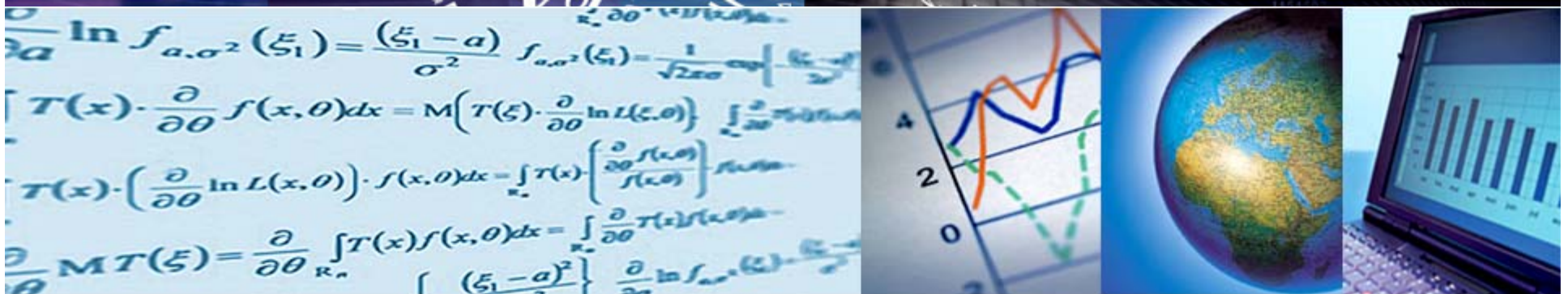


Portfolio Replication – Variable Annuity Case Study



Curt Burmeister
Senior Director
Algorithmics

Algorithmics

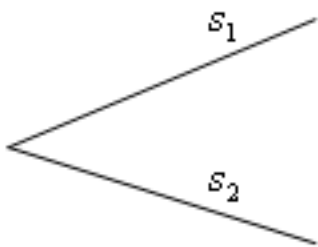


What is Portfolio Replication?

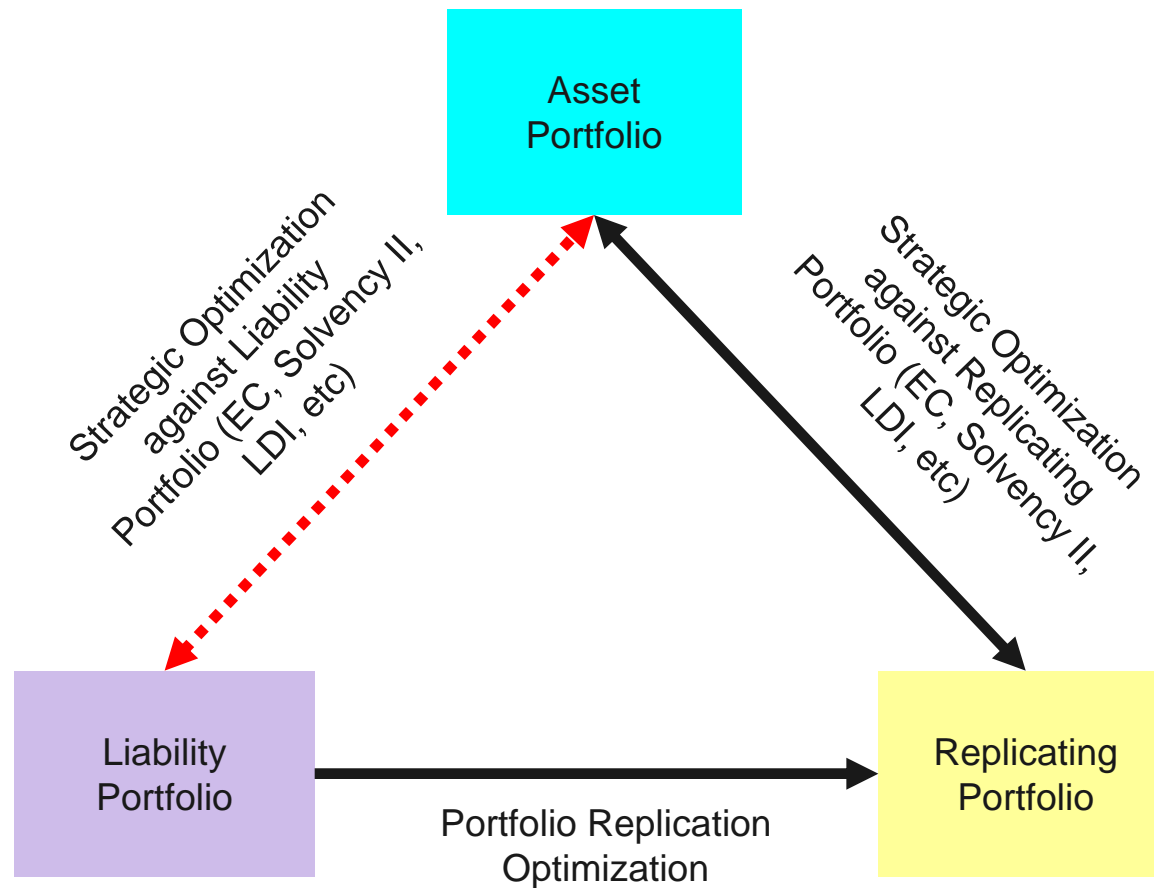
“To find a portfolio of assets whose value is equal to the value of a liability portfolio under today’s market conditions and future market conditions (scenarios.)”

