

$$\min_{L(z)} \sum_{i=1}^n z_i^2 = c \sum w_i B_i^T z_i \quad L = \sum x_i L_i$$



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Valuation and Risk of Structured Credit Products and Bespoke CDOs: A Scenario Framework



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Preface – in the News...



- \$232 billion in writedowns and credit losses since the beginning of 2007 *
 - UBS – additional \$19B
 - Deutsche Bank – additional \$3.9B
- Charges from collapse of the U.S. subprime mortgage market
 - Also reflect credit losses or writedowns of non-subprime and leveraged-loan commitments
- Some estimates that total losses may reach over one trillion US...

* Bloomberg April 1st 2008

	<i>Writedown</i>	<i>Credit Loss</i>	<i>Total</i>
TOTALS	206	25.8	231.8
1. UBS	38		38
2. Merrill Lynch	25.1		25.1
3. Citigroup	21.4	2.5	23.9
4. HSBC	3	9.4	12.4
5. Morgan Stanley	11.7		11.7
6. IKB Deutsche	9		9
7. Bank of America	7.3	0.9	8.2
8. Deutsche Bank		7.4	7.4
9. Credit Agricole	6.5		6.5
10. Credit Suisse	6.3		6.3
11. Washington Mutual	0.3	5.5	5.8
12. JPMorgan Chase	2.9	2.1	5
13. Wachovia	2.9	2	4.9
14. CIBC	4		4
15. Societe Generale	3.8		3.8
.....			
22. Bear Stearns	2.6		2.6
Canadian banks excluding CIBC	2.4	0.1	2.5

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Preface – in the News...



- Investors blaming a “1 in 10,000 years” event...

And then...

- *“It all happened exactly like he said it would happen... In every single detail...”*
 - *“The hedge fund creator's name was **John Paulson**... by making **between \$3 billion and \$4 billion** for himself in 2007, he appears to have set a Wall Street record... **no one has ever made so much so fast.**”*

Sources: Reuters, Risk, Banking Technology,, Dow Jones, FT., G&M, Bloomberg

Structured Credit – Valuation & Risk



1. Lack of integrated view of synthetic and cash products and single-name credit derivatives: pricing and risk management
2. Valuation of synthetic CDOs
 - Gaussian copula framework still prevalent
 - Pricing bespoke portfolios difficult – “mapping” models are generally ad-hoc
 - Dynamic models and detailed bespooke models still in infancy
3. Valuation of structured credit (cash CDOs, ABSs,...)
 - Difficult, non-standard, computationally intensive
 - IR, spreads, prepayment, credit (and correlation) risks
 - Structures are complex and opaque
 - Simple “bond models” and matrix pricing generally used – pricing based on ratings
 - Advanced models are fairly new or not fully developed for all asset classes
 - Standardized calibration is difficult to achieve

Structured Credit – Valuation & Risk



4. Risk modelling is immature (market and credit risk)

- Simple market risk sensitivities are generally used (e.g. CR01, etc.)
- Risk assessment and investment decision mainly driven by ratings
- VaR (market & credit) measures not easy to obtain and not used in general
- Risk contribution not well defined (non-linearities)
- Hedging has proven to be difficult and prone to large model errors
- Computationally intensive risk applications (e.g. name-specific sensitivities)
- Correlation is very important but difficult to assess
 - High systematic risk

Summary

Scenario framework – Valuation and Risk Profiling



- **Implied factor distributions** and **weighted MC** techniques
 - Multi-factor credit models – characterize correlations for different baskets
 - Weighted Monte Carlo techniques (used in options pricing)
 - CDO analytics and computational techniques
 - Ability to incorporate a full bottom-up approach
- Basic idea: set of scenarios where instruments are consistently valued
 - Imply “risk-neutral” distribution (process) for *underlying systematic risk factors*
 - Observed (liquid) prices (e.g. CDSs, index tranches)
 - Prior or “quality” preferences on distribution; subjective views
- Structured finance CDOs (ABS and CLO products)
 - Flexibly incorporate cash-flow waterfalls, prepayment and LGD models

Cash CDO = bespoke portfolio + complex cashflow waterfall
- Methodology is general (dynamic) – examples in paper use static models

**Introduction: Valuing CDOs
and Structured Credit**

Synthetic CDOs

- Underlying pool of credit default swaps – divided into “tranches”

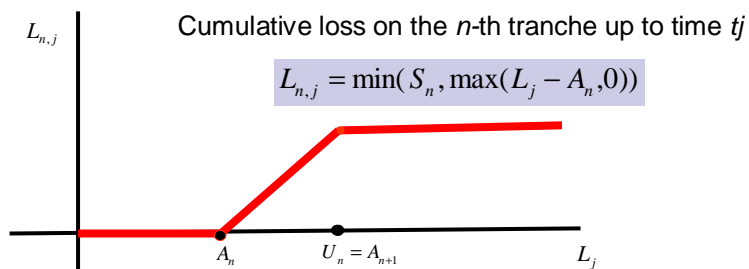
Tranche	Attachment	Detachment
Equity	0%	3%
1st Mezzanine	3%	7%
2nd Mezzanine	7%	10%
Senior	10%	15%
Super Senior	15%	30%

Size of the n -th tranche

$$S_n = U_n N - A_n N = N \cdot (U_n - A_n)$$

Cumulative portfolio loss of up to t_j

$$L_j = \sum_k N_k \cdot LGD_k \cdot 1_{\tau_k \leq t_j}$$



Background – Pricing Synthetic CDOs



- Standard model for pricing synthetic CDOs: single-factor Gaussian copula (Li 2001)
 - Codependence through a one-factor Gaussian copula of *times to default*
 - Single parameter to estimate (correlation for all obligors in portfolio)
- Basic model does not simultaneously match market prices of all traded tranches
 - “Correlation skew” – set of correlations that match the prices of all tranches
- Base correlations – alternative to tranche correlations
 - Implied correlations of equity tranches with different attachment points (mezzanine/senior tranches as difference between two equity tranches)
- Interpolation (or extrapolation) model
 - Calibrated to observed tranche prices (e.g iTraxx or CDX)
 - Pricing of bespoke portfolios – mapping (risk of bespoke vs. index portfolio)

Single-Factor Gaussian Copula (Times to Default)



$$L_T = \sum_k 1_{\tau_k \leq T} \cdot V_k(\tau_k) \cdot LGD_k(\tau_k)$$

- Cumulative default time distribution functions

$$F_k(t) = \Pr(\tau_k \leq t)$$
- Creditworthiness index
 - Z systematic factor (Gaussian)

$$Y_k = \sqrt{\rho_k} Z + \sqrt{1 - \rho_k} \varepsilon_k$$
- Default times
 - Mapping to Gaussian

$$Y_k \leq \Phi^{-1}(F_k(t)) \Leftrightarrow \tau_k \leq t$$
 - $$\tau_k = F_k^{-1}(\Phi(Y_k))$$
- Conditional on Z , default times are independent

$$p_k^Z(t) = \Pr(\tau_k \leq t | Z) \quad q_j^Z(t) = \Pr(\tau_k > t | Z)$$
- Explicit formulae for conditional default probabilities

$$p_k^Z(t) = \Phi\left(\frac{\Phi^{-1}(F_k(t)) - \sqrt{\rho_k} Z}{\sqrt{1 - \rho_k}}\right)$$

