



2011 ERM Symposium
March 14-16, 2011
Swissôtel Chicago
Chicago, IL

Mandelbrotian Grey Swan
Scenarios
Steve Craighead



Scenario Generation

$$dX = (AX+B)dt + V(G'dZ)$$


DMFBM MMAR(DMFBM)

Cascade

I to many Correlated many to many


Extreme to Severe

- No mean reversion
- Cluster Analysis
- Fund Threshold
- Simple Model
- Predictive/Replicating Portfolio Models

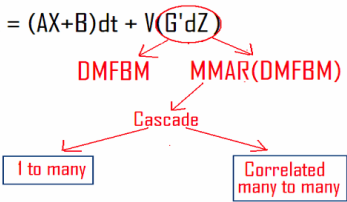


AAA Generator

- Long Normal Stochastic Volatility Generator
- 3 factor interest
 - (log(long rate)
 - yield curve slope
 - Volatility
- Wealth Factors
 - 9 funds – 6 used in paper
- www.actuary.org/life/phase3.asp#10




Scenario Generation

$$dX = (AX+B)dt + V(G'dZ)$$


```
graph TD; G((G'dZ)) --> DMFBM[DMFBM]; G --> MMAR[MMAR(DMFBM)]; DMFBM -- Cascade --> ItoMany[I to many]; MMAR -- Cascade --> Correlated[Correlated many to many];
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DMFBM

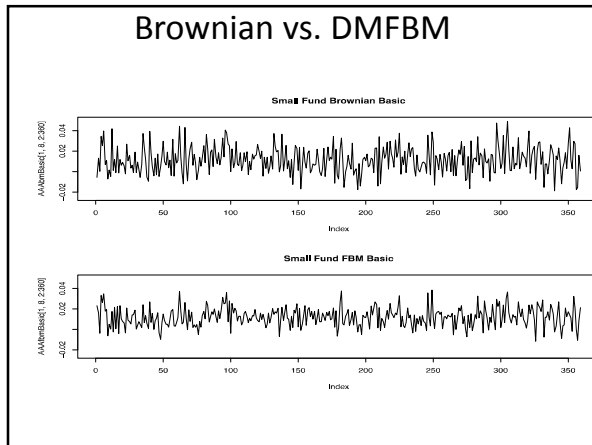
- Dependent Multivariate Fractional Brownian Motion
- Fractional – long history dependence – measured by Hurst exponent.
- Dependent Multivariate – Allow dependence modeling between separate states or assets.
- Merger of FBM and dependent noise generation.

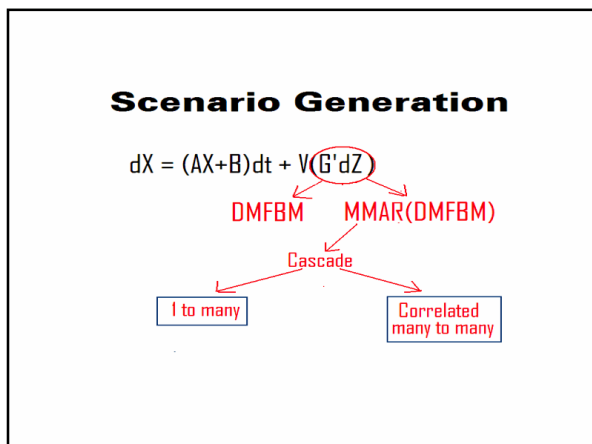


DMFBM - Algorithm

- Amblard et al. "Basic properties of the multivariate fractional brownian motion"
- Conceptually, a 3-dimensional array is created which auto-correlates states through time and with each other. This is done by using fast Fourier transforms (fft) and creating a circulant matrix (Generation Kernel) and applying this to complex random samples and then converting back using the fft into Fractional Gaussian samples (noise), which creates FBMs by cumulative summation of the samples. The noise (samples) are what are plugged in for G'dZ.






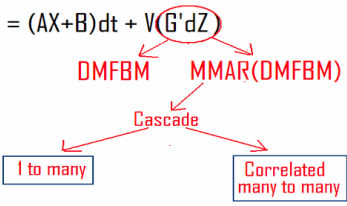


MMAR

- Multifractal Model of Asset Returns (MMAR)
- Two parts
- FBM – $B_H(t)$
- Stochastic trading time – $\theta(t)$ or cascade.
- MMAR - $X(t) = B_H[\theta(t)]$


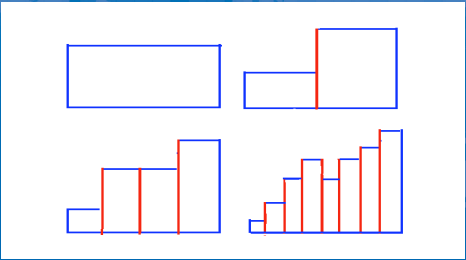


Scenario Generation

$$dX = (AX+B)dt + V(G'dZ)$$