Value today

Value at target date

instruments	scenarios	instruments	liability
q_1 q_2 q_3		q_{11} q_{21} q_{31}	l_1 scenario 1
		q_{12} q_{22} q_{32}	l_2 scenario 2

Why Replicating Portfolios?



Why Replicating Portfolios?

1. **Performance** - It can be difficult for existing actuarial projection systems run the 10,000 – 50,000 scenarios required for an economic capital calculation but it is typically much easier for asset systems to run 50,000 scenarios for an asset portfolio.
2. **Leverages existing liability models** - Most firms have invested years customizing their liability models. The portfolio replication approach uses the cash flows generated from the existing actuarial models.
3. **A common API** - Many firms use multiple actuarial projection systems throughout the organization and the portfolio replication process defines a “de facto” interface to all the actuarial projection systems

Uses of Replicating Portfolios

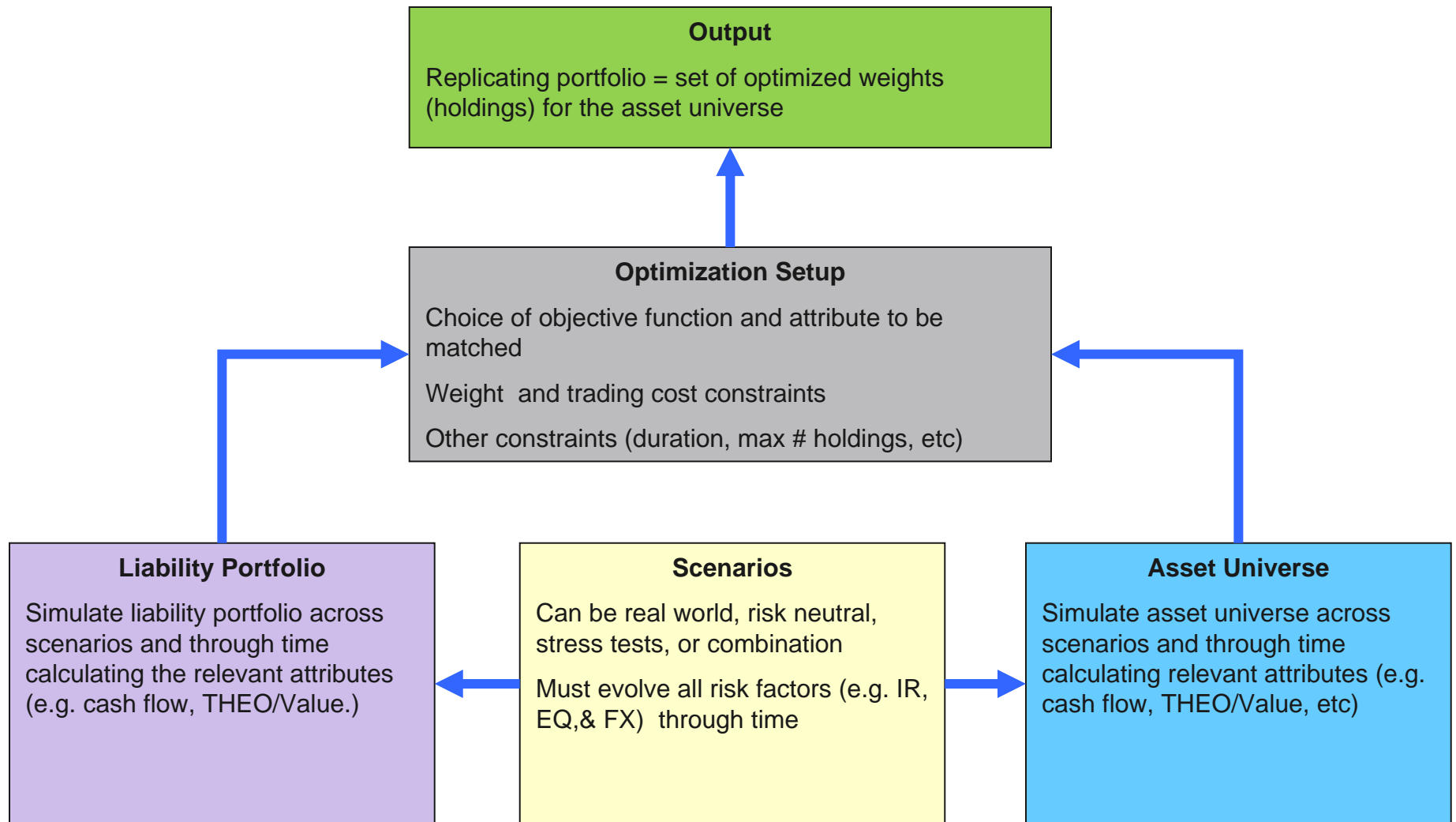
Current applications

1. Economic capital
2. Sensitivity analysis and hedging
3. Benchmarks for investment managers

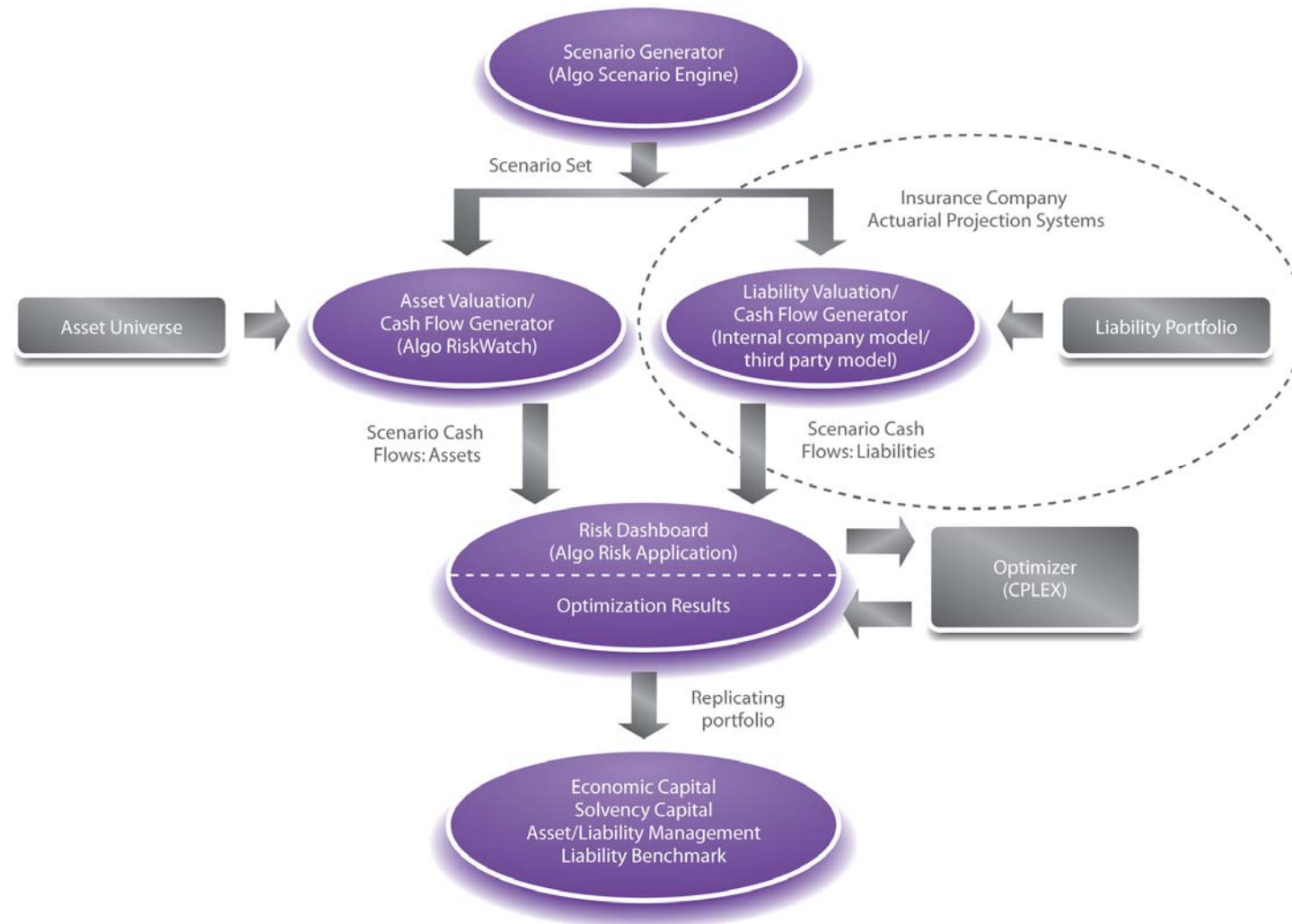
