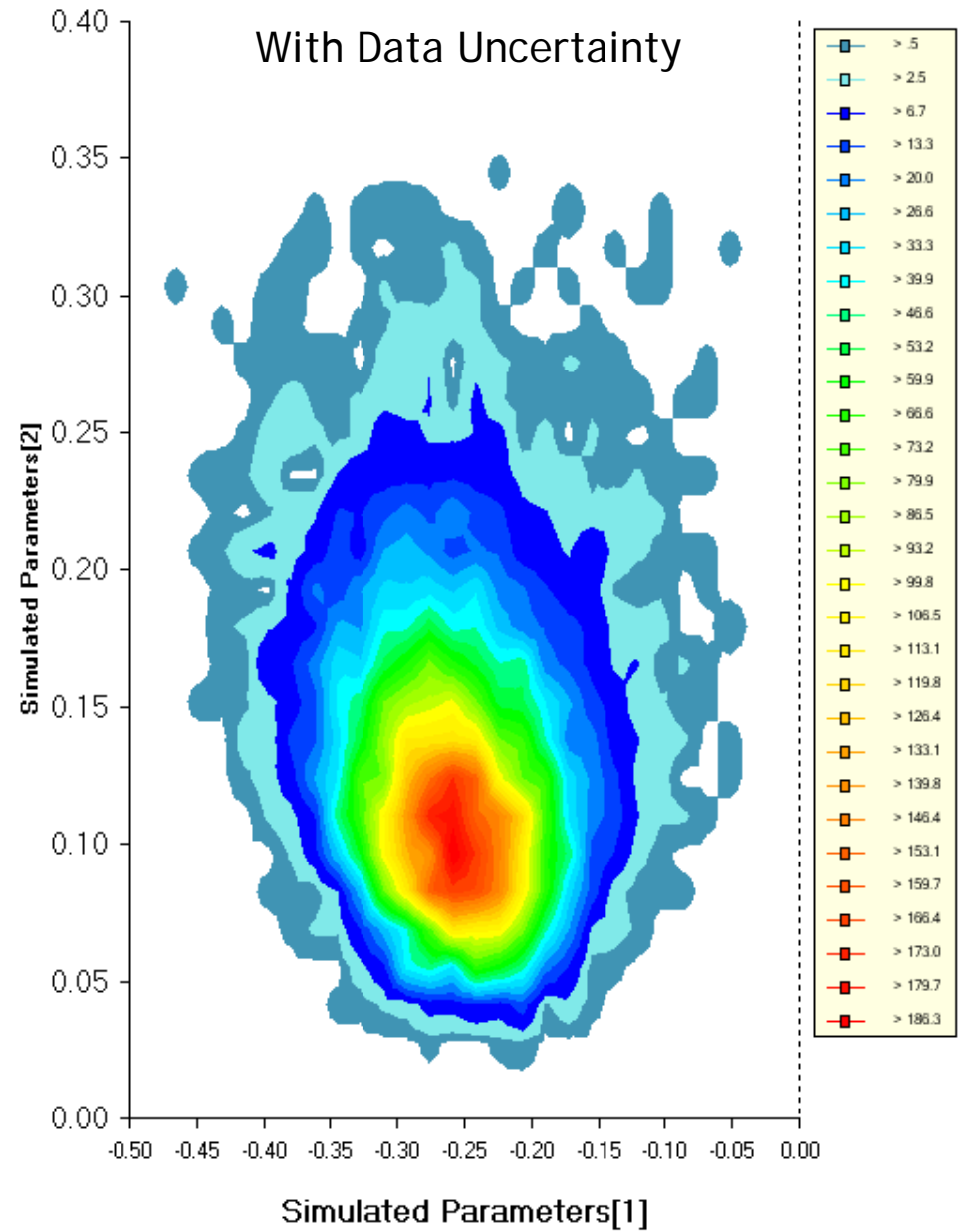
Value Scatter

Without Data Uncertainty



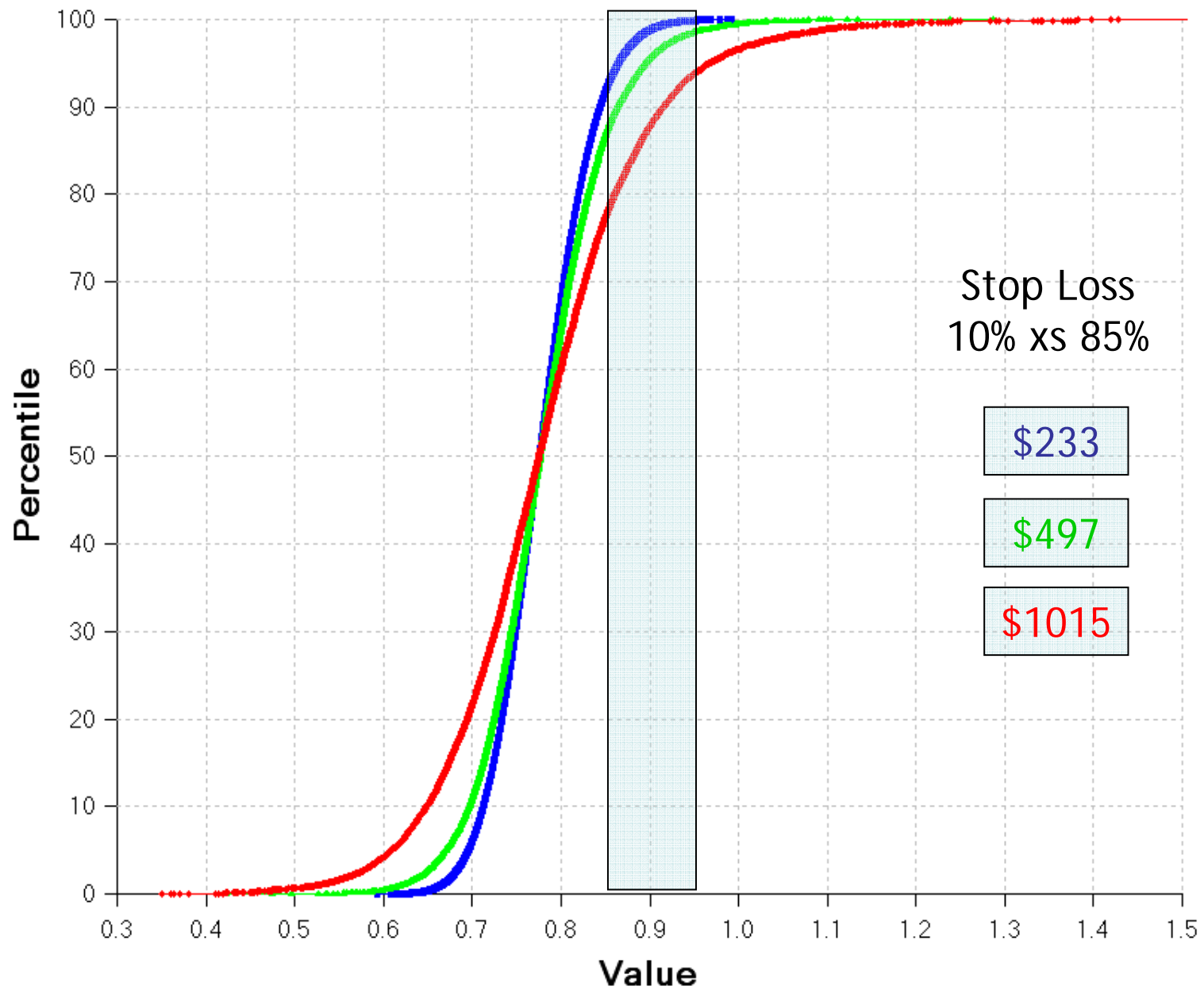
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With Data Uncertainty





Gross Loss Ratios (Lognormal with data and parameter uncertainty)



Conclusion

- Parameter uncertainty is often ignored
- Statistical methods exist to quantify this uncertainty
- The impact on model results can be significant
 - It has largest effect with small data sets...
 - ...especially in the tails of distributions
- Ignoring parameter uncertainty could therefore underestimate capital requirements, and underestimate (re-)insurance prices
- Ignoring data uncertainty will only make matters worse

