

A Dynamic Competing Risk Model for Subprime Security Pricing

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Outline

- **Current ABX and TABX Market**
- **Fundamental Driving Factors: Prepay and Default**
- **Competing Risk Model: Static Model**
- **Competing Risk Model: Dynamic model**
- **Cox Regression Model with GAM structure**
- **Dynamic Model with Calibration**

Mortgage Process

Borrower

- Purchase house to live, rent or flip
- Home equity loan to spend

Loan Originator/Broker/Appraiser

- Model based originator
- “Predator” brokers

ABS Originator

- Originator has income upon loan sale or securitization
- Bank earns fee for underwriting ABS bonds

CDO Originator/Rating Agency/CDO Managers

- Rating agency arbitrage allows CDO originator to book profit at closing
- CDO managers makes nominal investment, receives management fees
- Rating agency charges rating fee

CDO investor/Insurer

- Ultimate risk takers who rely on rating and investment bank; lack of understanding of the underlying risk

an Overview of All Products

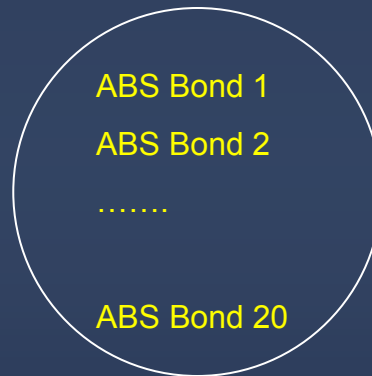
Loan Pool



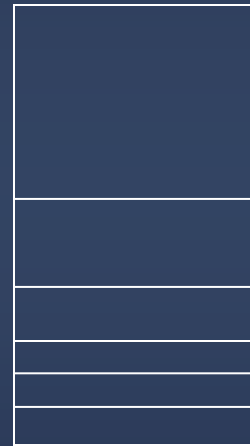
ABS Bond



ABX

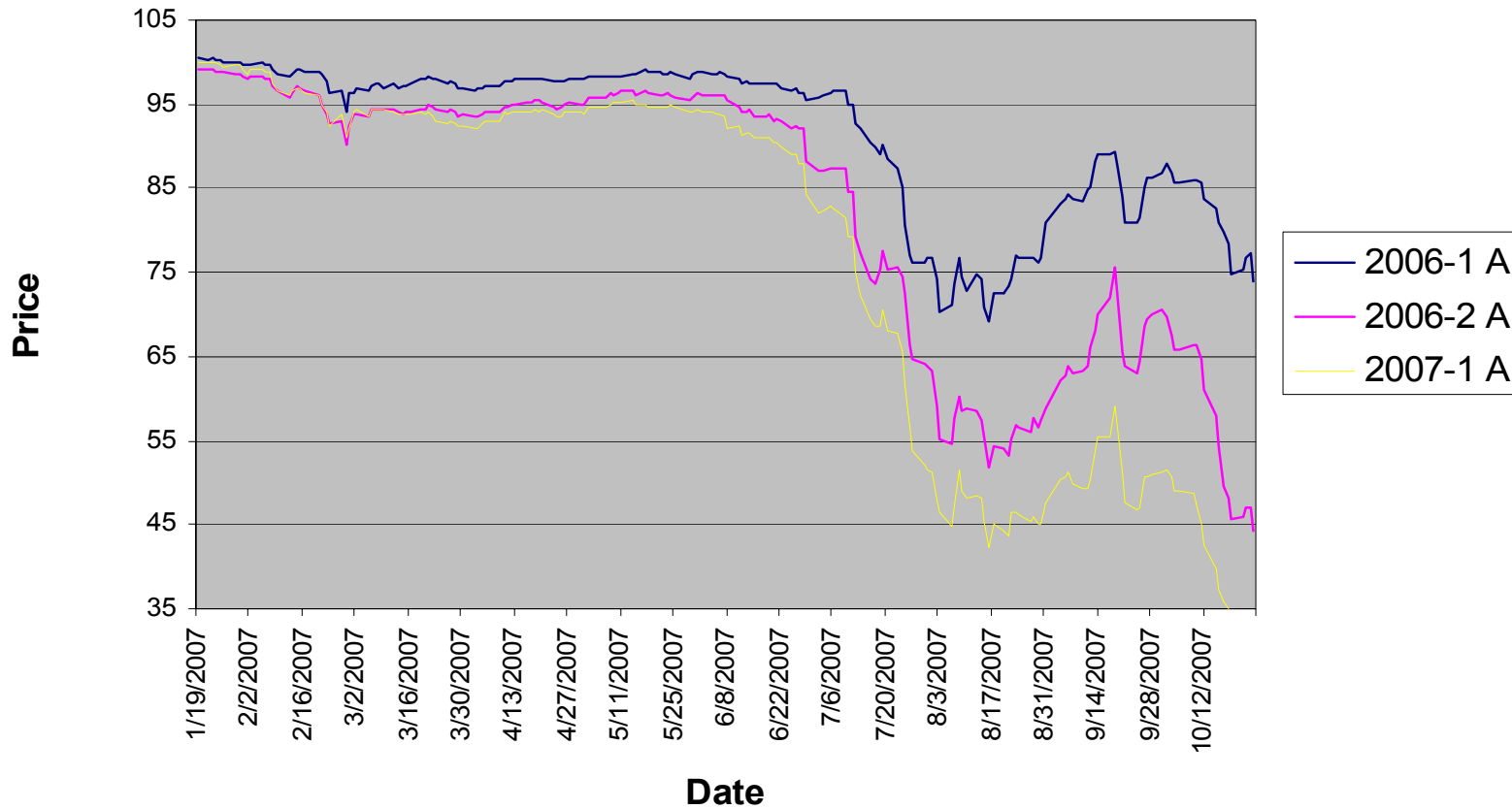


TABX/ABS CDO



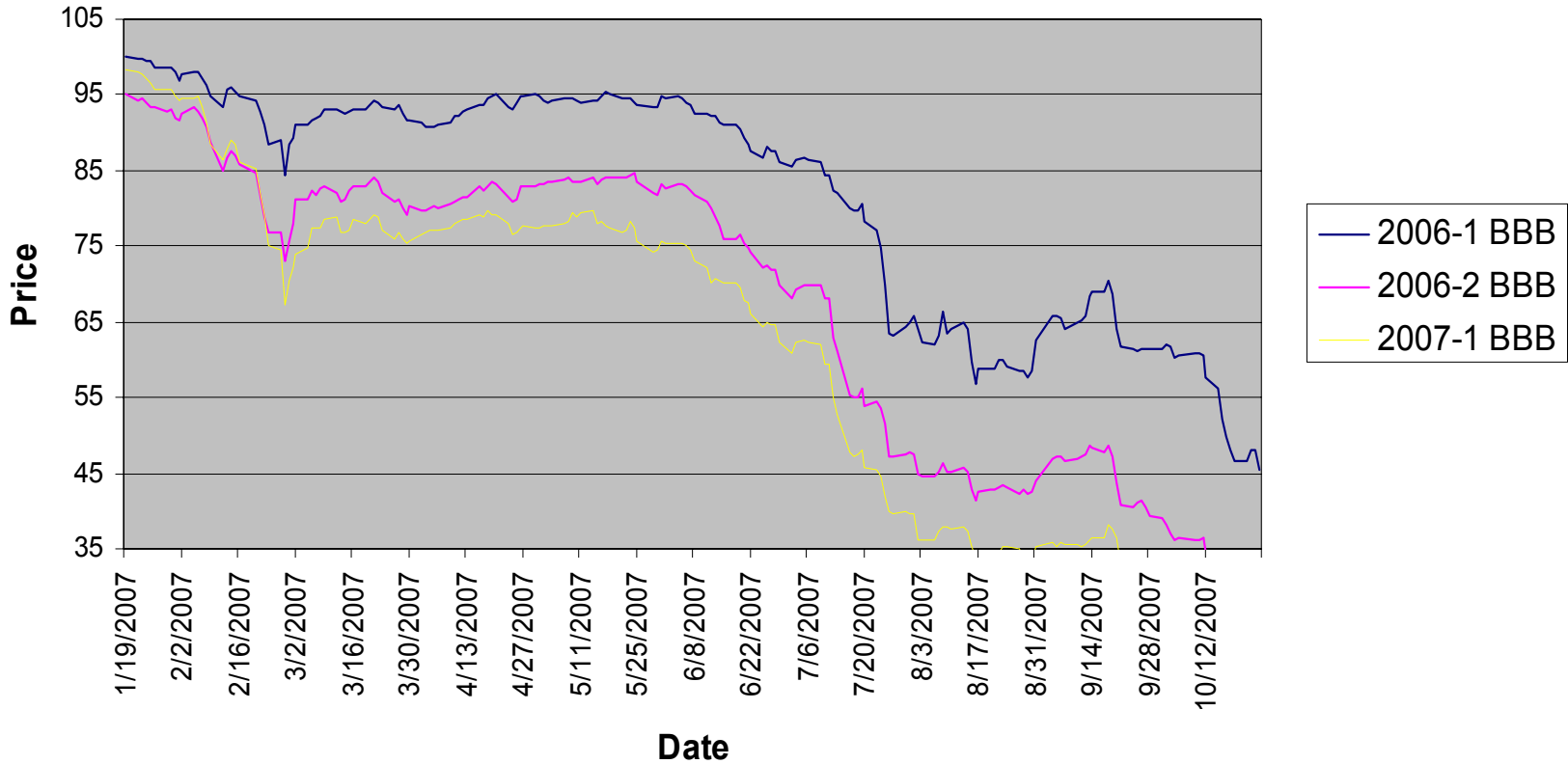
Recent Movement on ABX A (I)