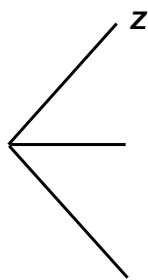
General Framework: Gaussian Copula Model



1. Scenarios: systematic factors

2. Conditional def. prob.

3. Conditional portfolio losses (discounted)



$$p_k^Z(t) = \Phi\left(\frac{\Phi^{-1}(F_k(t)) - \sqrt{\rho_k} Z}{\sqrt{1 - \rho_k}}\right)$$



$$L_j(Z) = \sum_k N_k \cdot LGD_k(Z) \cdot 1_{\tau_k \leq t_j}(Z)$$

- Conditionally independent obligor losses → convolution methods:
 - Recursions (Andersen et al, Hull-White)
 - Full simulation (independent Bernoulli variables)
 - LLN, or CLT approximation
 - Poisson approximation

General Framework: Gaussian Copula Model

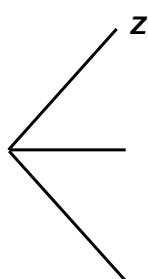


1. Scenarios: systematic factors

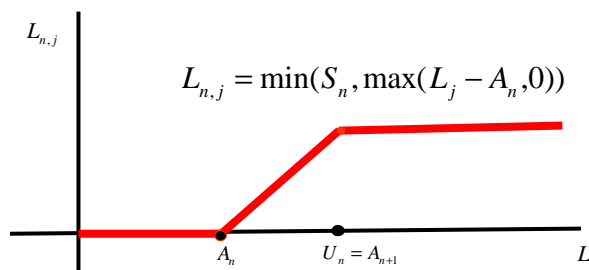
2. Conditional def. prob.

3. Conditional portfolio losses (discounted)

Conditional tranche losses



$$p_k^Z(t) = \Phi\left(\frac{\Phi^{-1}(F_k(t)) - \sqrt{\rho_k} Z}{\sqrt{1 - \rho_k}}\right)$$



General Framework: Implied Factor Distributions



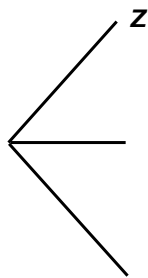
1. Scenarios: systematic factors

2. Conditional def. prob.

3. Conditional portfolio losses (discounted)

Conditional tranche losses

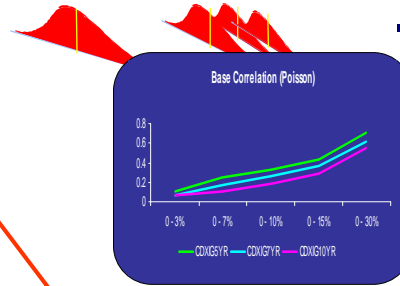
Conditional value of tranches



$$p_k^z(t) = \Phi\left(\frac{\Phi^{-1}(F_k(t)) - \rho_j Z}{\sqrt{1 - \rho_j^2}}\right)$$

Calibration:

- Correlation of each tranche j
- Base correlation
 - Equity tranches



4. Expected tranche losses & values

$$\sum_{\omega} p_{\omega} E[V | Z_{\omega}]$$

Bespoke Portfolios and Mappings



- Main idea: base correlations correspond to different levels of risk in the reference portfolio
 - By finding the same risk levels on the bespoke portfolio – transfer the base correlation structure from the standard portfolio to the bespoke

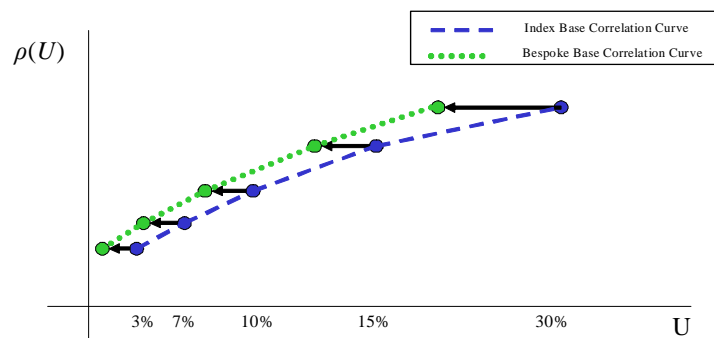
- Generally, solve an equation of the form:

$$S(\hat{P}, \hat{u}, \rho) = S(P, u, \rho)$$

S is a "risk statistic"; P the portfolio, u the detachment point), and ρ is the base correlation (the unknown is \hat{u}).

- Mapping: solve for the detachment point(s) in the bespoke portfolio which matches the equation:
 - Note: the base correlation for the standard portfolio is used on both sides

EL Mapping (Base Correlations)

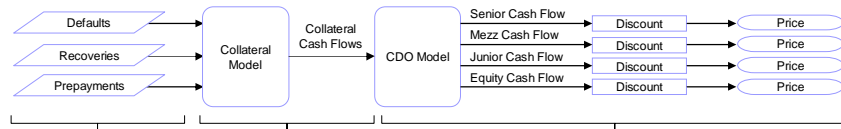


Understanding Bespoke Portfolios and Prices



- Differences in prices from bespoke portfolio and quoted (index) prices may arise from differences in
 - Credit quality (individual names spreads/PDs)
 - Concentration risk (sector/geographical, and name concentration)
 - Sector concentration (indices are well diversified)
 - Correlation (codependence)
 - Granularity
 - Reference to multiple indices (and different risk premia)
 - Liquidity
- e.g. for an equity tranche:
 - Higher quality (lower individual spreads) → lower tranche spread
 - Higher correlation → lower spread
 - Higher granularity (more names) → higher spread
 - Multiple indices → lower average correlation → higher spread

Cash CDOs: “Bond” Models (Single-Scenario)



- Step 1: Gather collateral cash flow assumption vectors
 - e.g. assumptions by type of CDO collateral (e.g. CRE, ABS, loan, bond)
 - For ABS CDOs, can use loan performance analysis on the loans in each ABS
- Step 2: Generate cash flows for each piece of collateral
- Step 3: Use collateral CFs and CDO waterfall to generate CDO cash flows
- Step 4: NPV – discount cash flows (appropriate discount rate)
 - Discount rate (spread) – from market quotes where available
 - Application of discount rate to all other tranches with the same rating, deal type, vintage, etc.

