ENTERPRISE RISK MANAGEMENT
ERM
Symposium
Where Cutting Edge Theory Meets State of the Art Practice

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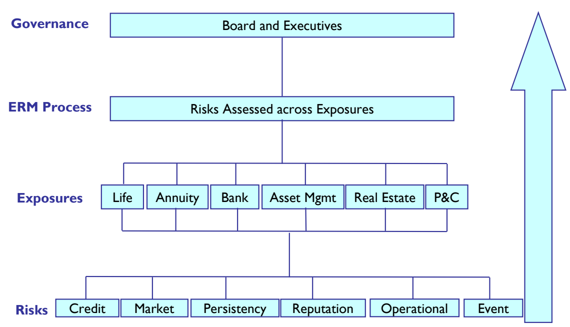
Economic Capital Modeling, a Faster (and Better?) Way

Agenda

- Why EC Modeling is Important, and Hard to Do Today
- A Faster Way: Extreme Value Theory
- Complications: Unforeseeable Events and Business Cycles
- Example: Product Line Drivers of TCE Underwriting Loss
- Example: Correlated Asset Classes
- Technical Support (if time and interest)

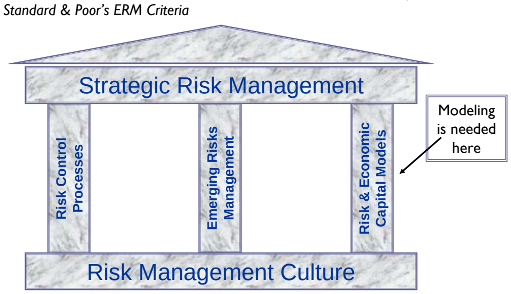
Why EC modeling is Important, and Hard to Do Today

What ERM Looks Like
Take Cross-Silo Views

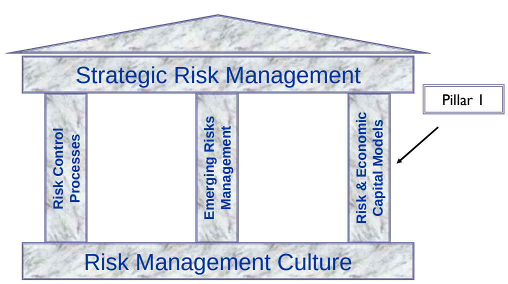


ERM and EC Modeling

"ERM is a subjective view of an insurer's or reinsurer's risk-management practices, focusing on how its loss tolerance is defined and measured and on the processes it undertakes to ensure that this tolerance is not exceeded. ERM criteria also assess how well management balances risk and returns within the context of overall corporate strategy." – *Standard & Poor's ERM Criteria*



Solvency II: Ditto



Use of Economic Capital to Make Business Decisions

- Calculate TVAR, VAR, RBC or BCAR, etc. contributions from individual operations
- Compute marginal capital and marginal profit for each operation
- Rank individual operations by profit / capital
- "Grow the winners."

But Dynamic Financial Analysis (DFA) is Slow, Expensive and Opaque

- Since DFA is a "bottom-up" view, the models require granular data on all aspects of a company's business: Cats, reserves, policy limit profiles, asset portfolios, payment patterns by AY by product line, etc, etc.
- Converging to a stable answer that tests multiple and correlated risk factors requires many, many simulation runs, often as "with and without".
- The model builder must be subjective in defining which scenarios to model.
- Asset portfolios in particular, but also some insurance risks, reflect management's decisions and market reactions during the forecast period. This is called "path dependence", and requires more runs to reflect these interim dates. So the models need even more inputs and even more subjectivities.

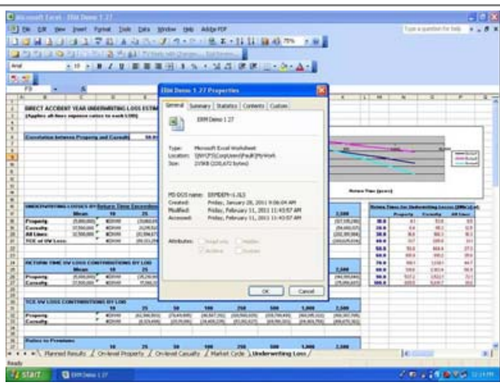
Commercially available packages can require 100,000's of input items, run for weeks, cost up to a million dollars per license, and still not be very transparent when done.

