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Insurance Companies

Optimal Layers for Catastrophe Reinsurance

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Agenda

- Introduction
- Optimal reinsurance: academics
- Optimal reinsurance: RAROC
- Optimal reinsurance: our method
- A case study
- Conclusions
- Q&A

1. Introduction

Reinsurance decision is a balance between cost and benefit

- Cost : reinsurance premium – loss recovered
- Benefit : risk reduction
    - Stable income stream over time
    - Protection again extreme events
    - Reduce likelihood of being downgraded

1. Introduction

How to measure risk reduction

- Variance and standard deviation
  - Not downside risk measures
  - Desirable swings are also treated as risk
- VaR (Value-at-Risk), TVaR, XTVaR
  - VaR: predetermined percentile point. PML (probable maximum loss per event) is a VaR measure at event level
  - TVaR: expected value when loss>VAR
  - XTVaR: TVaR-mean

1. Introduction

How to measure risk reduction

- Lower partial moment and downside variance

$$LPM(L \mid T, k) = \int_T^{\infty} (L - T)^k dF(L)$$

  - T is the maximum acceptable losses, benchmark for "downside"
  - k is the risk perception parameter to large losses, the higher the k, the stronger risk aversion to large losses
  - When k=1 and T is the 99th percentile of loss, LPM is equal to 0.01*VaR
  - When K=2 and T is the mean, LPM is semi-variance
  - When K=2 and T is the target, LPM is downside variance

2. Optimal reinsurance: academics

- Borch, K., 1982, “Additive Insurance Premium: A Note”, *Journal of Finance* 37(5), 1295-1298
- Froot, K. A., *2007,* “Risk Management, Capital Budgeting, and Capital Structure Policy for Insurers and Reinsurers,” *Journal of Risk and Insurance 74, pp. 273—299.*
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- Gajek, L., and D. Zagrodny, 2000, “Optimal Reinsurance Under General Risk Measures”, *Insurance: Mathematics and Economics*, 34, 227-240.
- Lane, M. N., 2000, “Pricing Risk Transfer Functions”, *ASTIN Bulletin* 30(2), 259-293.
- Kaluszka M., 2001, “Optimal Reinsurance Under Mean-Variance Premium Principles”, *Insurance: Mathematics and Economics*, 28, 61-67
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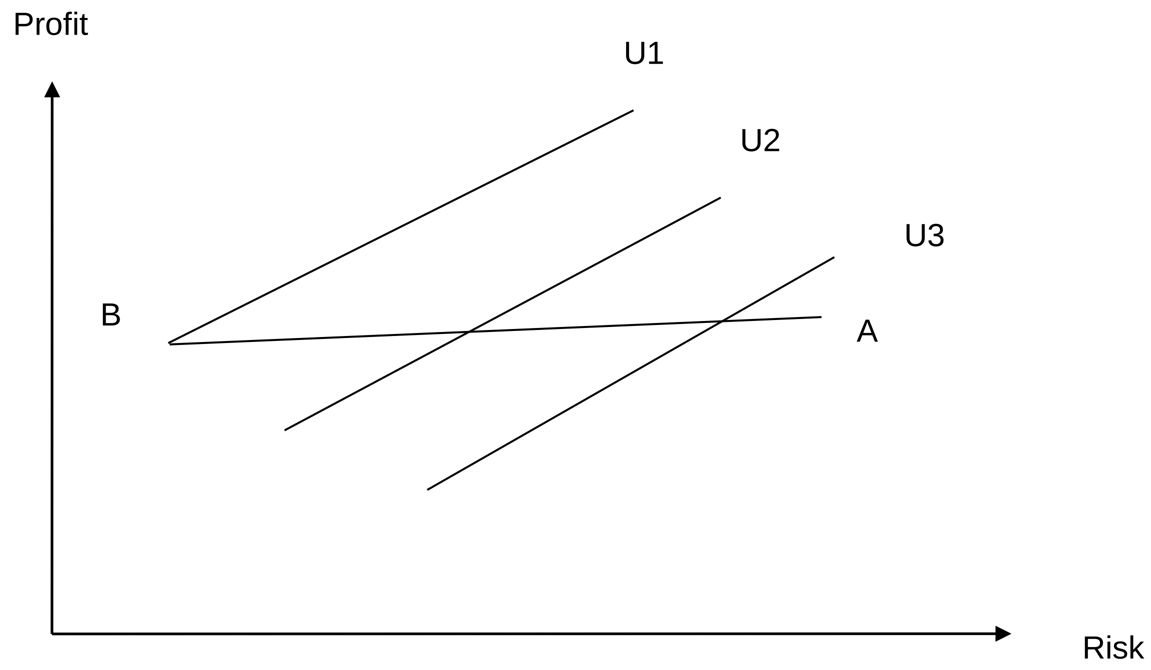
2. Optimal reinsurance: academics

