Tranching and Rating

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Why are we here?

- Approximately \$471 billion of the \$550 billion of collateralized debt obligations (CDOs) that were issued in 2006 were classified by the Securities Industry and Financial Markets Association (SIFMA) as 'Arbitrage CDOs'.
- These are defined by SIFMA as an 'attempt to capture the mismatch between the yields of assets (CDO collateral) and the financing costs of the generally higher rated liabilities (CDO tranches).
- The remaining issuance is classified as 'Balance Sheet' CDOs which 'remove assets or the risk of the assets off the balance sheet of the originator'.

Ratings play a dominant role in the marketing of CDO

- Financial Times (December 6. 2007):
  - "For many investors ratings have served as a universally accepted benchmark"
  - "Some funds have rued their heavy dependence on ratings"
- IMF Global Financial Stability Report, April 2008
  - "Credit ratings have been a key input for many investors in the valuation of structured credit products because they have been perceived to provide a common credit risk metric for all fixed-income instruments."
- Standard & Poor's, 2007
  - "Our ratings represent a uniform measure of credit quality globally and across all types of debt instruments. In other words, an 'AAA' rated corporate bond should exhibit the same degree of credit quality as an 'AAA' rated securitized debt issue."
- IMF "When Is a AAA not a AAA?"

Research Questions

- PROBLEM:
  Ratings (either based on default probabilities or on expected default losses) only (partially in the case of S&P) reflect the *total risk* of a security and do not say anything about *systematic risk*.

  In a more obscure language; these default probabilities and expected default losses are calculated under the physical measure and not risk neutral measure, i.e. they have not taken market price of risk into consideration.

- QUESTION:
  Mispricing occurs if tranched securities are treated to be the same as equally rated corporate bonds. The amount of mispricing then depends on
  - Diversification
  - Systematic risk
  - Tranching alone

Related Literature

- *Coval, Jurek and Stafford* (2007):
  → possible to exploit investors, who rely on default probability based ratings, by selling bonds whose default losses occur in high marginal utility states.
- *Cuchra* (2005):
  → launch spreads are mainly determined by tranche ratings.
- Further explanations for tranching debt securities:
  Brennan & Kraus (1989), *Boot & Thakor* (1993), *DeMarzo & Duffie* (1999), *DeMarzo* (2005), *Gaur et al.* (2004)

Rating Based on Default Probability

- Example: Standard & Poor's and Fitch
- Rating class k such that

$$k_1 = AAA, \quad k_2 = AA, \quad \cdots$$

- Probability of default, where

$$\Pi_{k_1} < \Pi_{k_2} < \cdots < \Pi_{k_n}$$

Rating Based on Expected Loss

- Example: Moody's
- Rating class k such that

$$k_1 = Aaa, \quad k_2 = Aa, \quad \cdots$$

- Probability of default, where

$$\Pi_{k_1} < \Pi_{k_2} < \cdots < \Pi_{k_n}$$

- Expected default loss is an increasing function

$$\Lambda_{k_1} < \Lambda_{k_2} < \cdots < \Lambda_{k_n}$$

Ratings Based Pricing (Corporate Bond)

Corporate Bond with rating k and maturity τ

$$\Lambda_k, \Pi_k \quad \xleftarrow{P_k^*} \quad B_k^* \quad \xrightarrow{Q_k^*} \quad \frac{W_k^*}{B_k^*} \equiv \phi_k^*$$

- Issued by a single firm
- All claims are equally senior; one tranche only
- Market Value W_k^*
- Face Value B_k^*
- Ratio of Market Value to Face Value:

$$\begin{aligned} \phi_k^* &\equiv W_k^*(V)/B_k^* \\ W_k^*(V) &= B_k^* \phi_k^* = B_k^* e^{-r_k^* T} \end{aligned}$$

Ratings Based Pricing (Structured Bond)