Future applications

1. Regulatory capital & Solvency II

Steps in the Portfolio Replication Process



Algo Risk Replicating Portfolio Architecture



Art versus Science

1. What type of scenarios and how many (i.e. risk neutral or real world?)
2. Which assets are included in the tradable universe (type, maturity, strike, underlying, etc?)
3. What is the objective function?
4. What are the time steps (i.e. bucketing?)
5. What are the constraints?

Minimize Cash Flow Deviations

$$\min \sum_{t \in \text{Times}} w_t \sum_{s \in \text{Scenarios}} p^s \left(\sum_{a \in \text{Assets}} x_a cf_a(t, s) - cf_L(t, s) \right)$$

w_t weight on the errors at time t

p^s probability of scenario s

x_a position in asset a

$cf_a(t, s)$ is the cash flow for asset a on scenario s and time t

$cf_L(t, s)$ is the cash flow for the liability portfolio on scenario s and time t

Variable Annuity Case Study

Variable Annuity Portfolio – # Policies by GMDB type

~15000 Total Policies

Policy Date	ROP	Roll-up	Ratchet	Combo	Total
< 2002	2428	1057	1960	664	6109
2002-2005	2165	444	879	1583	5071
2005-2007	1389	0	682	2147	4218
Total	5982	1501	3521	4394	15398