- Cat reinsurance has zero correlation with market index, and therefore zero beta in CAPM.
- Because of zero beta, reinsurance premium should be a dollar-to-dollar trade of loss recovered.
- Reinsurance reduces risk at zero cost. Therefore optimizing profit-risk tradeoff implies minimizing risk
    - buy largest possible protection without budget constraints
    - buy highest possible retention with budget constraints

2. Optimal reinsurance: academics

Academic Assumption



2. Optimal reinsurance: academics

Those studies do not help practitioners

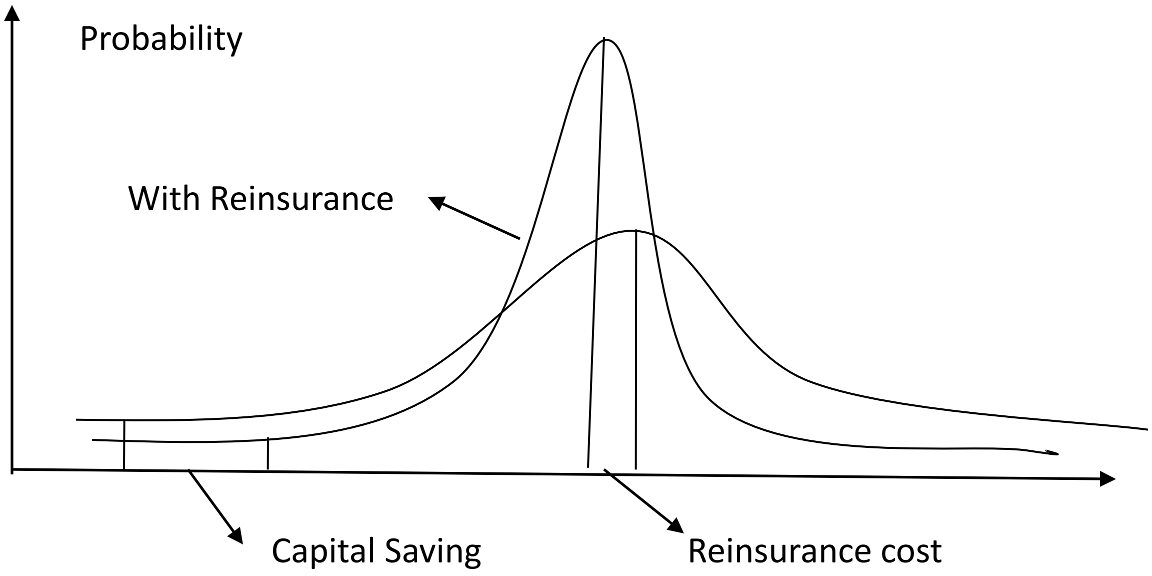
- Reinsurance is costly.
  - Reinsurers need to hold a large amount of capital and require a market return on such a capital.
  - Reinsurance premium/Loss recovered can be over 10 in reality
- No reinsurers can fully diversify away cat risk
- Only consider the risk side of equation and ignore cost side.

3. Optimal reinsurance: RAROC

RAROC (Risk-adjusted return on capital) approach is popular in practice

- Economic capital (EC) covers extreme loss scenarios
- Reinsurance cost = reinsurance premium – expected recovery
- Capital Saving = EC w/o reinsurance – EC w reinsurance
- Cost of Risk Capital (CORC) = Reinsurance cost / Capital Saving
- CORC balances profit (numerator) and risk (denominator)

3. Optimal reinsurance: RAROC



- No universal definition of economic capital
- Use VaR or TVaR to measure risk
  - Only consider extreme scenarios.
  - Linear risk perception.