- Complex cashflow from collateral and structure waterfall
- In addition to default, LGDs: prepayment (applies differently to tranches)
- Underlying: loans, bonds, retail loans (mortgages, credit cards, etc.), ABSs, CDOs (CDO²)

Cash CDOs: “Bond” Models (Single-Scenario)



- **Single-scenario** modelling
 - Deterministic cash-flow approach (scheduled amortization)
- No direct modelling of correlations, optionality, non-linearities
- Detailed cash-flow modelling of collateral pool and of CDO waterfall
 - Pool level assumptions and loan-level assumptions and clustering
- Comparative pricing via matrix approach (generally relies on ratings)
 - Scenario assumptions, discount spreads (premiums)
- Useful stress-testing framework

$$PV = \sum_{j=1}^T CF_j(Y) \cdot e^{-(r_j + s_j(X))t}$$

$$Y = (PP, Def, LGD, \dots) \quad X = (\text{rating, sector, vintage, } \dots)$$



***Scenario Framework:
Implied Factor Distributions
and Weighted Monte Carlo Methods***



- Multi-factor models are currently used extensively to assess portfolio credit risk and measure credit economic capital
 - Capital allocation – capture sector/geographical concentrations
- Over a decade of industry experience: KMV, CreditMetrics, CreditRisk+, CreditPortfolioView
 - Conditional independence framework – mathematical equivalence
- Extensive empirical studies on estimation of credit correlation parameters (Basel committee, rating agencies, vendors, financial institutions and academics)
- The origins of the Gaussian copula method to price CDOs trace back to the KMV and CreditMetrics model

Implied Factor Models & Weighted MC



Background

- Weighted MC approach used to price complex options
 - e.g. Avellaneda et al., 2001, Elices and Giménez, 2006
- Similar idea to fitting the implied distribution (or process) of underlying in a (discrete) lattice
- Hul-White's "implied copula" model (2006) is essentially an application of this concept
 - Homogeneous portfolio – cannot be used directly to price bespokes
 - Similar ideas (also for homogeneous portfolios) in Brigo et al (2006), Torresetti et al (2006)

Implied Factor Models & Weighted MC



Assumptions

- Correlations of names in portfolio: multi-factor model (systematic factors)
- MF model → joint default behaviour under real world measure P
- Coefficients of factor model for portfolio are known and fixed
- Difference between real measure P and RN-measure Q : joint distribution of the systematic factors
 - (Marginal) distribution of default times for each name under the risk-neutral measure based on CDS spreads
 - Conditional distribution of default times, as a function of the factor levels under the RN measure still given by the same formula

Solution

- Sample discrete "paths" (in this case, single values) for the systematic factors and adjust probabilities of paths to match prices

Weighted MC – GLLM Framework



- Portfolio model can be a Gaussian copula or, more generally, we can use other “link functions”
 - Logit, NIG, double-t, etc...
- **Generalized linear mixed models (GLMM)**

$$p_j^Z(t) = h\left(a(t) - \sum_k b_k Z_k\right)$$

□ Example of general multi-factor copula
$$p_j^Z(t) = G_j\left(\frac{H^{-1}(F_j(t)) - \sum_k \beta_k Z_k}{\sqrt{1 - \sum_k \beta_k^2}}\right)$$

- Match, for each name, the “unconditional” default probability term structure

$$p_j(t) = \int p_j^Z(t) df^z(z)$$

- ... and match quoted CDO prices

Weighted MC – GLLM Framework



- General formulation:

$$PD_{it}(Z^t) = h\left(a_{it} + \sum_{k=1}^K b_{ik}^t Z_k^t\right)$$

- Gaussian copula:

$$a_{it} = \frac{\Phi^{-1}(PD_{it})}{\sqrt{1 - \sum_{k=1}^K \beta_k^2}}, \quad b_{ik} = \frac{\beta_k}{\sqrt{1 - \sum_{k=1}^K \beta_k^2}}$$

- Poisson mixture (e.g. CreditRisk+)

$$\lambda_i(Z) = E[U_i | Z] = c_i \sum_{k=1}^K \beta_{ik} Z_k$$

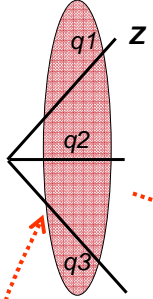
- Logit model:

$$h(x) = \frac{1}{(1 + \exp(-x))}$$

General Framework: Weighted MC



1. Scenarios: systematic factors



2. Conditional def. prob.

$$p_j^z(t) = \Phi\left(\frac{\Phi^{-1}(F_j(t)) - \sqrt{\rho_k} Z}{\sqrt{1 - \rho_k}}\right)$$

3. Conditional portfolio losses (discounted)



Conditional tranche losses



Conditional value of tranches

• • •

5. Optimization

Objective function (e.g. max. smoothness, max entropy, etc.)

$$\sum_{\omega} p_{\omega} E[V | Z_{\omega}] = 0$$

?

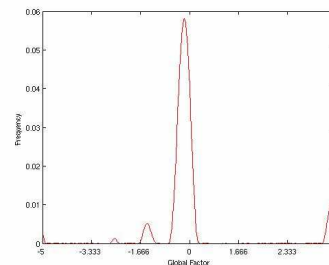
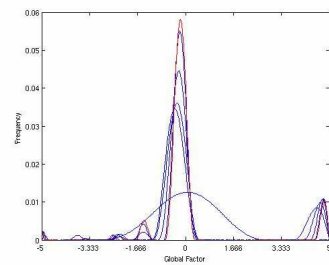
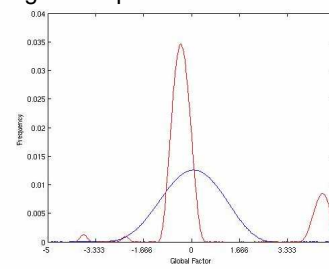
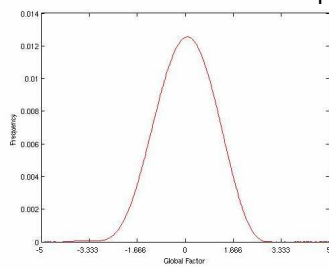
4. Expected tranche losses & values

$$\sum_{\omega} p_{\omega} E[V | Z_{\omega}]$$

Global Factor Implied Distribution



Evolution of distribution – from prior to tight fit of prices



Implied Factor Distributions – Intuition



Key objective: tractable distribution of joint default times – match marginal distributions and prices of CDSs and quoted CDO tranches

- In a Gaussian copula – conditioning on the systemic factor

$$p_j^z(t) = \Phi\left(\frac{\Phi^{-1}(F_j(t)) - \sqrt{\rho}Z}{\sqrt{1-\rho}}\right)$$

- **Base correlations** → the correlation ρ is a function of the detachment point
- **Implied copula** → model directly conditional PDs through discrete scenarios (on a hazard rate) for homogeneous portfolio
- **Implied multi-factor distribution** → model directly the **distribution of the systematic risk factor** through discrete scenarios → conditional default probabilities through the copula “mapping”
 - Extensible to multi-factor and applied to other portfolios

Weighted MC – Optimization problem



- Objective function → factor distribution “quality”:
 - min. distance from prior, max. entropy, max. smoothness
- Match tranche and index prices (can be more than one index at a time)
- Match CDS prices (cumulative default probabilities for all names)

$$\begin{aligned} \max G(q) \quad \text{subject to :} \\ \sum_{m=1}^M q_m PV_{Buy}^n(Z^m) &= \sum_{m=1}^M q_m PV_{Sell}^n(Z^m) \quad \text{for all } n \\ \sum_{m=1}^M q_m PD_{i,j}(Z^m) &= F_{i,j} \quad \text{for all } i, j \\ \sum_{m=1}^M q_m &= 1, \quad q_m \geq 0 \quad \text{for all } m \end{aligned}$$