Structured Bond (Tranche) with rating k and maturity τ

$$\Lambda_k, \Pi_k \quad \xleftarrow{P_k} \quad B_k \quad \xrightarrow{Q_k} \quad \frac{W_k}{B_k} \equiv \phi_k$$

- Issued by a Special Purpose Vehicle (against a portfolio of bonds)
- Typically subordinated to higher tranches
- Market Value W_k
- Face Value B_k
- Ratio of Market Value to Face Value:

$$\begin{aligned} \phi_k &\equiv W_k(V)/B_k \\ W_k(V) &= B_k\phi_k = B_k e^{-r_k T} \end{aligned}$$

Ratings Based Pricing Assumptions

The *sales price* of the structured bond bears the same relation to its face value as does the value of an equivalently rated corporate bond with the same maturity:

$$S_k = \phi_k^* B_k.$$

This price usually differs from the fair *market value* of the tranche given by:

$$W_k = \phi_k B_k$$

which gives rise to pricing error

$$\Omega = S_k - W_k$$

Justification

Reuters, March 12, 2008.

The Treasurer of the State of California , “If the state of California received the triple-A rating it deserved, we could reduce taxpayers' borrowing costs by hundreds of millions of dollars over the 30-year term of the still-to-be issued bonds...”

Single Tranche Securitization

1. Determination of Face Values:

   RATING BASED ON DEFAULT PROBABILITY:

$$B_k^* = F_{P^*}^{-1}(\Pi_k) \quad \text{or} \quad F_{P^*}(B_k^*) = \Pi_k$$

   RATING BASED ON EXPECTED LOSS:

$$\begin{aligned} B_k^* &= G_{P^*}^{-1}(\Lambda_k) \quad \text{or} \quad G_{P^*}(B_k^*) = \Lambda_k \\ \text{with} \quad \Lambda_k &= \frac{\mathcal{L}_k}{B_k^*}, \quad \text{and} \quad \mathcal{L}_k = \int_0^{B_k^*} (B_k^* - v) f_{P^*}(v) dv \end{aligned}$$

2. Calculation of Market Values

$$W_k^* = \int_0^{B_k^*} v f_{Q^*}(v) dv + B_k^* \int_{B_k^*}^{\infty} f_{Q^*}(v) dv \equiv \phi_k^* B_k^*$$

3. Amount of mispricing

$$\Omega = S_k - W_k = [\phi_k^* - \phi_k] B_k$$

Lemma 1: Rating Baed on Default Probability

(a)

$$\Omega > 0 \qquad \text{if} \quad P \geq^{FSD} P^* \quad \text{and} \quad Q^* \geq^{SSD} Q$$

The converse is true for $\Omega < 0$.

(b) If two SPVs have the same risk-neutral distribution Q and their physical distributions, P_1 and P_2, are such that

$$P_2 \geq^{FSD} P_1 \geq^{FSD} P^* \quad \text{and} \quad Q^* \geq^{SSD} Q$$

then

$$\Omega_2 > \Omega_1$$

for a given rating k.

Proof of Lemma 1 (a)

Note that $P \geq^{FSD} P^*$ implies that $B_k \geq B_k^*$. The amount gained can be written as:

$$\begin{aligned}
\Omega &= \frac{B_k}{B_k^*} E_{Q^*}\left\{\min[B_k^*, V^*]\right\} - E_Q\left\{\min[B_k, V]\right\} \\
&= E_{Q^*}\left\{\min[B_k, \frac{B_k}{B_k^*} V^*]\right\} - E_Q\left\{\min[B_k, V]\right\} \\
&\geq E_{Q^*}\left\{\min[B_k, V^*]\right\} - E_Q\left\{\min[B_k, V]\right\}
\end{aligned}$$

Ω is positive if $Q^* \geq^{SSD} Q$.
For the converse, note that $P^* \geq^{FSD} P$ implies $B_k < B_k^*$.