Example: 155 MB Download

A Faster Way: Extreme Value Theory

ERM

A Faster Way: 215K not 155MB



A Faster (and Better?) Way

- Instead of DFA, we can use a statistical tool called Extreme Value Theory to describe the risks that drive solvency risk, without using simulations.
- We can also build this so that as much as possible of the subjectivity is in the company's hands, not the model format.
  - ⇨ ***Illustration:*** *A spreadsheet that shows various contributions to solvency-level risks, that is small enough to e-mail, with no macros and fewer than 50 common-sense data items for an enterprise view, and not many more to include product line details, or equity sector exposures, etc.*
  - ⇨ ***Goal:*** *An easier way to embed EC models in ERM.*
- The specific example is for P&C, but the technique can be applied by any risk bearer.

A Faster (and Better?) Way

Extreme Value Theory describes the statistical properties of the largest samples from any random distribution.

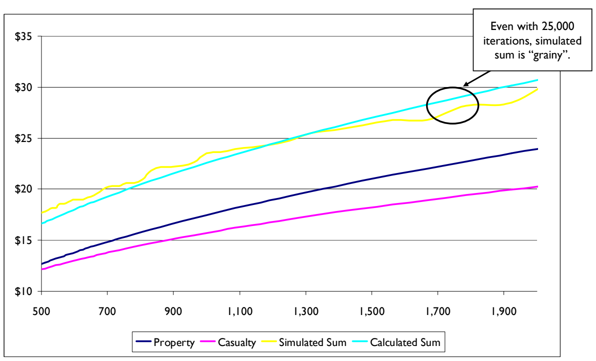
All you need to know is that: **The highest scenarios from any risk that is positive and unlimited can be almost exactly described by a simple formula in the Pareto family of curves.**

"Right-tailed" risks that drive solvency include:

- *Claim frequency*
- *Cat losses*
- *Adverse mortality*
- *WC lifetime medical claims*
- *Overall loss ratios*

To be unlimited, we need to look at exposures without considering the amount of current surplus. This also makes EPD and TVAR calculations natural.

When Parameters are Known, Combining Curves is Often Easy



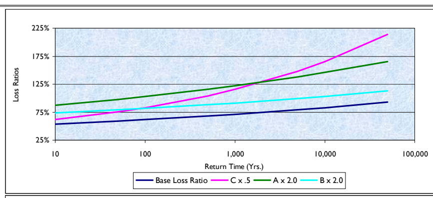
Examples of Pareto – Distributed Tails

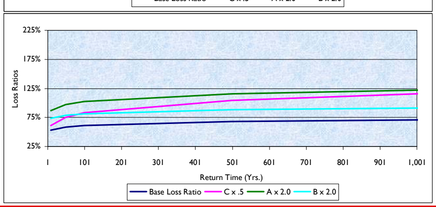
- Salaries within a profession
- Heights of NBA players
- City population rankings
- Dutch floods
- Individual health care costs
- Individual Risk losses in SIGMA database
- Popularity of Google searches
- Lengths of e-mail messages
- Size of traffic jams
- Log of asset portfolios

Some Nice Properties of Paretos

- $F(x) = 1 - \left(\frac{a}{x+b}\right)^c$, and the three parameters drive the curve in different ways:
  - *c is known as the "**shape**" parameter and is most influential in the shape of the tail*
  - *a is known as the "**scale**" parameter and is most influential in events near the mean*
  - *b is known as the "**shift**" parameter and can move the curve to lower or higher ranges of outcomes.*
- If d and e are constants, dx + e is another Pareto.
- x^e is, as well.

Parameters Drive Curves in Different Ways





Underlying Distributions of Risk Contributions

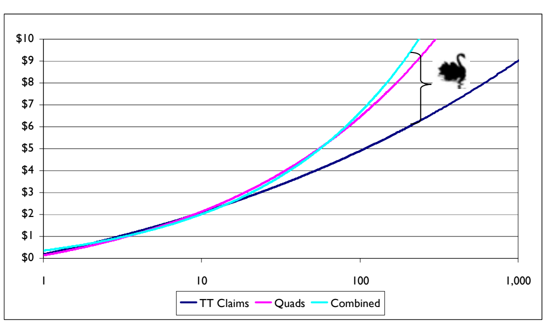
Types of Risk	Examples	Functional Forms	Shape Factors
Already extreme	EP's, or -Log (ROA)	Pareto	Observed shape
Unknown, unbounded	Depth of cycle, Reserve position	Gamma	Fit to tail values
Bounded	Individual policy events	Beta, Normal, Poisson, Neg Binomial	Cannot add to the tail