- Trade-off: well-behaved “smooth” solution might be preferred over perfect matching of prices (with some bounds)
 - Instead of perfect “perfect match” – minimize price differences

Weighted MC – Computation



■ Computational issues

- Each path requires computing the conditional portfolio loss distribution – trade-off between number of paths and accuracy of these distributions
- Number of paths
 - Low dimensions – numerical integration
 - More generally – Quasi-MC methods (low discrepancy sequences)
- Trade-off: nice behaved “smooth” solution might be preferred over perfect matching of prices (with some bounds of course)

Weighted MC – Scenario Generation

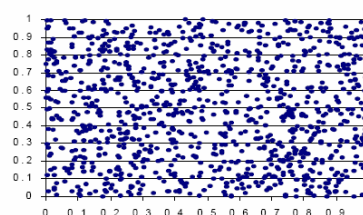


■ Quasi Monte Carlo methods

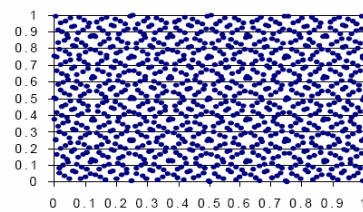
- Deterministic points generated from a type of mathematical vector sequences: *low discrepancy sequences (LDS)*

■ Basic idea:

- LDSs specifically attempt to cover the space of risk factors “evenly” - avoiding the clustering usually associated with pseudo-random sampling
- Number of scenarios necessary to achieve a desired level of accuracy in pricing or risk calculations is reduced



(a) Two-dimensional pseudo-random points



(b) Two-dimensional Sobol sequences

Source: Dembo et al. (1999), *Mark-to-Future*



Examples

Examples



1. ABX and cash CDO
2. CLO and HY Index
3. Bespoke synthetic CDO in Multi-factor model (sector concentrations)
4. Bespoke synthetic CDO with names in Europe and NA (two indices)

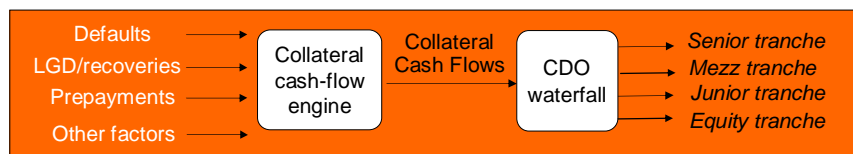
Example 1 – ABX



- ABX –referencing 20 Asset Backed CDS (ABX CDS)
 - Home Equity / Sub-prime Bonds (thousands of small loans)
 - Five indices: AAA / AA / A / BBB / BBB-
 - Trading Began January 2006

ABX	MktPrice
ABX-XHAAA72_INDEX	91.81
ABX-XHAA72_INDEX	71.06
ABX-XHA72_INDEX	44.31
ABX-XHBBB72_INDEX	26
ABX-XHBBBM72_INDEX	23

- Standard prices and quotes
- Modelling issues
 - Default risk and prepayment risk
 - Cash-flow generation
(underlying loans, bond (and CDS) waterfall)



Example – ABX Valuation Model



Simple valuation model (illustration purposes)

- Single systematic factor – drives

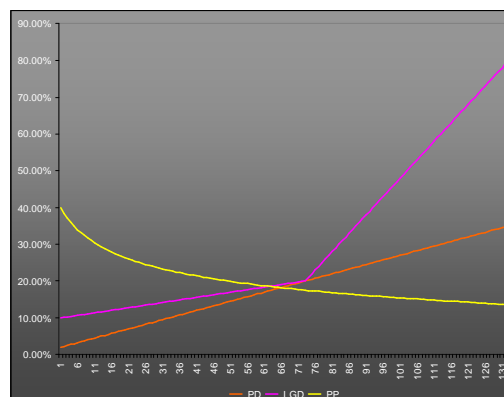
- Default rates
- Prepayment rates
- Recovery rates
(underlying loans in the pools)

$$p_j^Z(t) = h\left(a(t) - \sum_k b_k Z_k\right)$$

- Large homogeneous portfolio assumption

- Discretization

- 135 systematic factor scenarios

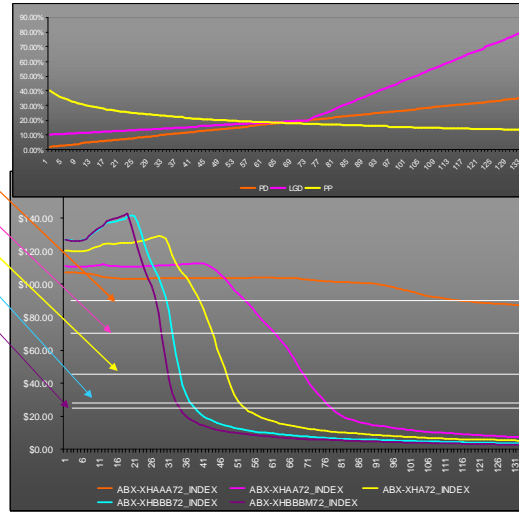


Example – ABX Valuation under Scenarios



- ABX bonds discounted Cashflows (values) under scenarios

ABX	MktPrice
ABX-XHAAA72_INDEX	91.81
ABX-XHAA72_INDEX	71.06
ABX-XHA72_INDEX	44.31
ABX-XHBBB72_INDEX	26
ABX-XHBBBM72_INDEX	23



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Example – ABX Valuation and Calibration

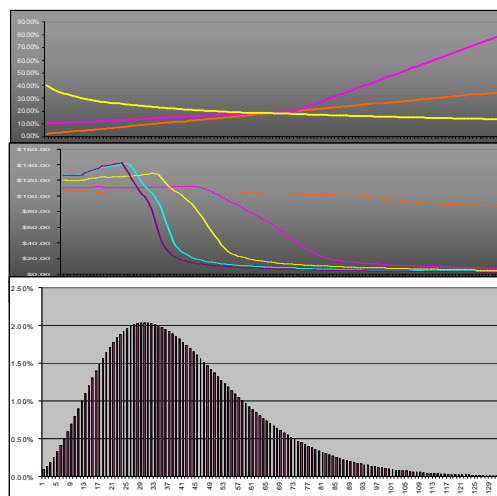


- Weighted MC → implied factor distribution (implied scenario weights)

Example:

- Vasicek PD distribution (Gaussian copula GLLM)
 - Implied avg. PD = 12%
 - Implied correlation = 7%

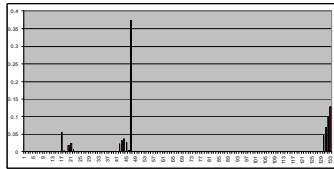
	Market Price	Estimated Prices
ABX-XHAAA72	91.81	103.06
ABX-XHAA72	71.06	94.40
ABX-XHA72	44.31	78.89
ABX-XHBBB72	26	57.19
ABX-XHBBBM72	23	48.47