ABX A Prices



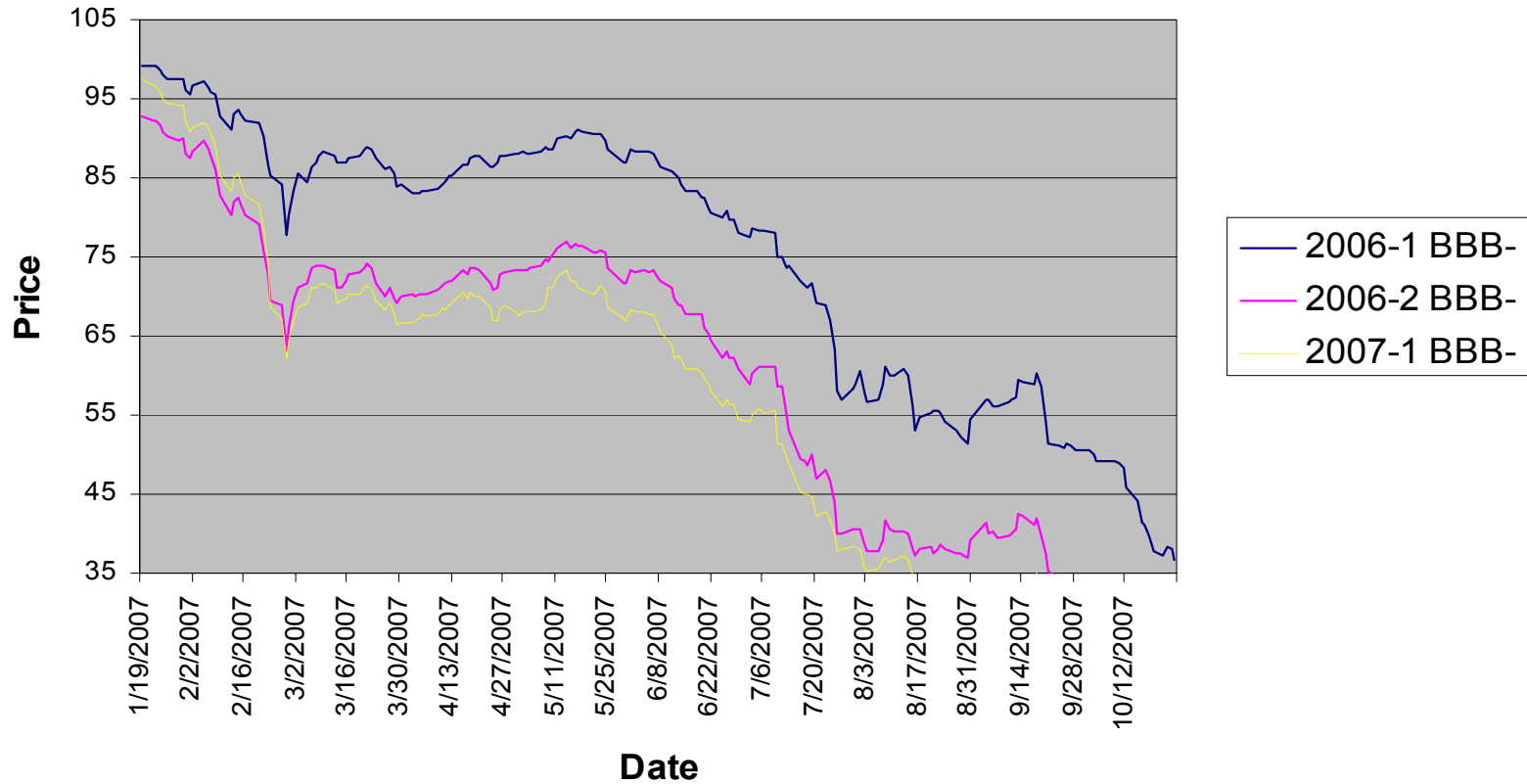
Recent Movement on ABX BBB (II)