4. Optimal Reinsurance: DRAP Approach

Downside Risk-adjusted Profit (DRAP)

$$DRAP = Mean(r) - \theta * LPM(r \mid T, k)$$

$$LPM(r \mid T, k) = \int_{-\infty}^{T} (T - r)^k \, dF(r)$$

- r is underwriting profit rate
- θ is the risk aversion coefficient
- T is the bench mark for downside
- K measures the increasing risk perception toward large losses

4. Optimal Reinsurance: DRAP Approach

Loss Recovery

$$G(x_i, R, L) = \begin{cases} 0 & if \quad x_i <= R \\ (x_i - R) * \phi & if \quad R < x_i <= R + L \\ L * \phi & if \quad x_i > R + L \end{cases}$$

- R is retention
- L is the limit
- Φ is the coverage percentage
- x_i is cat loss from the ith event

4. Optimal Reinsurance: DRAP Approach

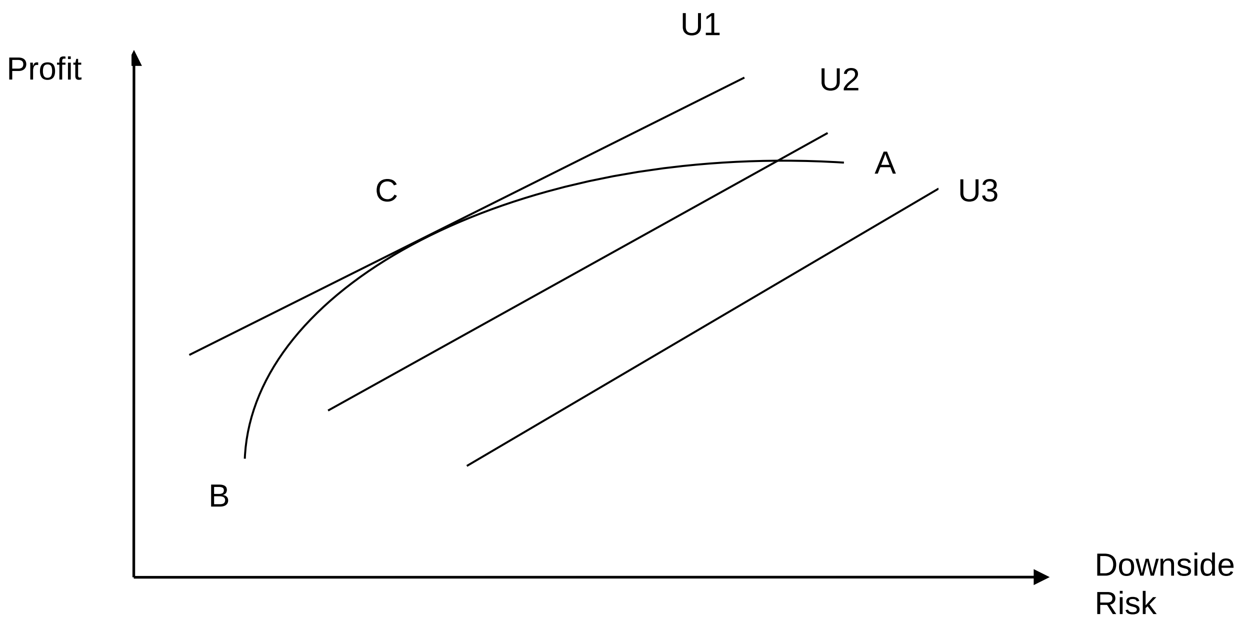
Underwriting profit

$$r = 1 - \frac{EXP + Y + RP(R, L)}{EP} - \frac{\sum_{i=1}^{N} x_i - G(x_i, R, L) + RI(x_i, R, L)}{EP}$$

- EP: gross earned premium
- EXP: expense
- Y non cat losses
- RP(R, L): reinsurance premium
- RI (xi, R, L): reinstatement premium
- N: number of cat events

4. Optimal Reinsurance: DRAP Approach

$$\underset{R,L}{Max} \quad Mean(r) - \theta * LPM(r \mid T, k)$$



AB is efficient frontier

U1, U2, U3 are utility curves

C is the optimal reinsurance that maximizes DRAP

4. Optimal Reinsurance: DRAP Approach

Advantages to conventional mean-variance studies in academics

- An ERM approach.
  - Considers both catastrophe and non-catastrophe losses simultaneously
  - Overall profitability impacts layer selection. High profitability enhances an insurer's ability to retain more cat risk.
- Use a downside risk measure (LPM) other than two-side risk measure (variance)