Variable Annuity Portfolio – # Policies by ITM Band

- 1: MGDB Guarantee / Account Value < 0.9
- 2: $0.9 \leq$ MGDB Guarantee / Account Value ≤ 1.1
- 3: $1.1 \leq$ MGDB Guarantee / Account Value

ITM Band	ROP	Roll-up	Ratchet	Combo	Total
1	4391	455	984	361	6192
2	1396	419	2241	3587	7645
3	195	627	296	446	1567
Total	5982	1501	3521	4394	15398

Variable Annuity Cash Flows

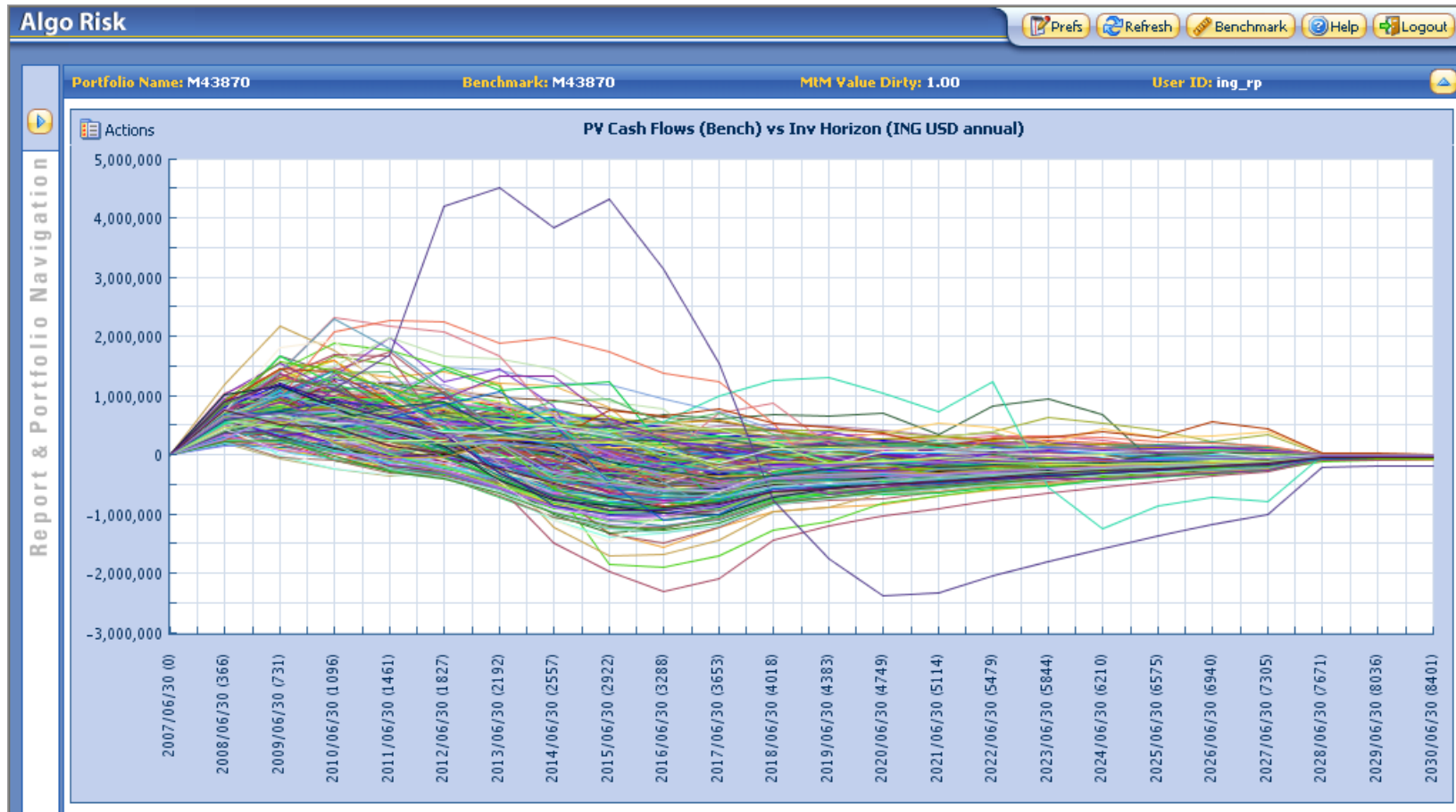
Scenario dependent cash flows include

1. guaranteed minimum death benefit
2. general account release
3. commissions
4. expenses
5. mortality/expense charge
6. revenue sharing
7. surrender charges
8. per policy fees

Market Indices

1. US Interest Rate Curve
2. Russell 1000
3. S&P 500
4. Nasdaq 100
5. MSCI EAFE Index
6. MSCI Emerging Market Free
7. MSCI REIT Index
8. Lehman US Aggregate