Default Probability vs. Expected Loss

Figure 2(a). Asset Distribution and
Payoff Profiles of A-rated (Based on Default Probability)
Corporate Bond and CDO Tranche, β=0.7, σ=14%

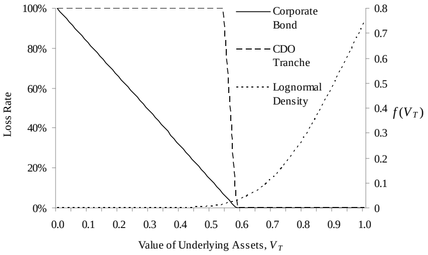

Figure 1(a). Asset Distribution and
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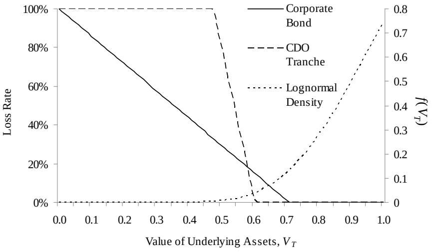


- One could also profit by, e.g., selling security with lower recovery rates and long in security that is otherwise the same but with a higher recovery rate.
- Since 2005, Standard & Poor's applies higher default probabilities for structured bonds (than corporate bond) which itself leads to a profit if pricing is strongly influenced by default probability.

Lemma 2: Rating Based on Expected Loss

(a)

$$\Omega > 0 \qquad \text{if} \quad P \geq^{SSD} P^* \quad \text{and} \quad Q^* \geq^{SSD} Q$$

The converse is true for $\Omega < 0$.

(b) If two SPVs have the same risk-neutral distribution Q and their physical distributions, P_1 and P_2, are such that

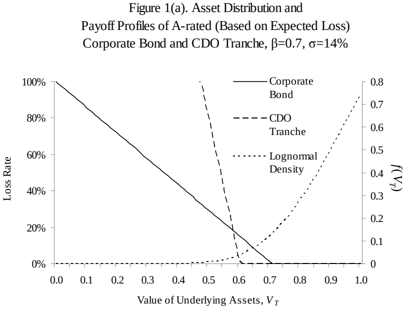
$$P_2 \geq^{SSD} P_1 \geq^{SSD} P^* \quad \text{and} \quad Q^* \geq^{SSD} Q$$

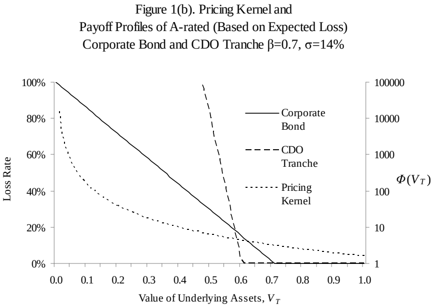
then

$$\Omega_2 > \Omega_1$$

for a given rating k.

Expected Loss: Effect of Pricing Kernel





- One could profit, e.g., by selling securities whose default losses are allocated to states with the highest state price per unit probability (e.g. high beta)

Effect of Tranching

Single tranche is split into two tranches:

$$B_k = B_{1,k_1} + B_{2,k_2}$$

DEFAULT PROBABILITY RATING SYSTEM:

- Junior tranche:

  $$\Pi_{k_2} = \Pi_k \Rightarrow \phi^*_{k_2} = \phi^*_k$$

  The junior tranche will have a same yield as the corporate bond.
- Senior tranche:

  $$\Pi_{k_1} < \Pi_k \Rightarrow \phi^*_{k_1} > \phi^*_k$$

  The senior tranche will have a lower yield (and hence higher price) as the corporate bond.
- Extra gain due to having two tranches instead of one

$$\Omega_{k_1+k_2} - \Omega_{k_1} = (\phi^*_{k_1} - \phi^*_k)B_{1,k_1}$$

Multiple Tranche Securitization

- **(Lemma 3)** Under a default probability rating system it is optimal to subdivide a given tranche into a junior and a senior tranche with different ratings.
- **(Lemma 4)** When rating is based on expected loss, it is optimal to subdivide the tranche into a junior and a senior tranche with different ratings, whenever the pricing kernel, $m^*(v)$, is a decreasing function of the underlying asset value.
- Under both rating systems, it is optimal to have as many tranches as there are different rating classes.