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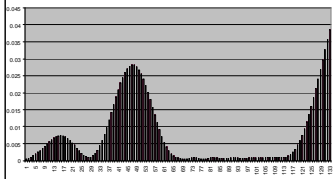
Example – ABX Valuation and Calibration



	Market Price	Model Prices
ABX-XHAAA72_INDEX	91.81	97.66
ABX-XHAA72_INDEX	71.06	69.91
ABX-XHA72_INDEX	44.31	44.69
ABX-XHBBB72_INDEX	26	25.84
ABX-XHBBBM72_INDEX	23	23.02

Best fitted prices

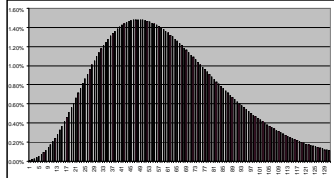
5.99%
-1.65%
0.86%
-0.63%
0.08%



	Market Price	Model Price
ABX-XHAAA72_INDEX	91.81	98.46
ABX-XHAA72_INDEX	71.06	69.59
ABX-XHA72_INDEX	44.31	44.94
ABX-XHBBB72_INDEX	26	26.04
ABX-XHBBBM72_INDEX	23	22.70

Smoothed distribution (non-parametric)

7.2%
-2.1%
1.4%
0.2%
-1.3%
7.8%



	Market Price	Model Price
ABX-XHAAA72	91.81	101.46
ABX-XHAA72	71.06	69.07
ABX-XHA72	44.31	45.35
ABX-XHBBB72	26	27.21
ABX-XHBBBM72	23	21.70

Optimal parametric Distribution
PD=16.5
R=7%

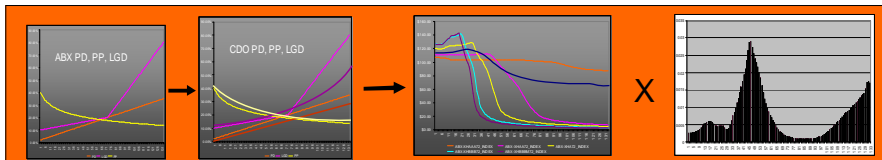
10.5%
-2.8%
2.3%
4.7%
-5.7%
13.3%

Example – Valuing ABS CDO

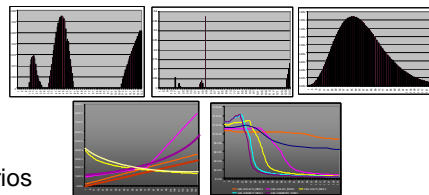


Four Steps

1. Factor scenarios → PD, PP, LGD scenarios for new CDO (factor models)
2. CDO PV scenarios from cash-flow and waterfall engines
3. Valuation using pre-calibrated model



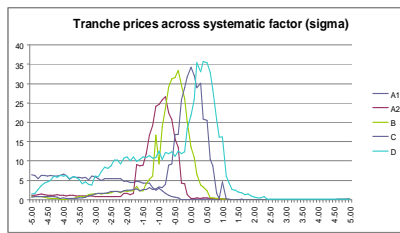
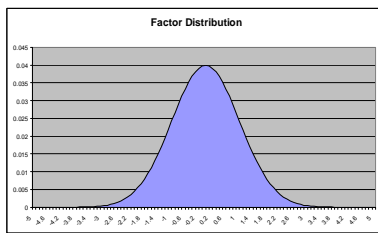
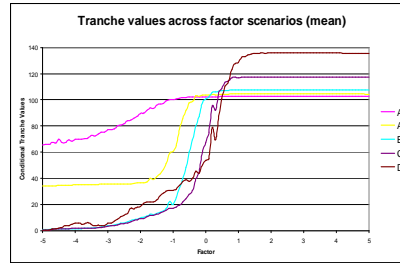
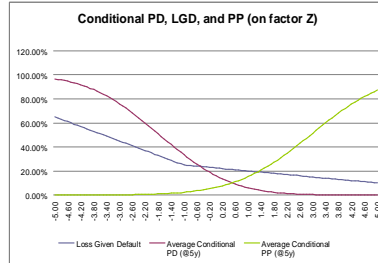
4. Sensitivities and risk measures
5. Model risk assessment
 - “Plausible” factor distributions
 - PD, PP, LGD model assumptions
 - Stress testing and extreme scenarios



Example 2 – CLO Valuation



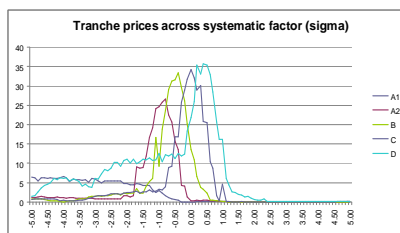
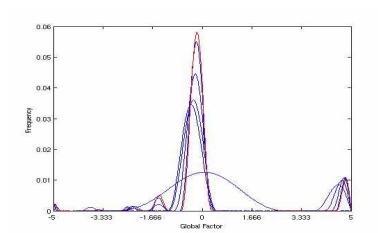
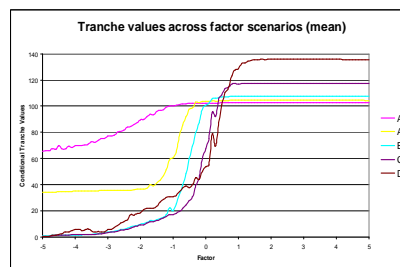
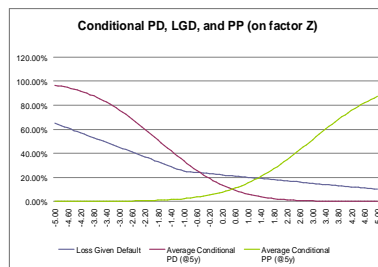
Tranche	Price
A1	\$ 101.02
A2	\$ 90.97
B	\$ 77.16
C	\$ 67.01
D	\$ 70.77



Example – CLO Valuation



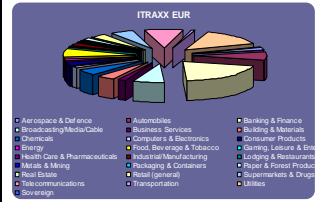
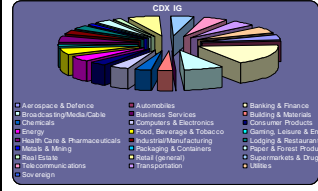
Tranche	Price
A1	\$ 76.60
A2	\$ 67.24
B	\$ 43.32
C	\$ 22.50
D	\$ 23.81