ABX BBB Prices



Recent Movement on ABX BBB- (III)

ABX BBB- Prices



Recent ABX Quote (29 October, 2007)

Series	Fixing Spread	Close Price
2006-1 AAA	18	96 29/32
2006-1 AA	32	88 3/4
2006-1 A	54	69 9/32
2006-1 BBB	154	44 13/32
2006-1 BBB-	267	35 7/16
2006-2 AAA	11	89 7/8
2006-2 AA	17	70 21/32
2006-2 A	44	42 3/8
2006-2 BBB	133	23 7/8
2006-2 BBB-	242	21 15/16
2007-1 AAA	9	83 3/8
2007-1 AA	15	52 1/32
2007-1 A	64	28 9/32
2007-1 BBB	224	18 15/16
2007-1 BBB-	389	18 9/16
2007-2 AAA	76	86 1/4
2007-2 AA	192	58 1/16
2007-2 A	369	35 27/32
2007-2 BBB	500	23 1/8
2007-2 BBB-	500	20 5/32

Recent TABX Quote

TABX-HE 07-1 06-2 BBB
Quote:

29-Oct-07

Tranche	Running Spread	Low	High
40 - 100%	72	19.38	96.70
25 - 40%	267	17.67	80.80
15 - 25%	500	18.17	60.30
10 - 15%	500	17.19	45.80
5 -10%	500	15.85	37.70
0-5%	500	14.46	31.70

Loan Types

- Interest Rate is fixed or floating
 - Fixed Loans
 - ARM, 2/28, 3/27, 5/25
- First Lien or Second Lien
 - Silent second
- Teaser Rate and rate after reset
- Prepayment penalty

Multiple Decrement: Tabulation

Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Total Num at Risk (Thousand)	204	204	204	204	202	199	196	190	184	177	171	164	156	148	140	132	125	119	113	107
Num of Default	-	-	-	0.0	0.1	0.4	0.7	0.9	1.1	1.1	1.2	1.2	0.9	0.9	0.8	0.9	0.8	0.9	0.9	0.8
Num of Prepay	0.0	0.2	0.6	1.5	2.5	3.5	4.7	5.3	5.5	5.6	5.5	6.2	7.5	7.4	6.6	6.1	5.6	5.2	4.5	3.9
Total Number of Decrement	0.0	0.2	0.6	1.5	2.7	4.0	5.4	6.3	6.6	6.8	6.7	7.4	8.5	8.3	7.4	6.9	6.5	6.1	5.4	4.7
Marginal Default Rate	-	-	-	0.0	0.1	0.2	0.3	0.5	0.6	0.6	0.7	0.7	0.6	0.6	0.6	0.6	0.7	0.7	0.8	0.7
Marginal Prepay Rate	0.0	0.1	0.3	0.7	1.2	1.8	2.4	2.8	3.0	3.2	3.2	3.8	4.8	5.0	4.7	4.6	4.5	4.4	4.0	3.7
Total Marginal Decrement Rate	0.0	0.1	0.3	0.7	1.3	2.0	2.7	3.3	3.6	3.8	3.9	4.5	5.4	5.6	5.3	5.3	5.2	5.1	4.8	4.4
Total Marginal Survival Rate	100.0	99.9	99.7	99.3	98.7	98.0	97.3	96.7	96.4	96.2	96.1	95.5	94.6	94.4	94.7	94.7	94.8	94.9	95.2	95.6
Total Cumulative Survival Prob	100.0	99.9	99.6	98.9	97.6	95.7	93.1	90.0	86.8	83.5	80.2	76.6	72.4	68.3	64.7	61.3	58.2	55.2	52.6	50.3