Gains in the CAPM Framework

- For corporate bond $(*)$, the value of the underlying assets is

$$\begin{aligned} dV^* &= \mu^* V^* dt + \sigma^* V^* dz^* \\ \mu^* &= r_f + \beta^*(r_m - r_f) \end{aligned}$$

- For the SPV (no $*$), the value of the collateral is:

$$\begin{aligned} dV &= \mu V dt + \sigma V dz, \\ \mu &= r_f + \beta(r_m - r_f) \end{aligned}$$

- Given σ_m^2, r_m and r_f, the price dynamic of the corporate bond is completely described by (β^*, σ^*) and that of the collateral portfolio is completely characterized by (β, σ):

$$\underbrace{\sigma^2}_{\text{Total Risk}} = \underbrace{\beta^2 \sigma_m^2 + \sigma_\mu^2}_{\text{Conduit}} = \underbrace{(\beta^*)^2 \sigma_m^2 + \left(\sigma_\mu^*\right)^2}_{\text{Corporate Bond}}$$

Single Tranche Securitization

1. Determination of B_k and B_k^*:

   Rating Based on Default Probability:

$$\Pi_k = \mathcal{N}\left(-\frac{ln(V/B_k) + (\mu - 0.5\sigma^2)\tau}{\sigma\sqrt{\tau}}\right)$$

   Rating Based on Expected Loss:

$$\Lambda_k = \frac{\mathcal{L}_k}{B_k} = \frac{B_k\mathcal{N}(-d_2^P) - Ve^{\mu\tau}\mathcal{N}(-d_1^P)}{B_k}$$

2. Market Values given by Merton (1974) formula:

$$W_k^*(V^*) = B_k^* e^{-r_f\tau}\mathcal{N}(d_2^{Q^*}) + V^*\mathcal{N}(-d_1^{Q^*})$$

3. Sales Price of Tranche:

$$S_k = \phi_k^* B_k = \frac{W_k^*(V^*)}{B_k^*} B_k$$

4. Amount of mispricing: $\Omega = S_k - W_k(V)$

Multiple Tranche Securitization

1. Values for the corporate bond are determined as before.
2. Face Value of *ith* tranche (B_{i,k_i}):

   RATING BASED ON DEFAULT PROBABILITY:

$$B_{i,k_i} = B_{k_i} - B_{k_{i-1}}$$

   RATING BASED ON EXPECTED LOSS:

$$\Lambda_{k_n} = \frac{\mathcal{L}_{i,k_i}}{B_{i,k_i}} = \frac{\mathcal{L}_{k_i} - \mathcal{L}_{k_{i-1}}}{B_{k_i} - B_{k_{i-1}}} \quad with \quad \Lambda_{k_1} = \frac{\mathcal{L}_{k_1}}{B_{k_1}}$$

3. Market Value of *ith* tranche: $W_{i,k_i}(V) = W_{k_i}(V) - W_{k_{i-1}}(V)$
4. Sales Price of *ith* tranche: $S_{i,k_i} = \phi^*_{k_i} B_{i,k_i} = \frac{W^*_{k_i}(V^*)}{B^*_{k_i}} B_{i,k_i}$
5. Total mispricing: $\Omega = \sum_i \Omega_i = \sum_i S_{i,k_i} - W_{i,k_i}(V)$

Valuation & Pricing by Rating Class: Effect of Tranching

$\beta^* = \beta = 0.7$, $\sigma^* = 18\%$, $\sigma = 12\%$, $r_f = 3.5\%$, $r_m - r_f = 8\%$