Annual Variable Annuity Cash Flows



Replicating Universe and Optimization Setup

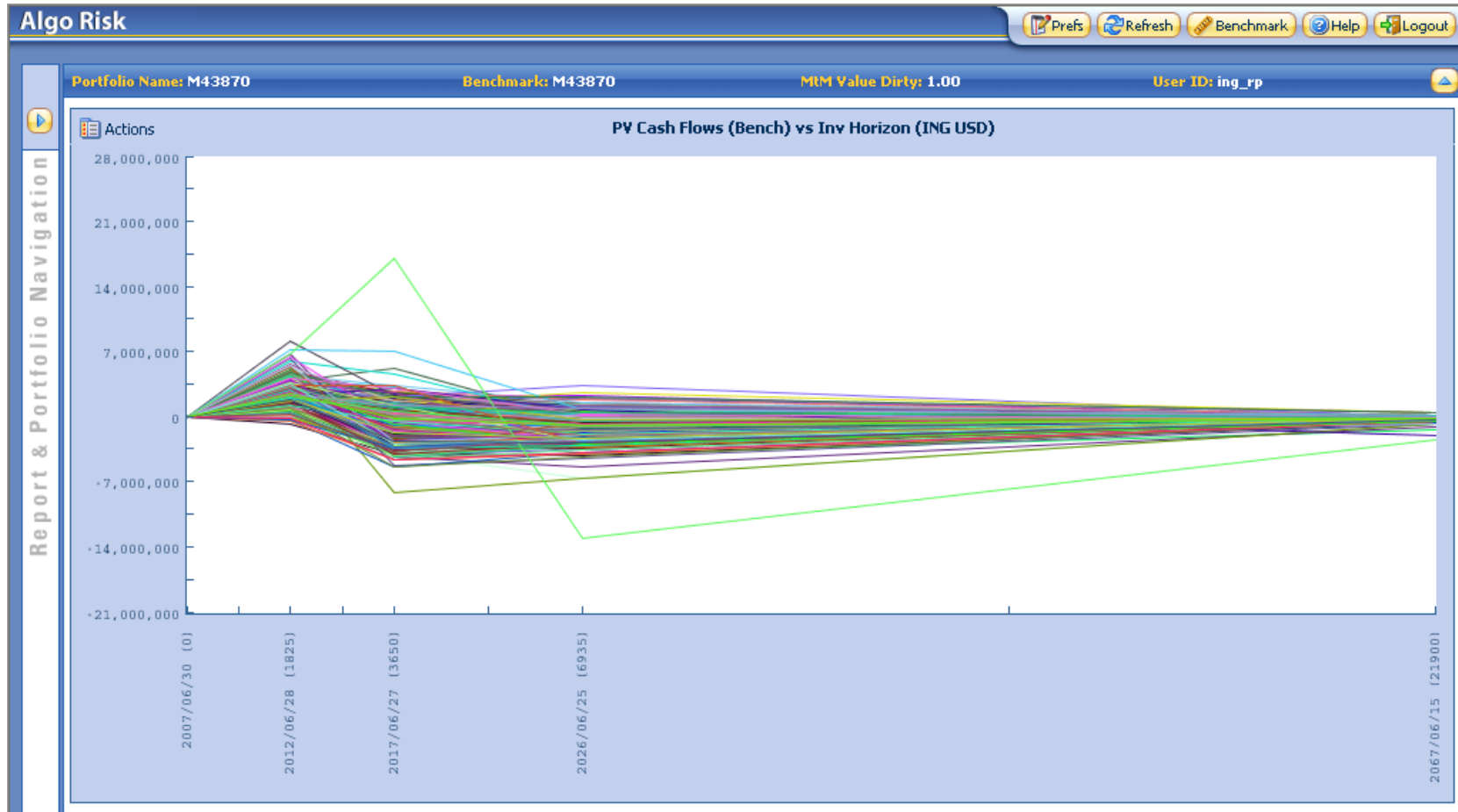
Replicating Instruments

- Zero Coupon Bonds
- Swaptions (physical settlement),
- Equity Forwards (on each index)
- European Equity Options (on each index)

Optimization Setting Choices

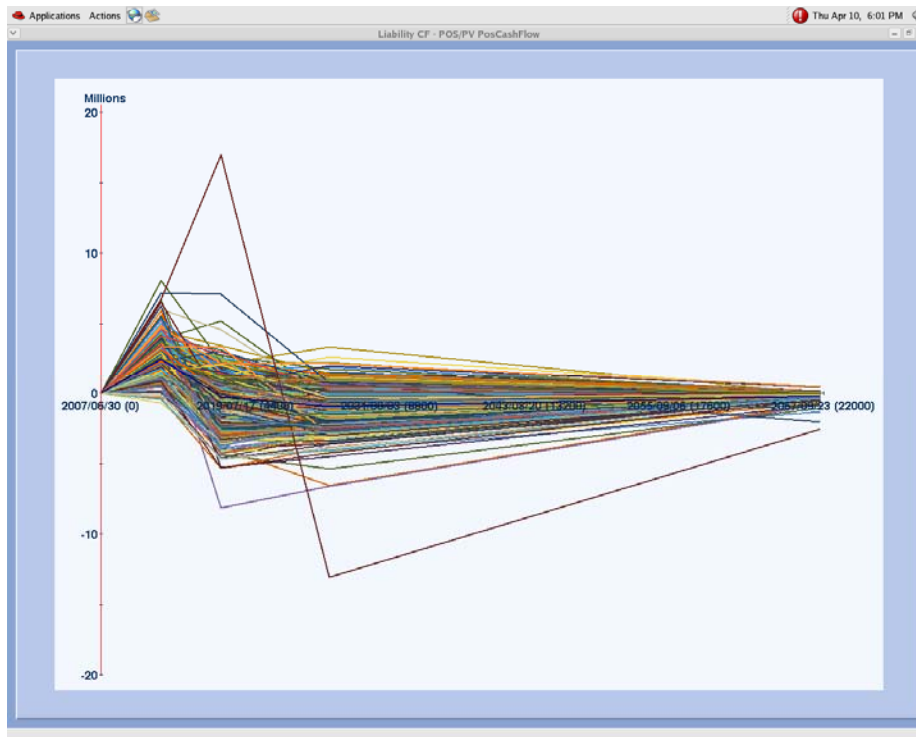
- *Bucketing:* Low/**High** (match each cash flow vs. match bucketed cash flow)
- *Value Constraint* Yes/**No** (match replicating portfolio value to RN liability value)
- *Trade Penalties* **Yes/No**