Parameter Uncertainty

FSA Comment



"We do not think it appropriate to ignore this risk altogether. In particular, informal discussions with market participants suggest that applying parameter uncertainty can have a significant impact on the underlying ICA"

FSA Insurance Sector Briefing: ICAS - one year on
(Nov. 2005)

Reverend Thomas Bayes (1702-1761)



“Modern computer simulation techniques open up a wide field of practical applications for risk theory concepts, without the restrictive assumptions, and sophisticated mathematics, of many traditional aspects of risk theory”.

- Daykin, Pentikainen and Pesonen (1996)

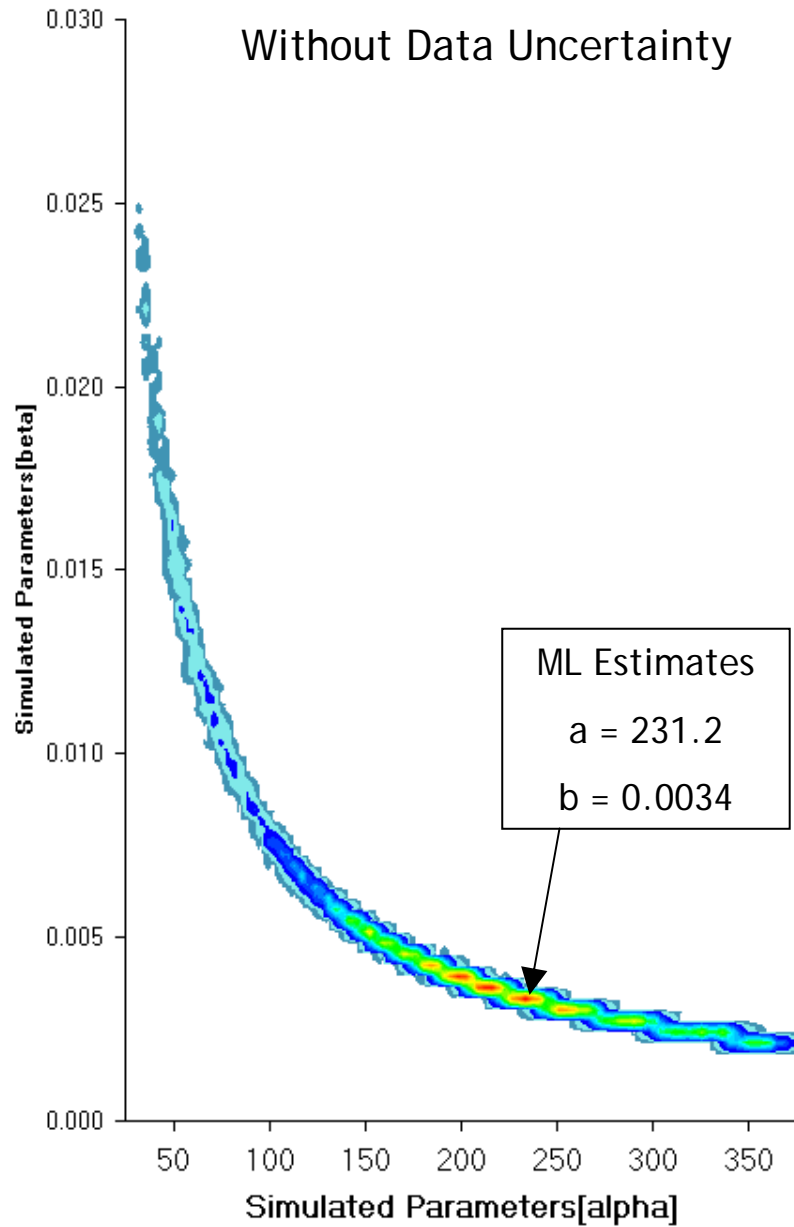
Model Uncertainty

- Try different distributions
- For example, using a Gamma(a, b) distribution

	ML Estimate	Std Err
a	231.2	103.6
b	0.0034	0.0015

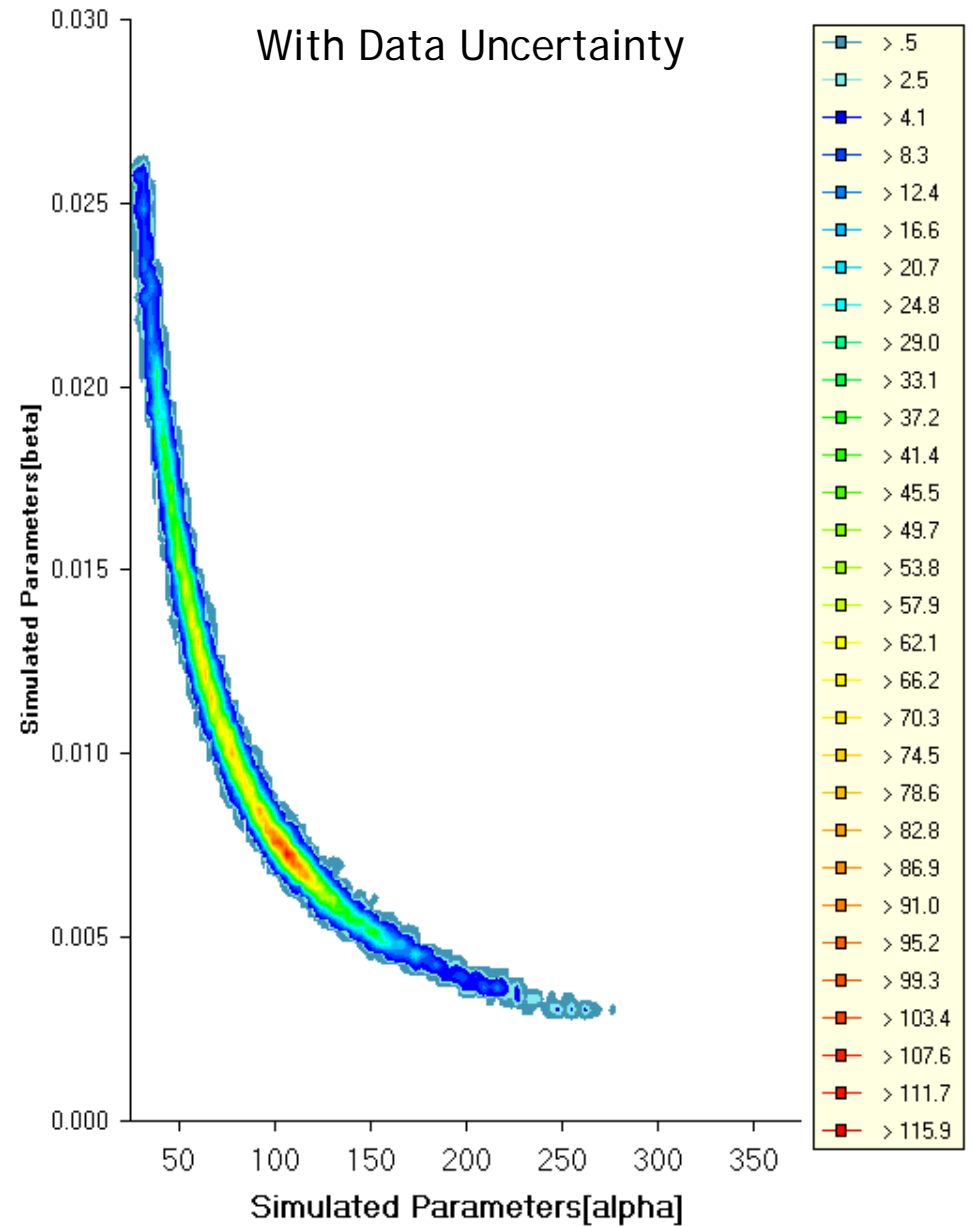
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Without Data Uncertainty



Value Scatter

With Data Uncertainty





Gross Loss Ratios (Gamma with data and parameter uncertainty)