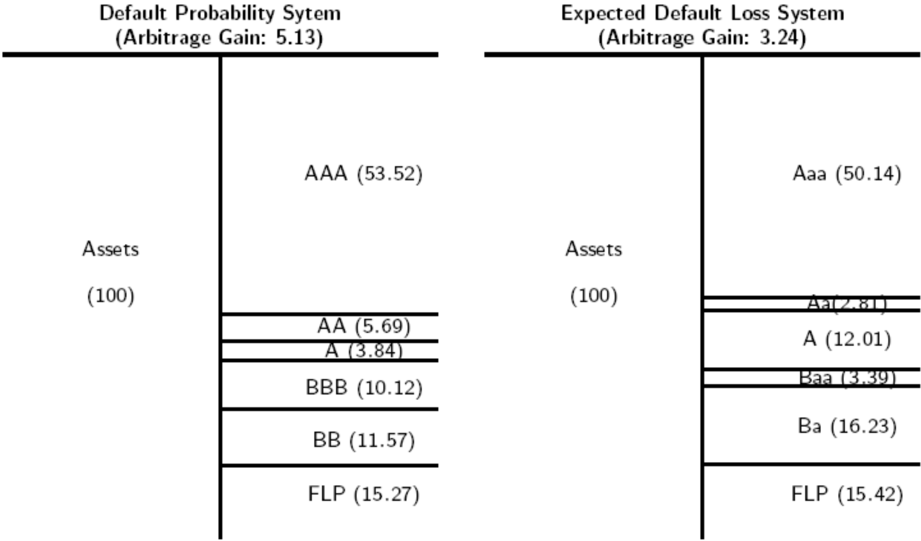
Panel A: Corporate Bond*

S&P Rating (k_i)	Probability of Default Π_{k_i}	Face Value $B^*_{k_i}$	Market Value $W^*_{k_i}$	Yield to Maturity	$\phi^*_k \equiv W^*_{k_i}/B^*_{k_i}$
AAA	0.061%	39.55	33.17	3.51%	0.839
AA	0.219%	46.17	38.68	3.54%	0.838
A	0.459%	50.94	42.60	3.58%	0.836
BBB	2.323%	65.22	53.88	3.81%	0.826
BB	10.424%	87.61	69.18	4.73%	0.790

Panel B: Tranche Valuation and Sales Prices

S&P Rating (k_i)	Cum. Value B_{k_i}	Face Value B_{i,k_i}	Market Value W_{i,k_i}	Yield to Maturity	Sales Price S_{i,k_i}	Gain
AAA	63.84	63.84	53.52	3.52%	53.55	0.03
AA	70.78	6.94	5.69	3.98%	5.82	0.13
A	75.57	4.79	3.84	4.46%	4.01	0.17
BBB	89.11	13.54	10.12	5.83%	11.18	1.07
BB	108.49	19.38	11.57	10.32%	15.30	3.73
FLP			15.27		15.27	0.00
		Total:	100.00		105.13	5.13

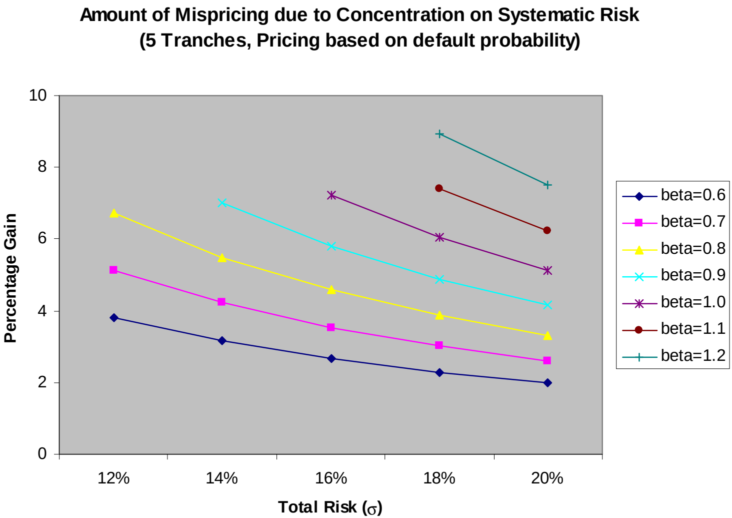
Example Structure: Beta=0.7, sigma=12%, sigma*=18%