4. Optimal Reinsurance: DRAP Approach

Theta estimations

$$DRAP = Mean(r) - \theta * LPM(r \mid T, k)$$

- Theta may not be constant by the size of loss
- Theta is time variant
- Theta varies by individual institution
- How much management is willing to pay to mitigate risk?
- How much do investors require to take the risk?
  - index risk premium = index return – risk free rate
  - Insurance risk premium= insurance return-risk free rate
  - cat risk premium= cat bond yield- expected loss-risk free rate

4. Optimal Reinsurance: DRAP Approach

K and T estimations

$$LPM(r \mid T, k) = \int_{-\infty}^{T} (T - r)^k \, dF(r)$$

- k may not be constant by the size of loss
  - For smaller loss, loss perception is close to 1, k=1; for severe loss, k>1
  - Academic tradition: k=2
- T is the bench mark for “downside”
  - Zero: underwriting loss is risk
  - Zero ROE: underwriting loss larger than investment income is risk
  - Large negative: severe loss is treated as risk

5. Case Study

A hypothetical company

- Gross earned premium from all lines:10 billion
- Expense ratio: 33%
- Lognormal non-cat loss from actual data

mean=5.91 billion; std=402 million

- Lognormal cat loss estimated from AIR data
    - mean # of event=39.7; std=4.45
    - mean loss from an event=10.02 million; std=50.77 million
    - total annual cat loss mean=398 million; std=323 million

5. Case Study

- K=2
- T=0%
- Theta is tested at 16.71, 22.28, and 27.85, which represents that primary insurer would like to pay 30%, 40%, and 50% of gross profit to hedge downside risk, respectively.
- UW profit without Insurance is 3.92%
- Variance 0.263%
- Downside variance is 0.07% (T=0%)
- Probability of underwriting loss is 18.41%
- Probability of severe loss (<-15%) is 0.48%

5. Case Study

Reinsurance quotes (million)

Retention	Upper Bound of Layer	Reinsurance Limit	Reinsurance Price	Rate-on-line
305	420	115	20.8	18.09%
420	610	190	21.7	11.42%
610	915	305	19.8	6.50%
610	1,030	420	25.2	5.99%
1,030	1,800	770	28.7	3.72%
1,800	3,050	1,250	39.1	3.13%

5. Case Study

Recoveries and penetrations by layers

Retention (million)	Upper Limit (million)	Mean	Standard Deviation	Recovery/reinsurance Premium	Penetration Probability
305	420	8,859,074	29,491,239	42.59%	10.18%
420	610	8,045,968	35,917,439	37.08%	6.04%
610	915	6,496,494	41,009,356	32.81%	3.15%
610	1,030	7,923,052	51,899,244	31.44%	3.15%
1,030	1,800	4,858,545	55,432,115	16.93%	1.11%
1,800	3,050	2,573,573	48,827,021	6.58%	0.40%

5. Case Study

Reinsurance Price Curves Fitting

- (x1, x2) represents reinsurance layer
- f(x) represent rate-on-line

$$p(x_1, x_2) = \int_{x1}^{x2} f(x)dx$$

- Add quadratic term. Logrithm, and inverse term to reflect nonlinear relations

$$f(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 \log(x) + \beta_4 x^{-1}$$

$$p(x_1, x_2) = \beta_0(x_2 - x_1) + \frac{1}{2}\beta_1(x_2^2 - x_1^2) + \frac{1}{3}\beta_2(x_2^3 - x_1^3)$$
$$+ \beta_3(x_2 \log(x_2) - x_1 \log(x_1)) + \beta_4(\log(x_2) - \log(x_1))$$

5. Case Study

Reinsurance Price Fitting

Retention	Upper Bound of Layer	Reinsurance Limit	Reinsurance Price	Rate-on-line	Fitted rate	Fitted Rate-on-line
305	420	115	20.8	18.09%	20.84	18.12%
420	610	190	21.7	11.42%	21.69	11.41%
610	915	305	19.8	6.50%	19.87	6.51%
610	1,030	420	25.2	5.99%	25.18	6.00%
1,030	1,800	770	28.7	3.72%	28.73	3.73%
1,800	3,050	1,250	39.1	3.13%	39.10	3.13%
305	610	305	42.5	13.93%	42.52	13.94%
305	915	610	62.3	10.22%	62.39	10.23%
305	1,030	725	67.7	9.33%	67.70	9.34%
305	1,800	1,495	96.5	6.45%	96.43	6.45%
305	3,050	2,745	135.6	4.94%	135.53	4.94%
420	915	495	41.5	8.39%	41.55	8.39%
420	1,030	610	46.9	7.68%	46.87	7.68%
420	1,800	1,380	75.6	5.47%	75.60	5.48%
420	3,050	2,630	114.7	4.36%	114.69	4.36%
610	1,800	1,190	53.9	4.53%	53.91	4.53%
610	3,050	2,440	93	3.81%	93.01	3.81%
915	1,030	115	5.3	4.64%	5.32	4.62%
915	1,800	885	34	3.85%	34.04	3.85%
915	3,050	2,135	73.1	3.42%	73.14	3.43%
1,030	3,050	2,020	67.8	3.36%	67.83	3.36%