Complications: Unforeseeable Events and Business Cycles

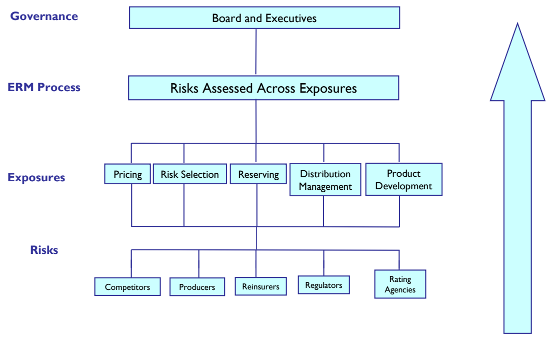
A Complication:
Using Any Distribution to Evaluate the Future Brings the Black Swan

- Data fits and model designs can never consider the "next big thing" that we will learn.
- So, every statistical model understates risks. This is also a problem with the more familiar DFA/DUA/VAR/CPCU tools.
- Whatever we do, it will only be a measure of the floor of their risk level. But it can be a relative measure.

WC Example: What Severity Do You Model Before Your First Quadriplegia?



A Complication: Business Cycles
Silos That Matter in P&C



"Driving Through the Back Window"

- f ″ = - f (Turn sharper when you hear gravel)
- Both the industry and every company have reaction lags
- Under-reserving forces bad pricing, risk selection, coverage grants, distribution management and hiring/firing
- **The cycle has killed off far more P&C companies than Cats, credit, operational failures and ALM combined**

Moving Forward

All EC models:

- Understate the risk of ruin, but can give a floor measure
- Are objective
- Can provide a relative measure inside a company

But:

- Need to reflect a wider view of risk: Cycles or Bubbles. Although not currently knowable, their distributions are also right-tailed.

Example: Product Line Drivers of TCE Underwriting Loss

Product Line Drivers of TCE Underwriting Loss

P&C example

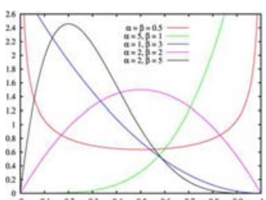
- Two lines of business
- Partly correlated
- Common business cycle
- Two classes of expenses

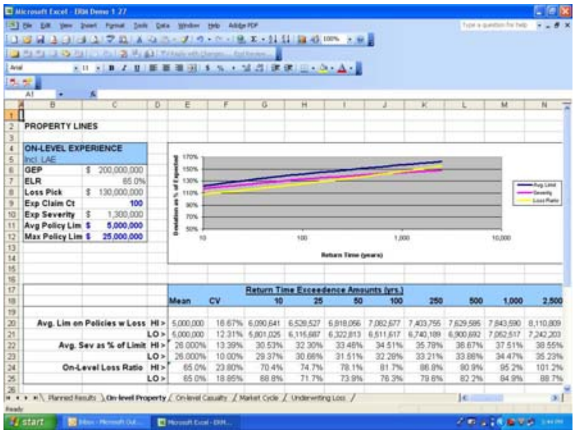
For simplicity here, we omit:

- Catastrophes
- Reinsurance
- ALM
- Taxes
- Growth
- Reserve uncertainty
- Dividends

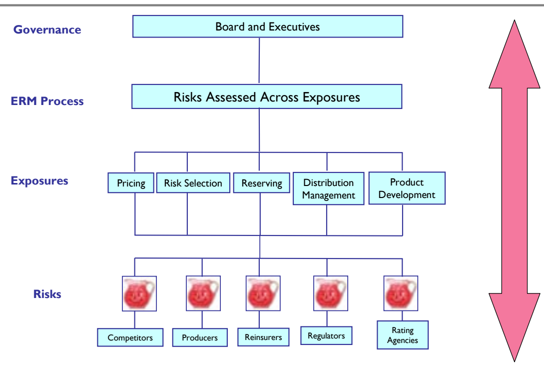
Modeling On-Level Frequency and Severity

- Set aside for now the related questions of cycles and unforeseeable scenarios and look at what we expect the random volatility to be around claim frequency and severity. We will assume here that both are random samples from independent distributions.
- **Tool:** A bounded random variable that has a continuous density with limits of zero at the bounds, one positive mode and a closed-form, can be expressed as a **Beta Distribution.**
- In modeling on-level losses, the Beta is the natural description for two key questions: average claim size relative to average policy limit; and average policy limit relative to maximum policy limit. Given mean estimates for both questions, we can find bounds on the possible variances.
- Some complications not addressed here are lines exposed to multiple perils, cross correlations, loss adjustment and bad-faith losses above limits.