Example 3: Multi-Factor Model & Bespoke Synthetic



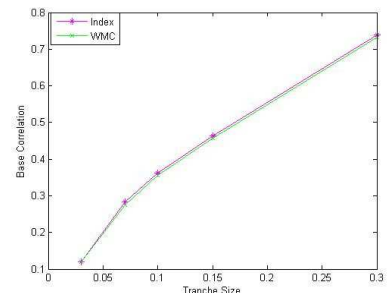
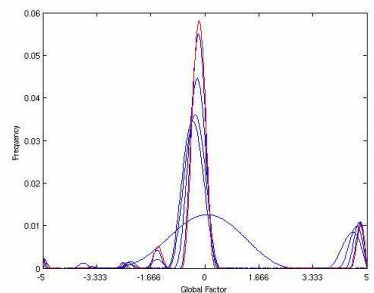
Industry concentration by Notional					
	CDX HY	CDX IG	ITRAXX EUR	ITRAXX CJ	ITRAXX ASIA
Exposure Per Name	10MM	8MM	8MM	20MM	1B
Number of Names	100	125	125	50	50
Industry (Fitch)					
Aerospace & Defence	3.00%	4.00%	2.40%		4.00%
Automobiles	9.00%		7.20%	4.00%	4.00%
Banking & Finance	3.00%	17.60%	20.00%	20.00%	24.00%
Broadcasting/Media/Cable	7.00%	8.00%	7.20%		
Business Services	4.00%		1.60%	10.00%	
Building & Materials	4.00%	3.20%	4.00%	6.00%	
Chemicals	6.00%	3.20%	4.80%	2.00%	2.00%
Computers & Electronics	10.00%	4.00%	1.60%	8.00%	6.00%
Consumer Products	5.00%	3.20%	3.20%	4.00%	
Energy	10.00%	4.80%	1.60%		10.00%
Food, Beverage & Tobacco	4.00%	5.60%	7.20%	4.00%	
Gaming, Leisure & Entertainment	3.00%	1.60%	0.00%		2.00%
Health Care & Pharmaceuticals	3.00%	5.60%	0.80%	0.00%	
Industrial/Manufacturing	2.00%	3.20%	2.40%	8.00%	6.00%
Lodging & Restaurants	3.00%	3.20%	0.80%		
Metals & Mining	2.00%	1.60%	0.80%	12.00%	4.00%
Packaging & Containers	2.00%				
Paper & Forest Products	5.00%	3.20%	1.60%		
Real Estate		1.60%	0.00%	4.00%	
Retail (general)	3.00%	6.40%	4.00%	4.00%	8.00%
Supermarkets & Drugstores	1.00%	4.80%	4.80%		
Telecommunications	4.00%	4.80%	8.80%	2.00%	12.00%
Transportation	3.00%	4.80%	0.80%	8.00%	4.00%
Utilities	4.00%	5.60%	14.40%	4.00%	4.00%
Sovereign					14.00%
Herfindahl	5.7%	7.0%	9.5%	9.8%	12.2%
Effective number of sectors	17.5	14.2	10.5	10.2	8.2



Weighted MC and CDX Index Prices



Penalties	Global factor statistics				Tranche Prices				
	MEAN	STD	SKEW	EX.KURT	0-3%	3-7%	7-10%	10-15%	15-30%
None	0.00	0.99	-0.11	-0.27	28.47%	229.06	45.97	9.81	1.12
level 1	0.37	1.90	1.49	1.02	32.31%	118.39	32.85	23.27	9.47
level 2	0.30	1.81	1.68	2.05	32.22%	111.58	34.8	18.67	10.37
level 3	0.22	1.69	1.90	3.33	32.04%	106.54	30.78	16.12	10.77
level 4	0.14	1.54	2.23	5.01	31.89%	101.58	24.46	12.47	7.5
Final	0.12	1.47	2.42	5.98	31.82%	99.35	21.47	10.77	5.77
MARKET					31.81%	99	21	10.5	5.5



Bespoke Portfolios and Concentration

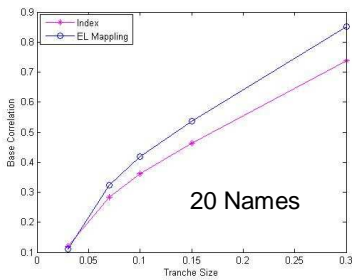


Sector (aggregate)	CDX Index		Bespoke Portfolio (40)		Bespoke Portfolio (20)	
	Weight (Notional)	Weight (EL)	Weight (Notional)	Weight (EL)	Weight (Notional)	Weight (EL)
TECH	20.8%	21.4%	20%	14.5%	50%	72.3%
SERVICE	9.6%	10.9%	15%	16.7%		
PHARMA	5.6%	3.6%	5%	3.9%		
RETAIL	20.0%	29.0%	17.5%	27.2%		
FINANCE	19.2%	11.8%	10%	6.6%	50%	27.7%
INDUSTRY	9.6%	9.9%	15%	14.2%		
ENERGY	15.2%	13.5%	17.5%	16.9%		
<i>HI</i>	0.16	0.19	0.16	0.18	0.50	0.60
No. Eff. sectors	6.07	5.40	6.30	5.63	2.00	1.67
Avg 5yr PD	Index = 3.65%		40 Name = 3.92%		20 Name = 3.36%	

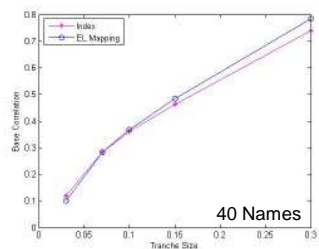
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EL Mapping



CDX Point	EL Mapped Point (40)	EL Mapped Point (20)
3%	3.45%	3.17%
7%	7.01%	5.89%
10%	9.75%	7.98%
15%	14.01%	11.55%
30%	27.77%	24.65%



PRICES			
Tranche	Index	Bespoke (40)	Bespoke (20)
0 - 3%	31.81%	34.04%	28.26%
3 - 7%	99	139	113
7-10%	21	43	29
10-15%	9.9	18	11
15-30%	5.5	0	0

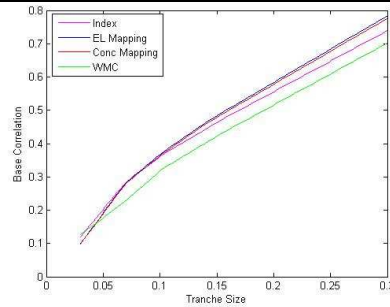
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Example – Bespoke (40 Names)



Penalties	Tranche Prices				
	0-3%	3-7%	7-10%	10-15%	15-30%
None	29.75%	302.4	77.78	19.53	1.83
level 1	31.58%	244.48	39.2	25.16	10.65
level 2	31.93%	232.1	39.48	22.28	10.85
level 3	32.22%	219.38	38.48	19.67	10.99
level 4	32.48%	207.92	35.2	15.41	7.78
Final	32.57%	203.51	33.07	13.37	6.09
EL Mapping	34.04%	139	43	18	0
WMC Index	31.82%	99.35	21.47	10.77	5.77
Market Index	31.81%	99	21	10.5	5.5



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Example – Bespoke (40 Names)



	0-3%	3-7%	7-10%	10-15%	15-30%
MARKET	31.81	99	21	10.5	5.5
WMC INDEX	31.81	99.53	19.96	10.86	5.48
BESPOKE (WMC MF)	33.13	214.54	24.66	11.18	5.53
BESPOKE (WMC SF)	32.57	203.51	33.07	13.37	6.09
EL Mapping	34.04	139	43	18	0