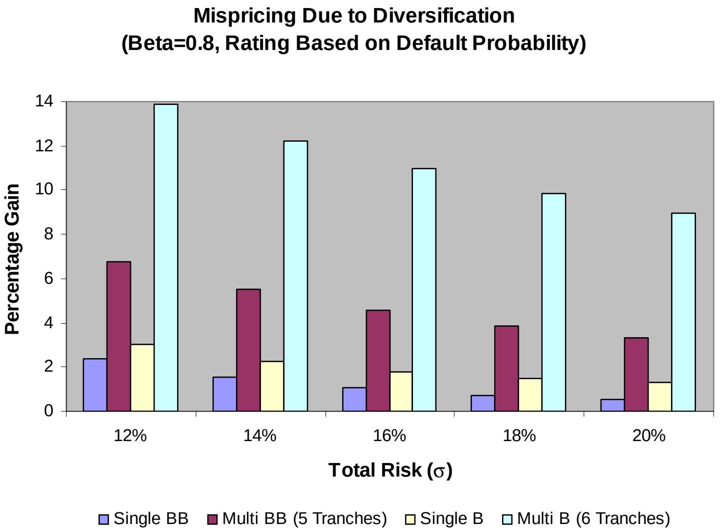


Errors when pricing is based on Default Probability

Corporate Bond $(\beta^*, \sigma^*) = (0.7, 18\%)$, $r_f = 3.5\%$, $r_m - r_f = 8\%$

SPV Collateral			Five Tranches			Six Tranches		
β	σ	Lemma 1	Debt	Ω^M_{BB}	Ω^S_{BB}	Debt	Ω^M_B	Ω^S_B
0.6	0.12	×	82.5	3.80	-0.24	90.1	8.73	-1.31
	0.16	×	72.1	2.67	-0.58	82.6	7.07	-1.38
0.7	0.12		84.7	5.13	0.93	91.8	11.14	0.69
	0.18		69.2	3.02	0.00	80.6	8.06	0.00
	0.20		64.3	2.60	-0.09	76.6	7.36	-0.01
0.8	0.12		86.8	6.74	2.36	93.2	13.89	3.02
	0.18		71.3	3.88	0.74	82.4	9.87	1.48
	0.20	✓	66.3	3.32	0.52	78.5	8.96	1.30
1.0	0.18		75.4	6.05	2.64	85.7	14.19	5.11
	0.20	✓	70.2	5.13	2.09	81.9	12.78	4.48
1.2	0.18		79.3	8.93	5.24	88.7	19.53	9.69
	0.20	✓	74.1	7.52	4.23	85.1	17.49	8.50


Amount of Mispricing due to Concentration on Systematic Risk
(5 Tranches, Pricing based on default probability)
Percentage Gain
Total Risk (σ)
0
2
4
6
8
10
12%
14%
16%
18%
20%
beta=0.6
beta=0.7
beta=0.8
beta=0.9
beta=1.0
beta=1.1
beta=1.2



Mispricing Due to Diversification
(Beta=0.8, Rating Based on Default Probability)
Percentage Gain
14
12
10
8
6
4
2
0
12%
14%
16%
18%
20%
Total Risk (σ)
Single BB
Multi BB (5 Tranches)
Single B
Multi B (6 Tranches)