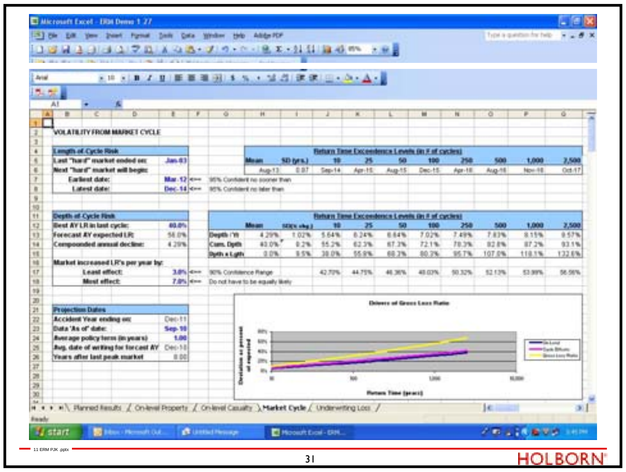


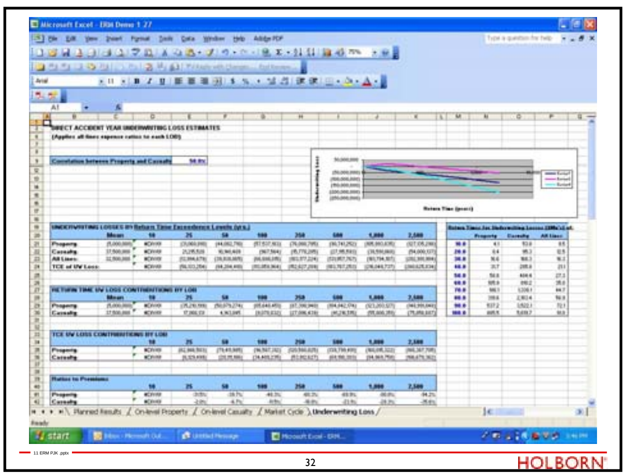
Silos That ***Really*** Matter in P&C



Business Cycles and Black Swans

- Any measure of the probability of bad outcome has to grapple with how much we can't know. We do our best to gauge the on-level exposures to loss, but know that we are mislead by systemic market forces, by the randomness of prior loss experience, and simply the next "big one" that has never occurred before.
- Our model assumes that there are occasional "Road to Damascus" moments when we understand, for once, what we have been missing. We know when those have occurred, but we can't know when the next one will be. If we believe that f " = -f, the response to shock is a sine wave, but with unknown amplitude and period. We reduce the question of "how much we can't know" to describing the distributions of the amplitude and period.
- Similar to using the Beta as the description of bounded closed forms, the **Gamma** is the closed-form, right-tailed description for these questions.
- Some complications not addressed here are different periods for different lines and the possibilities of multiple unknowns causing multiple shocks and cycles.


VOLATILITY FROM MARKET CYCLE
31
HOLBORN


DIRECT ACCIDENT YEAR UNDERWRITING LOSS ESTIMATES
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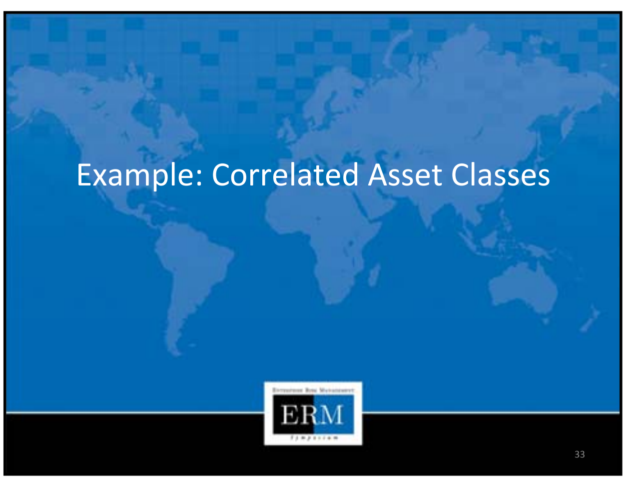
HOLBORN