Mathematical Formulation

- Denote T as time-until-termination
- Denote J as the cause of termination: prepay, default, scheduled repayment
- Joint distribution of T and J , whose p.d.f is $f(t, j)$

$${}_t q_x^{(j)} = \int_0^t f(s, j) ds$$

$${}_t q_x^{(\tau)} = \Pr[T \leq t] = \sum_{j=1}^m {}_t q_x^{(j)}$$

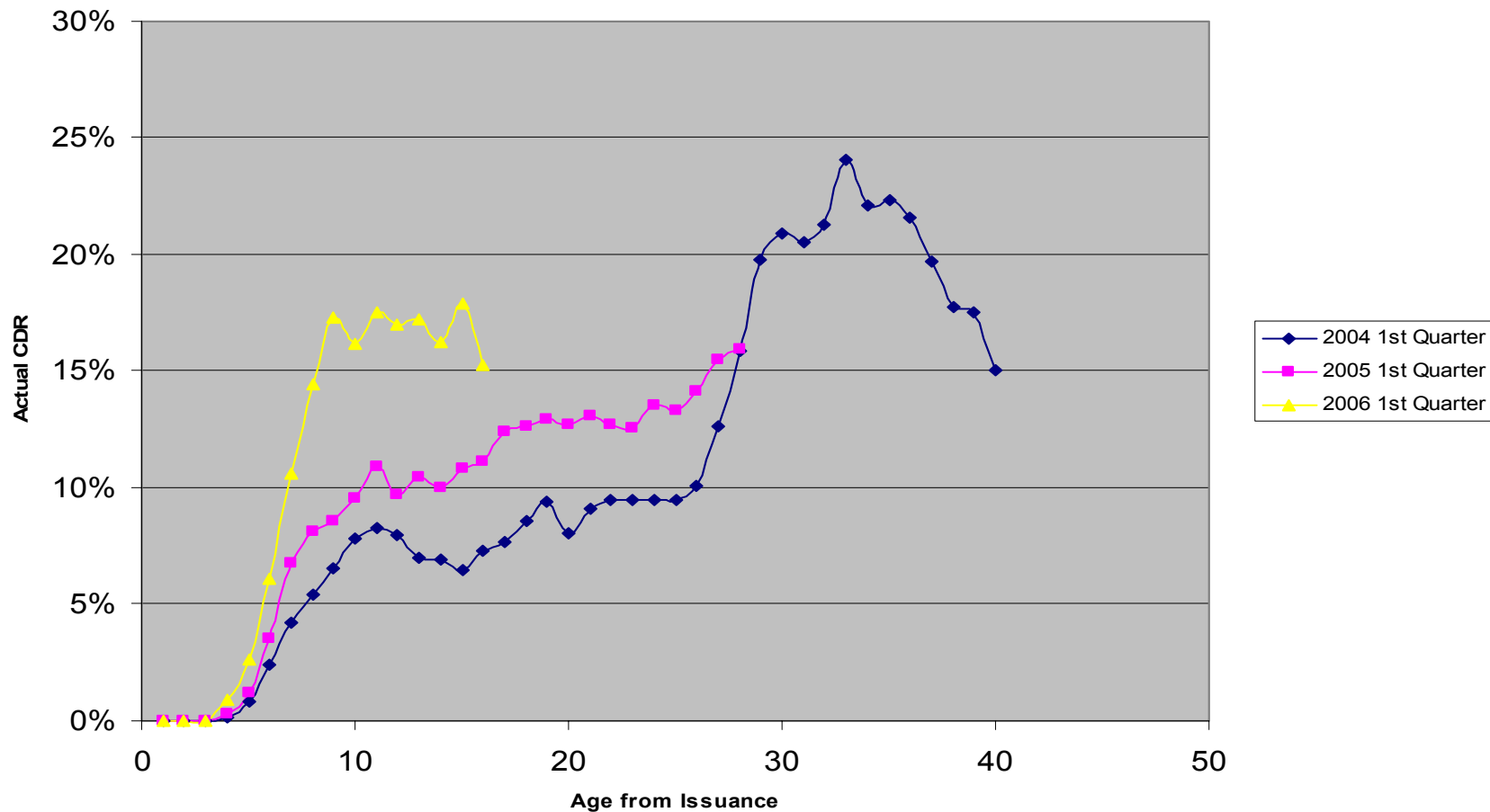
$$h_{x+t}^{(j)} = \frac{f(t, j)}{1 - G(t)} = \frac{f(t, j)}{{}_t p_x^{(\tau)}}$$