Expected Default Loss Rating System

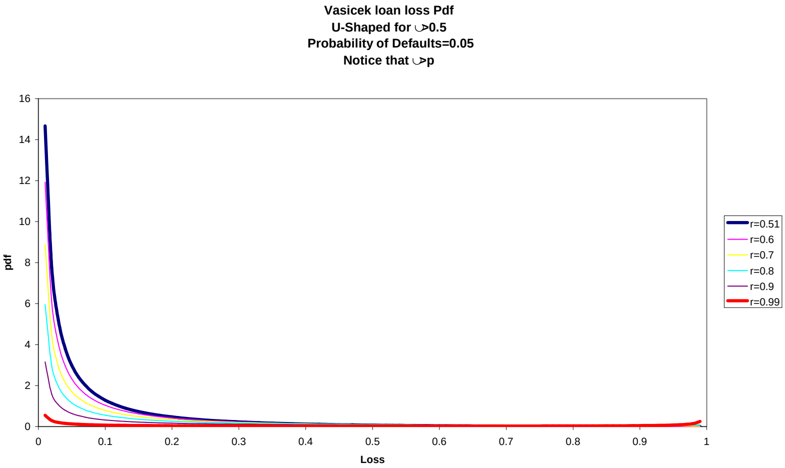
Corporate Bond $(\beta^*, \sigma^*) = (0.7, 18\%)$, $r_f = 3.5\%$, $r_m - r_f = 8\%$

SPV Collateral			Five Tranches			Six Tranches		
β	σ	Lemma 2	Debt	Ω^M_{Ba}	Ω^S_{ba}	Debt	Ω^M_B	Ω^S_b
0.6	0.12		82.4	1.99	0.97	84.2	2.41	1.11
	0.16		71.5	0.65	-0.23	74.6	0.93	-0.30
0.7	0.12	✓	84.6	3.24	2.18	86.3	3.85	2.49
	0.18		68.3	0.84	0.00	72.0	1.22	0.00
	0.20		63.2	0.42	-0.35	67.5	0.69	-0.45
0.8	0.12	✓	86.7	4.74	3.64	88.2	5.56	4.16
	0.18	✓	70.4	1.56	0.69	74.0	2.13	0.88
	0.20		65.2	1.01	0.21	69.4	1.48	0.29
1.0	0.18	✓	74.6	3.46	2.51	78.0	4.53	3.16
	0.20		69.2	2.52	1.66	73.3	3.47	2.18
1.2	0.18	✓	78.7	6.07	5.04	81.9	7.73	6.25
	0.20		73.2	4.58	3.64	77.1	6.10	4.70

Concluding Remarks

- For both rating systems, we derive general conditions under which single and multiple tranche securitizations will be overpriced.
- The conditions depend on the risk characteristics of the collateral portfolio relative to those of the typical firm for which bonds ratings apply.
- In the CAPM-Merton framework, we show that overpricing is largest when the collateral has high systematic risk and low total risk relative to a typical firm.
- There are significant gains from multi-tranching.
- Overpricing is greater when securities are valued using default probability based ratings than if rating is based on expected loss.
- Our analysis highlights the current weakness in a vakuation system that is so strong influenced by ratings which only reflect the *total risk* of fixed income securities.

Vasicek portfolio loss distribution, $X_i = \sqrt{\rho} Y + \sqrt{1-\rho} Z_i$



When Is a AAA not a AAA?

- Vasicek portfolio loss distribution, $X_i = \sqrt{\rho}Y + \sqrt{1-\rho}Z_i$
- In a "fully diversified" portfolio, $\rho \to 1$, and Y could be a single exposure to e.g. housing.
- Defaults of tranched securities of a specified rating will tend to be much more highly correlated than defaults of securities of the same rating issued by a typical undiversified firm - in the limit the defaults of the tranched securities will be perfectly correlated.
- A portfolio of n 'A' rated CLO tranches will in general be much more risky than a portfolio of n 'A' rated bonds issued by corporations.

Models are important (heavy decisions at stake)... but not that important (to some extent, models work as interpolation tools). Above all, there is no such thing as the “perfect” model (believing in one is dangerous).

Thank You!

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