Example: Correlated Asset Classes
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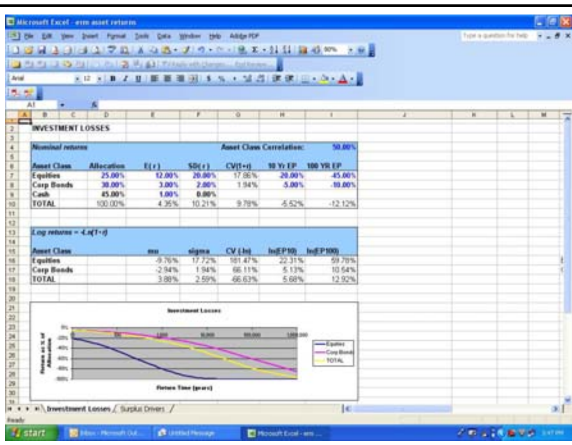
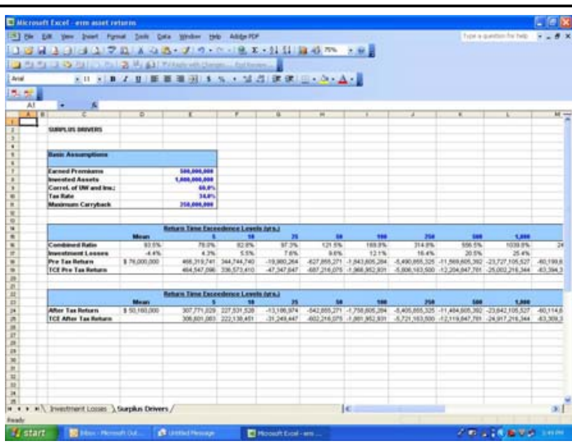
Correlated Asset Classes

- For illustration, 1-year horizon, pre-tax total return perspective, with two risky asset classes.
- We can map the return on an asset class to the relative decline in its value, specifically = - ln (1+r(a)).
- This is unbounded and right-tailed, so in the limit case it is a Pareto.
- The underwriting variables were defined to have means and variances and the combination used their means, variances and shapes to solve for the combined distribution.
- The mapped asset return do not have to have movements. (Just ask Lehman or LTCM). We cannot rely on the method-of-moments to combine curves.
- The illustration matches at chosen points and fits a pareto though them.
- A work in progress, but useful to see the relative scale of underwriting and investment risks.





Final Thoughts

- The Extreme Value approach is only useful in looking at tail-risk events. No insights about central tendencies.
- Algebraic solutions work best when combining risks with similar shapes. Very different shapes (EQ and Auto Collision) may still need numerical or simulation solutions, but we can start with known curves, not processes. Much faster.
- Business cycles and other popular delusions are bigger drivers of uncertainty than foreseeable scenarios. All solvency measures understate the probabilities of trouble.
- Answers are only as good as the questions asked. Asking good questions is part art and part science.

For Comments or Questions

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Technical Support

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The "Shape Operator"

If x is a positive, unbounded random variable, then the Lim x → ∞ of F(x) can be described as a pareto, with shape parameter c. This defines an **operator from the random variable x to the specific value c(x)** in x's limit-case pareto distribution.

Theorems about the Shape Operator:

1. If n is a positive constant, c(n·x) = c(x),
2. and $c(x^n) = c(x) / n$
3. If c(x) < c(y) then c(x+y) = c(x) = c(x-y),
4. and c(max(x,y)) = c(x)
5. If B is a bounded random variable, c(x+B) = c(x),
6. and c(x · B) = c(x)
7. If x and y are independent, and c(x) < c(y), then c(x·y) = c(x),
8. and if they are inversely correlated, then c(x·y) = c(x),
9. but if they are positively correlated, then c(x·y) = c(x) / 2
10. If x is already a pareto, c(x) = c

Economic Capital Charge for Cycles

If f" = - f, the cycle response is a sine function.

Risk level at a time is:

R(t) = a(1+cos(t/b+c))

a = amplitude (observed, guessed)
b = period (guessed), key risk factor in decision making
c = time since last trough (observed)

Charge for business cycle risk = dR/db = a(1-sin(t/b+c)) · t/b^2

EC charge for business cycles reflects your estimate of the amplitude (a), but even more how long you think the period is (b).

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