$$h_{x+t}^{(\tau)} = \frac{g(t)}{1 - G(t)} = - \frac{d}{dt} \ln {}_t q_x^{(\tau)} = \sum_{j=1}^m h_{x+t}^{(j)}$$

$${}_t q_x^{(j)} = 1 - e^{-\int_0^t h_{x+s}^{(j)} ds}$$

Comparison of Different Vintage Years (Kaplan-Meier Estimate) (2/28 ARM)

CDR for Different Vintages



Cox Model with GAM Structure

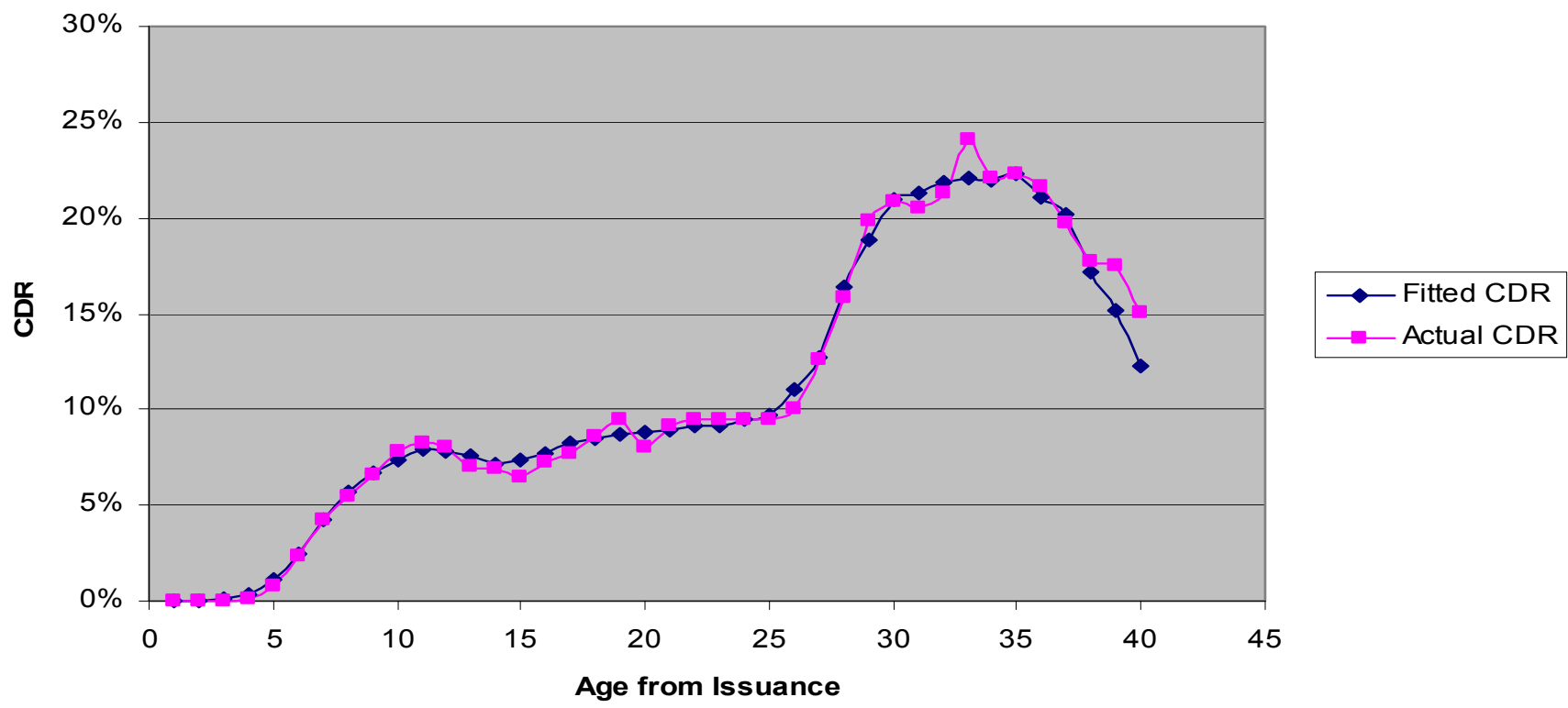
$$h_i^{(J)} = h_{i0}^{(J)} e^{\sum_{k=1}^K f(X_k)}$$