Bucketed Variable Annuity Cash Flows

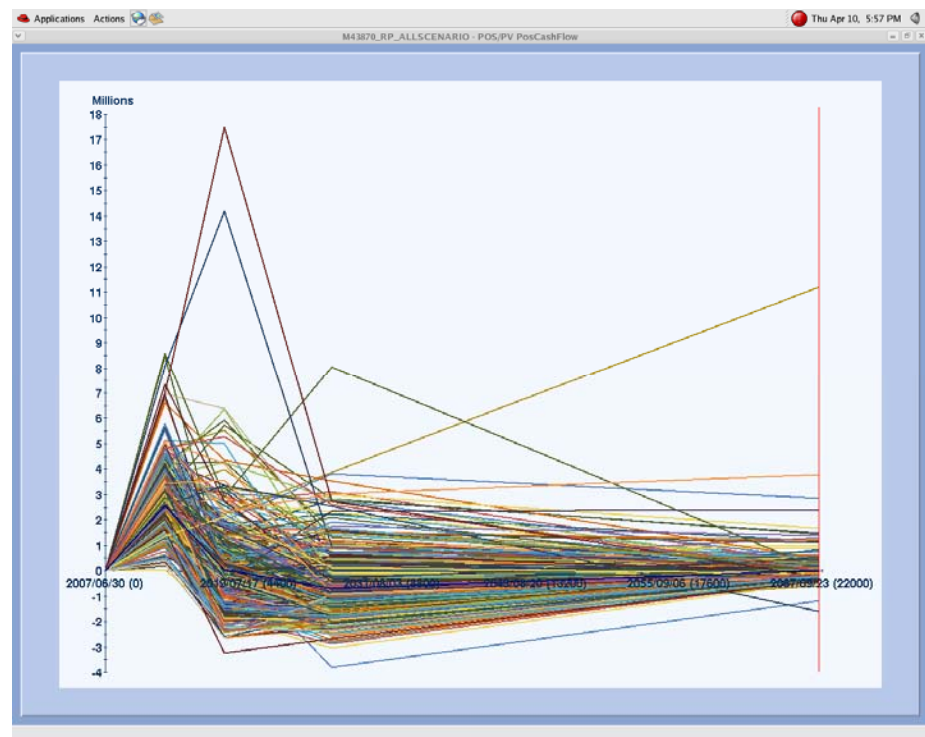


Cash Flow Comparison

Variable Annuity Cash Flows



Replicating Portfolio Cash Flows



Replicating Portfolio

AlgoRisk dev v.2.3.1 Prefs Refresh Benchmark Help Log

Natural Hierarchy **Asset Type** Country Currency Maturity (5yr buckets) Strike

Maturity (years)

Select a Multilevel Aggregation... Select a Custom Sector Aggregation... Custom Ratings