SWMC pricing of bespoke portfolio (40 names) – multi-factor model

Moments of implied multi-factor distributions

Factor	MEAN	STD	SKEW	KURT
Global	0.66	1.26	1.35	2.89
TECH	0.15	2.91	- 0.14	- 1.13
SERVICE	0.13	2.86	- 0.07	- 1.09
PHARMA	0.31	2.59	- 0.09	- 1.14
RETAIL	- 0.36	2.73	0.13	- 1.01
FINANCE	0.13	2.41	0.06	- 0.72
INDUSTRY	0.39	2.78	- 0.21	- 1.13
ENERGY	0.37	2.83	- 0.16	- 1.10

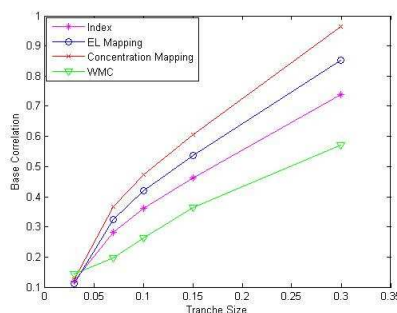
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Example – Bespoke (20 Names)



	0-3%	3-7%	7-10%	10-15%	15-30%
MARKET	31.81	99	21	10.5	5.5
WMC INDEX	31.81	99.53	19.96	10.86	5.48
BESPOKE					
(40 name) WMC	32.57	203.51	33.07	13.37	6.09
BESPOKE					
(20 Name) WMC	20.98	266	81	30	8
BESPOKE EL Mapping	28.26	113	29	11	0



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Example 4: Bespoke CDO on Two Indices



Index Prices (5Y)

CDX Tranche	Spread	iTraxx Tranche	Spread
0-3%	0.27617	0-3%	0.11615
3-7%	112.25	3-6%	59.44
7-10%	21.16	6-9%	14.88
10-15%	9.96	9-12%	6.56
15-30%	4.26	12-22%	2.63
Index	38.52	Index	24.33

Implied Default Probabilities

Index	Avg. Hazard Rate	Avg. annual Def. Prob.
CDX	0.00632	0.00629
iTraxx	0.00448	0.00447

Sector Concentrations

Sector	CDX	iTraxx
Comm. and Tech.	14.4%	16%
Financial	18.4%	20%
Materials	8.8%	10.4%
Consumer Stable	12.8%	14.4%
Utilities	6.4%	12%
Energy	4.8%	2.4%
Industrial	11.2%	5.6%
Consumer Cyclical	21.6%	17.6%
Government	1.6%	1.6%
Eff. Num. Sectors	6.92	6.83

(March 31st, 2007)

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Bespoke CDO – Super Senior (CDX & iTraxx)



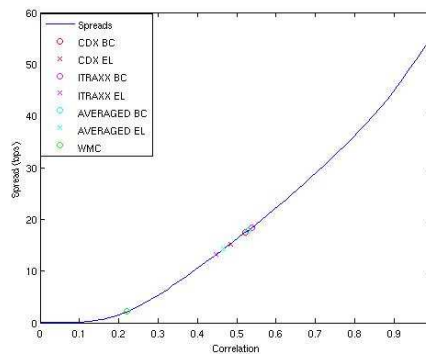
Characteristics	Values
Attachment Point	15%
Detachment Point	60%
Underlying Portfolio	100 names, 51 NA (28 CDX, 23 bespoke), 49 Euro (= non NA, 28 iTraxx, 21 bespoke).
Bespoke Average Hazard Rate	0.0115
Bespoke Average 1 Yr. Implied Default Probability	0.0114

Sector	Bespoke	CDX	iTraxx
Comm. and Tech.	17%	14.4%	16%
Financial	14%	18.4%	20%
Materials	19%	8.8%	10.4%
Consumer Stable	11%	12.8%	14.4%
Utilities	2%	6.4%	12%
Energy	1%	4.8%	2.4%
Industrial	3%	11.2%	5.6%
Consumer Cyclical	32%	21.6%	17.6%
Government	1%	1.6%	1.6%
Eff. Num. Sectors	4.985	6.92	6.83

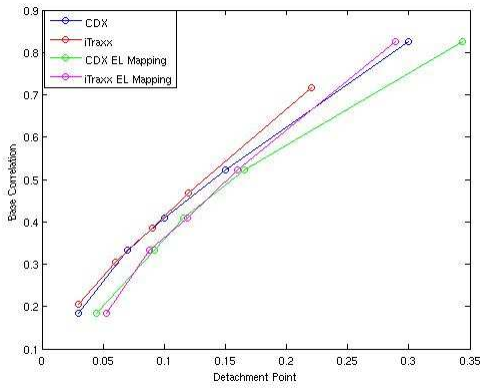
Bespoke CDO Pricing



Method	Bespoke Spread	CDX comp Spread	iTraxx comp Spread
Weighted Monte-Carlo	2.19 ($\rho=0.22$)	3.78 ($\rho=0.41$)	2.05 ($\rho=0.43$)
CDX Base Correlation	17.4 ($\rho=0.52$)	8.46 ($\rho=0.52$)	4.56 ($\rho=0.52$)
CDX EL Mapping	15.25 ($\rho=0.49$)	6.76 ($\rho=0.49$)	3.38 ($\rho=0.49$)
iTraxx Base Correlation	18.38 ($\rho=0.54$)	9.24 ($\rho=0.54$)	5.16 ($\rho=0.54$)
iTraxx EL Mapping	13.16 ($\rho=0.45$)	5.19 ($\rho=0.45$)	2.46 ($\rho=0.45$)
Avg. Base Correlation	17.88 ($\rho=0.53$)	8.84 ($\rho=0.53$)	4.85 ($\rho=0.53$)
Averaged EL Mapping	14.21 ($\rho=0.47$)	5.96 ($\rho=0.47$)	2.89 ($\rho=0.47$)



Bespoke CDO Pricing and EL Mapping



CDX Detachment	Bespoke Mapped	iTraxx Detachment	Bespoke Mapped
0.03	0.0444	0.03	0.0527
0.07	0.0921	0.06	0.0883
0.1	0.1156	0.09	0.1183
0.15	0.1660	0.12	0.1598
0.3	0.3439	0.22	0.2888



Concluding remarks

Modelling ABS CDO Structured Credit Products



- Important to understand contents and model collateral details (inhomogeneous portfolios)
 - Scenario vectors with specific loan assumptions
- Key to model concentration risk
 - Within a product – impact on valuation and risk
 - Across products – portfolio credit risk
- Risk metrics
 - Sensitivities
 - Scenarios and stress testing
 - Statistical measures (VaR) and risk contributions

Detailed Asset Modelling and Discrimination

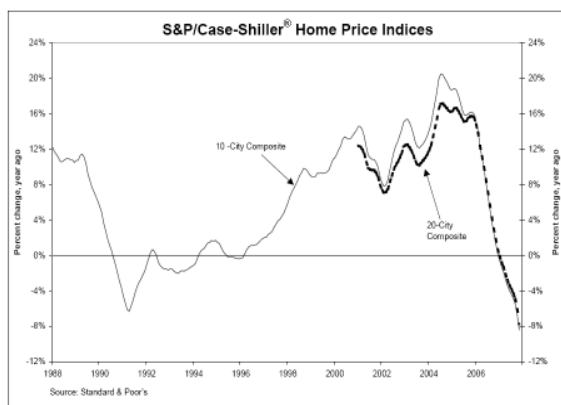


Important to understand underlying pools and model collateral details (bottom-up)
 - sub-prime vs. prime, vintage, region, etc...