```
graph TD; A["V(G'dZ)"] --> B["DMFBM"]; A --> C["MMAR(DMFBM)"]; C --> D["Cascade"]; D --> E["1 to many"]; D --> F["Correlated many to many"];
```

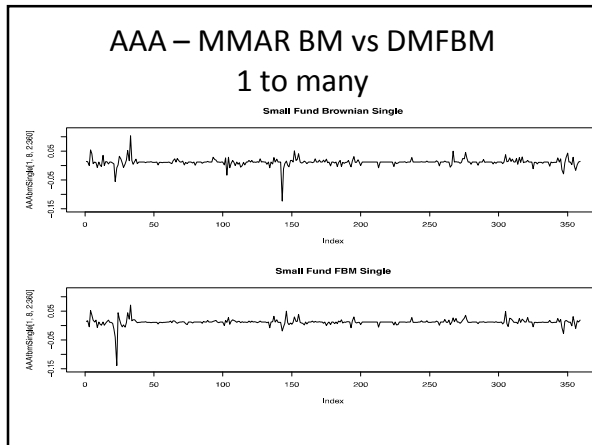
Cascade 2^3 -deterministic



Cascade - 1 to many

- This is when only one sample of a cascade distribution is used for every state
- This assumes that the volatility structure is consistent across all funds and returns.
- This is another dependence structure that could be used

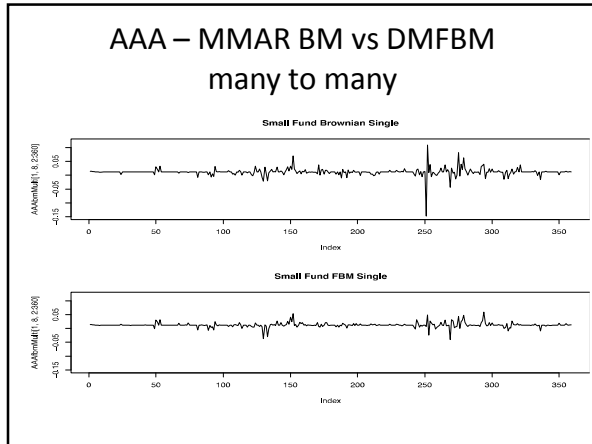




Cascade Many to Many

- Here there is a cascade model for each state and could be independent or correlated with each one another.
- This assumes that each fund or return has a different volatility regime.



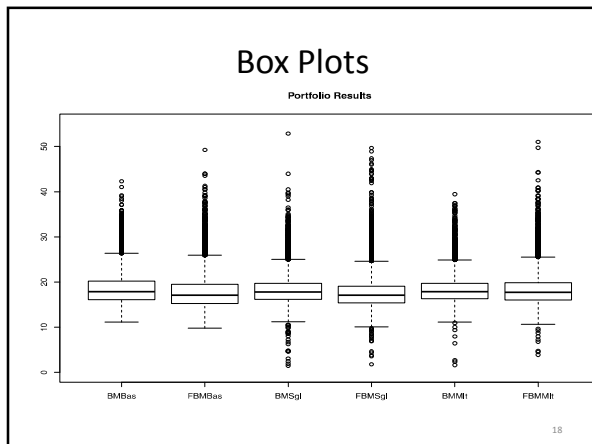


Simple Portfolio

- \$1 invested in each of 6 funds at time 0.
- Hold for 30 years.
- US, Small, AGGR, Money, ITGVT and LTCorp

ERM

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- Cluster Analysis




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