5. Case Study

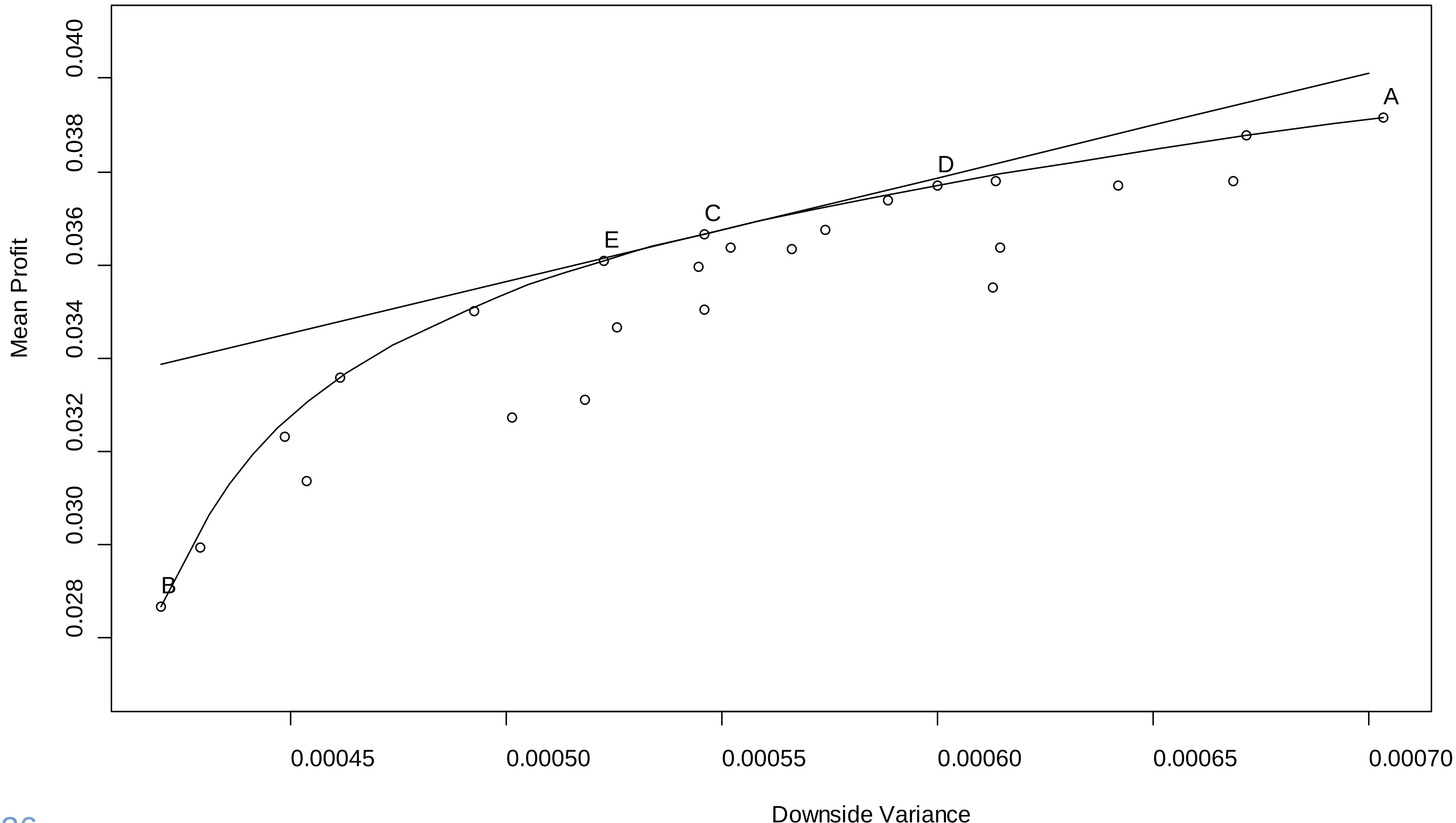
Performance of Reinsurance Layers theta=22.28