Currency USD

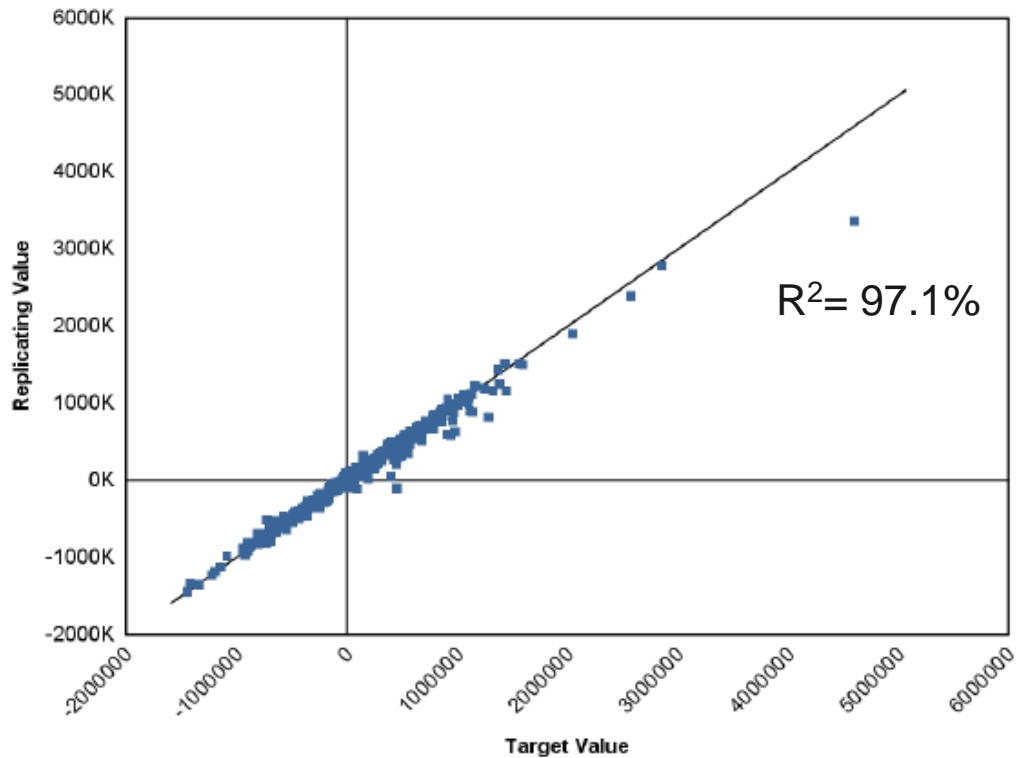
Portfolio Name: Replicating Portfolio M43870 **MtM Value Dirty:** 1,557,760,595.45 **User ID:** ing_rp

Actions Maturity (years) - Asset Type **Output:** MtM Value Clean

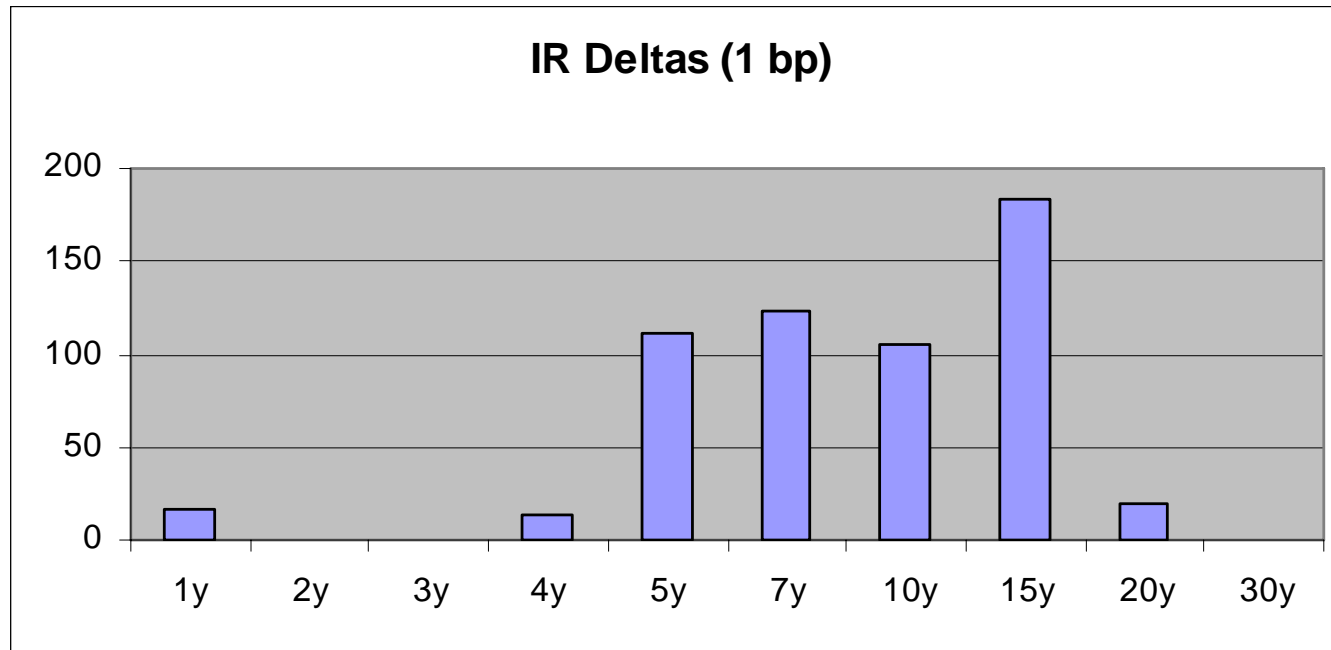
	Equity Forward	Equity Option	Swaption	Zero Coupon Bond
0yr	371,177,089.16			
1yr	130,664,615.52	9,721,641.41	114,838.14	
2yr	33,101,648.12			
3yr	52,242,401.10	22,153,957.25	16,882.05	
4yr	54,042,005.76			
5yr	29,462,197.90	18,314,523.48	20,749.12	
6yr	15,191,274.47	634,718.28		541,698,208.97
7yr		52,243,555.99		
8yr	3,973,878.58			2,495,345.03
9yr		583,794.55		
10yr	12,453,125.38	34,957,636.52	1,437,271.55	
12yr	1,682,244.83			109,483,816.06
15yr	9,992,750.10	45,258,811.63	2,577.74	
18yr	450,143.00			3,136,672.62
20yr	3,131.50		6,512.97	343,054.09
25yr	114,226.69	538,741.87		
30yr	1,141.17		3,770.25	
35yr				34,553.87
40yr	765.96			
45yr				4,563.68
50yr	500.14			
60yr	1,161.82			97.11