Metropolitan Area	November 2007 Level	November/October Change (%)	October/September Change (%)	1-Year Change (%)
Atlanta	131.46	-1.8%	-1.2%	-2.0%
Boston	167.40	-1.1%	-0.8%	-3.0%
Charlotte	132.68	-1.0%	-0.9%	2.9%
Chicago	161.61	-0.9%	-0.8%	-3.9%
Cleveland	113.29	-2.3%	-1.2%	-5.8%
Dallas	122.38	-1.6%	-0.8%	-1.2%
Denver	133.36	-2.0%	-1.7%	-3.1%
Detroit	105.24	-2.7%	-2.4%	-13.0%
Las Vegas	201.95	-3.2%	-2.2%	-13.2%
Los Angeles	240.43	-3.6%	-2.1%	-11.9%
Miami	237.99	-2.6%	-2.1%	-15.1%
Minneapolis	158.57	-1.7%	-1.4%	-6.6%
New York	203.88	-0.8%	-0.5%	-4.8%
Phoenix	194.45	-3.1%	-2.2%	-12.9%
Portland	183.65	-0.8%	-0.3%	1.3%
San Diego	209.60	-3.4%	-2.6%	-13.4%
San Francisco	195.49	-3.2%	-2.1%	-8.6%
Seattle	187.14	-1.4%	-0.9%	1.8%
Tampa	203.45	-1.4%	-1.8%	-12.6%
Washington	223.45	-1.7%	-0.8%	-7.8%
Composite-10	205.09	-2.2%	-1.4%	-8.4%
Composite-20	188.82	-2.1%	-1.4%	-7.7%

Source: Standard & Poor's
 Data through November 2007

Remarks – Systematic Risk and Cash CDOs



- Strong conditional systematic factor → falling home prices
 - Default rates of subprime mortgages
 - Correlation of default of ABS tranches
- Home prices in the US could not continue to increase indefinitely - regime switch
- Default rates and ABS tranche's correlations based on benign period (1996-2006 – prices continually rising) not applicable in a falling price environment

Summary



- Systematic approach – robust and practical CDO valuation framework
 - CDO analytics and computational techniques
 - Multi-factor credit models
 - Weighted Monte Carlo techniques (used in options pricing)
- Value consistently bespoke tranches, CDOs of bespoke portfolios, products on multiple indices, structured finance CDOs, CDO-squared,
 - Arbitrage-free prices
 - Flexible calibration – fits prices and makes effective use of market, historical and portfolio information
 - Characterize and model explicitly the effect concentration/diversification
 - Transparent, easy to understand and relate to market practices
 - Integration of other risks (prepayment) and complex structures (ABS, cashCDO)
- Practical advantages of working through factor models, rather than directly on a common hazard rate (or a set of them)

Presenter's Bio



Dr. Dan Rosen is the co-founder and President of *R² Financial Technologies* and acts as an advisor to institutions in Europe, North America, and Latin America on derivatives valuation, risk management, economic and regulatory capital. He is a research fellow at the *Fields Institute* for Research in Mathematical Sciences and an adjunct professor at the *University of Toronto's* Masters program in Mathematical Finance.

Up to July 2005, Dr. Rosen had a successful ten-year career at *Algorithmics Inc.*, where he held senior management roles in strategy and business development, research and financial engineering, and product marketing. In these roles, he was responsible for setting the strategic direction of Algorithmics' solutions, new initiatives and strategic alliances as well as heading up the design, positioning of credit risk and capital management solutions, market risk management tools, operational risk, and advanced simulation and optimization techniques, as well as their application to several industrial settings.

Dr. Rosen lectures extensively around the world on financial engineering, enterprise risk and capital management, credit risk and market risk. He has authored numerous papers on quantitative methods in risk management, applied mathematics, operations research, and has coauthored two books and various chapters in risk management books (including two chapters of PRMIAs Professional Risk Manager Handbook). In addition, he is a member of the Industrial Advisory Boards of the Fields Institute, and the Center for Advanced Studies in Finance (CASF) at the University of Waterloo, the Academic Advisory Board of Fitch, the Advisory Board and Credit Risk Steering Committee of the IAFE (International Association of Financial Engineers) and the regional director in Toronto of PRMIA (Professional Risk Management International Association). He is also one of the founders of RiskLab, an international network of research centres in Financial Engineering and Risk Management.

He holds several degrees, including an M.A.Sc. and a Ph.D. in Chemical Engineering from the University of Toronto.

Selected Recent Publications



- Rosen D. and Saunders D., 2007, *Valuing CDOs of Bespoke Portfolios with Implied Multi-Factor Models*, Working Paper Fields Institute for Mathematical Research and University of Waterloo
- Rosen D. and Saunders D., 2006a, *Analytical Methods for Hedging Systematic Credit Risk with Linear Factor Portfolios*, Working Paper, Fields Institute for Mathematical Research and University of Waterloo
- Rosen D. and Saunders D., 2006b, *Measuring Capital Contributions of Systemic Factors in Credit Portfolios*, Working Paper Fields Institute for Mathematical Research and University of Waterloo
- Garcia Céspedes J. C., Keinin A., de Juan Herrero J. A. and Rosen D., 2006, *A Simple Multi-Factor "Factor Adjustment" for Credit Capital Diversification*, Special issue on Risk Concentrations in Credit Portfolios (M. Gordy, editor) *Journal of Credit Risk*, Fall 2006
- Garcia Céspedes J. C., de Juan Herrero J. A., Rosen D., Saunders D., 2007, *Effective modelling of Alpha for Regulatory Counterparty Credit Capital*, Working Paper, Fields Institute
- Mausser H. and Rosen D., 2007, *Economic Credit Capital Allocation and Risk Contributions*, forthcoming in *Handbook of Financial Engineering* (J. Birge and V. Linetsky Editors)
- De Prisco B., Rosen D., 2005, *Modelling Stochastic Counterparty Credit Exposures for Derivatives Portfolios*, Counterparty Credit Risk (M. Pykhtin, Editor), Risk Books, London
- Aziz A., Rosen D., 2004, *Capital Allocation and RAPM*, in *Professional Risk Manager (PRM) Handbook*, Chapter III.0, PRMIA Publications
- Rosen D., 2004, *Credit Risk Capital Calculation*, in *Professional Risk Manager (PRM) Handbook*, Chapter III.B5, PRMIA Publications

$$\min_{z_i} L(z) \\ \text{s.t. } \sum z_i^2 = C \quad \sum w_i \beta_i z_i \quad L = \sum x_i L_i$$



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