Retention (million)	Upper Limit (million)	Prob r<0	Prob r<-15%	Mean	Variance	Downside Variance	Risk-adjusted Profit
No Reinsurance		18.41%	0.48%	3.916%	0.263%	0.070%	2.350%
305	420	19.02%	0.42%	3.781%	0.253%	0.067%	2.291%
420	610	19.17%	0.35%	3.771%	0.249%	0.064%	2.341%
610	915	19.31%	0.30%	3.779%	0.247%	0.061%	2.412%
610	1030	19.53%	0.27%	3.739%	0.243%	0.059%	2.428%
1030	1800	19.95%	0.26%	3.676%	0.243%	0.057%	2.397%
1800	3050	20.44%	0.41%	3.551%	0.247%	0.061%	2.186%
305	610	19.63%	0.33%	3.637%	0.241%	0.061%	2.268%
305	915	20.50%	0.25%	3.503%	0.228%	0.055%	2.287%
305	1,030	20.76%	0.22%	3.465%	0.224%	0.053%	2.293%
305	1,800	22.31%	0.13%	3.231%	0.210%	0.045%	2.231%
305	3,050	24.77%	0.04%	2.869%	0.200%	0.042%	1.934%
420	915	19.85%	0.25%	3.634%	0.235%	0.057%	2.373%
420	1,030	20.06%	0.22%	3.595%	0.232%	0.054%	2.382%
420	1,800	21.79%	0.14%	3.358%	0.216%	0.046%	2.330%
420	3,050	24.25%	0.05%	2.995%	0.206%	0.043%	2.038%
610	1,800	21.05%	0.16%	3.500%	0.226%	0.049%	2.402%
610	3,050	23.35%	0.11%	3.135%	0.215%	0.045%	2.124%
915	1,030	18.63%	0.40%	3.877%	0.258%	0.067%	2.380%
915	1,800	20.14%	0.21%	3.637%	0.239%	0.055%	2.407%
915	3,050	22.44%	0.17%	3.272%	0.226%	0.050%	2.155%
1030	3050	22.15%	0.20%	3.311%	0.230%	0.052%	2.156%
680	1390	20.00%	0.21%	3.667%	0.237%	0.055%	2.451%

5. Case Study

Efficient Frontier

Figure 3: Reinsurance Efficient Frontier



5. Case Study

- Optimal Reinsurance Layers theta =16.71, 22.28, 27.85

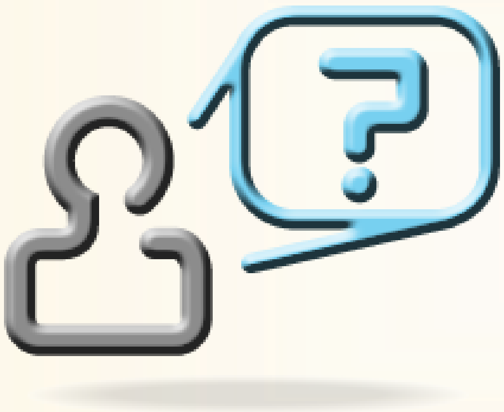
Theta	Retention (million)	Upper Limit (million)	Mean	Downside Variance	Risk-Adjusted Profit theta=16.71	Risk-Adjusted Profit theta=22.28	Risk-Adjusted Profit theta=27.85
16.71	795	1220	3.771%	0.060%	2.768%	2.434%	2.100%
22.28	680	1390	3.667%	0.055%	2.755%	2.451%	2.147%
27.85	615	1460	3.610%	0.052%	2.736%	2.445%	2.154%

- If the overall profit rate increases 2% and theta remains at 22.28, the optimal layers becomes (740, 1420)

6. Conclusions

- The overall profitability (both cat and noncat losses) impacts optimal insurance decision
- Risk appetites are difficult to measure by a single parameter.
- DRAP capture risk appetites comprehensively though theta (risk aversion coefficient), T (downside bench mark), and moment k (increasingly perception toward large loss)
- DRAP provides an alternative approach to calculate optimal layers.

- For technical details, please refer to Fu, Luyang, and C.K. Stan Khury, "Optimal Layers for Catastrophe Reinsurance," *Variance* 4:2, 2010, pp. 191-208, http://www.variancejournal.org/issues/04-02/191.pdf



Q & A

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