PV Liabilities/PV Replicating Portfolio across scenarios

Graph of PV liabilities/PV Replicating Portfolio for all scenarios



Market Risk Sensitivities



IR Delta (1bp)	IR Gamma (1 bp)	IR Vega (1%)	EQ Delta (\$1)	EQ Gamma (\$1)	EQ Vega (1%)
507	-.05	0	884.76	-1.16	-9428.05

Stress Test Scenarios (not used in optimization)

RN liability value and RP values are also computed under 8 instantaneous shocks

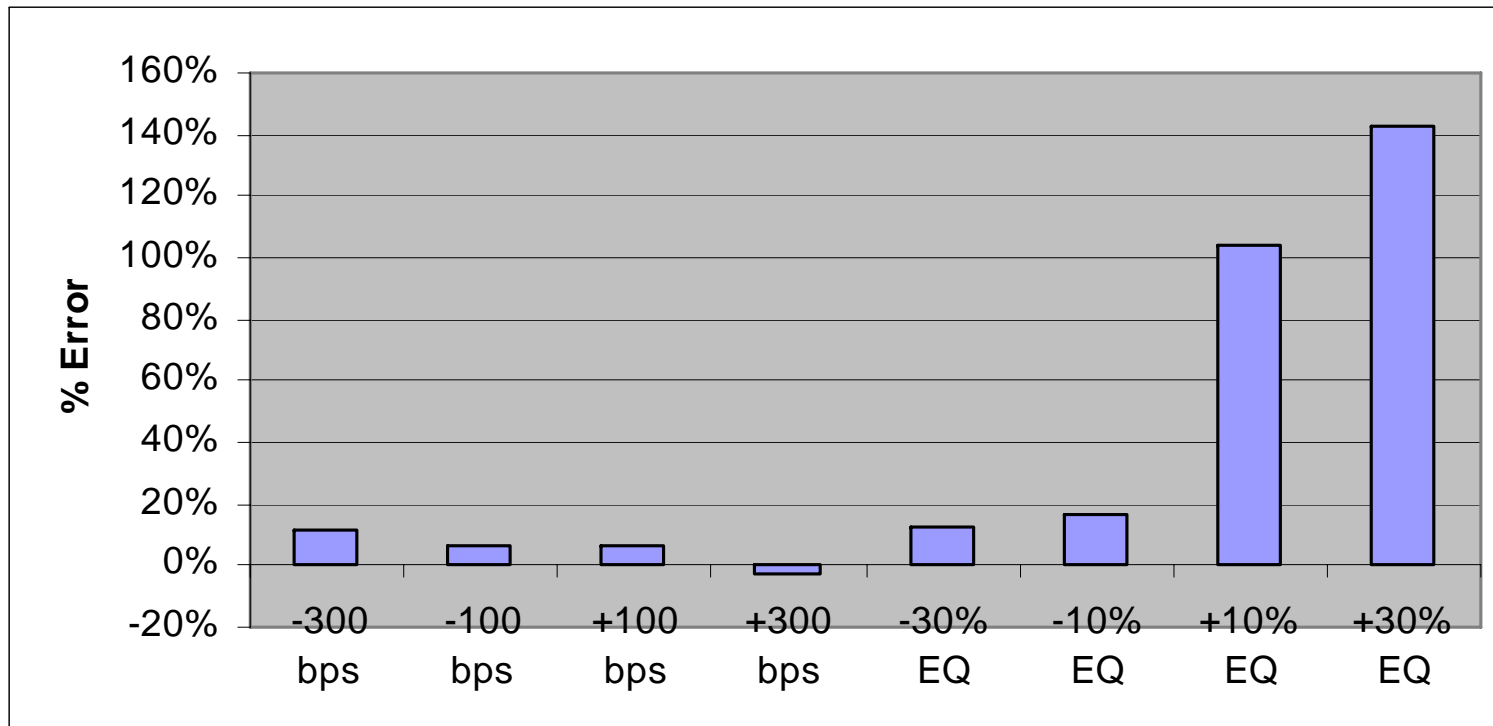
Equity shocks

- -10%
- +10%
- -30%
- +30%

IR shocks

- -100bp
- +100bp
- -300bp
- +300bp

Relative Changes for Shock Scenarios



	-300bps	-100bps	+100bps	+300bps	-30% EQ	-10% EQ	+10% EQ	+30% EQ
Replicating Portfolio	-236	-64	50	125	-608	-180	163	452
VA Portfolio	-211	-60	47	128	-539	-154	80	186
Difference	-25	-4	3	-3	-29	-26	83	266

Ideas for improving the replicating portfolio

1. Adding new instrument types to the asset universe (i.e. composite indices)
2. Reduce the number of market indices
3. Increase the number of scenarios
4. Adjust the strikes to better match the guarantees

Q&A