- X are all covariates that impact default or prepayment
- If X are static and the function f is linear, it is a traditional Cox model
- If f takes general form of cubic spline. It is the generalized additive model (GAM)
- When X's are functions of time t, then it is a dynamic model

- Common driving factor: static variable: LTV, FICO, Documentation, loan purpose, margin, loan size, property type etc; dynamic variables: interest rate environment, HPA, refinance incentive, liquidity measure (spread between prime and sub-prime mortgage rate), employment number.

Result for Subprime Loans Issued 2004 1st Quarter

CDR Fitted Value vs Actual
(2004 1st Quarter)



Dynamic Competing Risk Model

$$h_{im}^{(P)}(t) = h_{im0}^{(P)}(t) \cdot e^{\sum_{k=1}^K f_k^P(X_k^P) + f_M^P(X^M)}$$

$$h_{im}^{(D)}(t) = h_{im0}^{(D)}(t) \cdot e^{\sum_{k=1}^K f_k^D(X_k^D) + f_M^D(X^M)}$$

Valuation: ABS Bond, ABX and ABS CDO

- ABS bonds: not linear products; it depends on prepayment and default and volatility of prepayment and default and correlation between them. The total loss of the loan pool can be expressed as:
- ABX indices of different ratings need to be priced consistently
- TABX tranches of different ratings also need to be priced consistently
- All above can be achieved through cash flow aggregation generated from CDR, CPR scenarios from the dynamic model specification

$$L(T) = \sum_{i=1}^N N_i (1 - R_i) \cdot \int_0^T {}_t p_x^{(\tau)} h_{x+t}^{(D)} e^{-rt} dt$$

Cash Flow of Bond Under a CDR and CPR Assumption

Period	Payment Date	Balance	Principal	Interest	Write Down	Interest Shortfall	Cap Shortfall	Payback Cap Shortfall
1	Tue, 30-Oct-07	7,500,000	0	0	0	0	0	0
2	Mon, 26-Nov-07	7,500,000	0	49,150	0	0	0	0
3	Wed, 26-Dec-07	7,500,000	0	45,686	0	0	0	0
4	Fri, 25-Jan-08	7,500,000	0	44,408	0	0	0	0
5	Mon, 25-Feb-08	7,500,000	0	45,313	0	0	0	0
6	Tue, 25-Mar-08	7,500,000	0	41,505	0	0	0	0
7	Fri, 25-Apr-08	7,500,000	0	44,370	0	0	0	0
8	Tue, 27-May-08	7,500,000	0	45,577	0	0	0	0
9	Wed, 25-Jun-08	7,500,000	0	40,684	0	0	0	0
10	Fri, 25-Jul-08	7,500,000	0	42,088	0	0	0	0
11	Mon, 25-Aug-08	7,500,000	0	43,356	0	0	0	0
12	Thu, 25-Sep-08	7,500,000	0	42,967	0	0	0	0
13	Mon, 27-Oct-08	7,500,000	0	44,348	0	0	0	0
14	Tue, 25-Nov-08	7,500,000	0	40,165	0	0	0	0
15	Fri, 26-Dec-08	7,500,000	0	42,859	0	0	0	0
16	Mon, 26-Jan-09	7,500,000	0	42,857	0	0	0	0
17	Wed, 25-Feb-09	7,500,000	0	41,546	0	0	0	0
18	Wed, 25-Mar-09	5,363,999	0	38,992	2,136,001	0	0	0
19	Mon, 27-Apr-09	2,587,097	0	32,862	2,776,901	0	0	0
20	Tue, 26-May-09	0	0	13,983	2,587,097	0	0	0

Price	10.08	Running	500	Upfront	86.3%
Premium PV	1,093.4	Upfront	0%	Running	6,421.5
Loss PV	7,021,087.5				
Spread	6,421.5				
Cum Loss %	100.0%				