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Elements of After-Tax Risk Management

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Abstract
Most risk management is done on a pre-tax basis with tax issues often treated as an after-thought. This paper will outline what an after-tax risk management process should look like and how it can differ from a pre-tax model. One of the paper’s key conclusions is that a tax authority is often an implicit participant in many business transactions and this can have material implications for risk management. The paper starts by developing a simple three-step model of an income tax structure and then uses that model to understand a tax authority as special class of equity investor. The paper then goes on to consider the impact of the tax structure on economic capital, the fair value of liabilities and after-tax A/L M. In many cases the impact of adding tax into the A/L M process is to lengthen liability durations while reducing convexities. Another impact is to make insurance liabilities sensitive to interest rate volatility in a way that tends to offset the interest rate volatility of interest rate options and guarantees.

Introduction and Three Risk Management Questions

What is real money? In the life insurance industry this question can have many different answers. For some people the answer could be pre-tax IFRS or US GAAP earnings. This would be an understandable answer coming from an executive whose incentive compensation was tied to one of those metrics.

Even if pre-tax earnings are the basis for day to day decision making there can be situations where tax issues can play a more significant role. Some examples are

\begin{enumerate}
\item Merger and Acquisition valuations are almost always done on an after-tax basis using an approach consulting actuaries call embedded value analysis\(^2\). This can result in a value being assigned to a block of life insurance liabilities that is materially different from any accounting or regulatory value.

\item New Product Pricing is almost always done using a variation on the after-tax embedded value model mentioned above. Pricing actuaries discovered many decades ago that a conservative approach to setting tax reserves creates the financial equivalent of an interest free loan from the tax authority to the insurance company. The value of that loan can be used to put a more competitive price on the insurance product.
\end{enumerate}

\(^1\) The authors are both Directors at GGY a Moody’s Analytics Co. The views and opinions expressed here are those of the authors and not Moody’s Analytics.
\(^2\) For more background on embedded value see the American Academy of Actuaries March, 2011 Public Policy Practice Note on Market Consistent Embedded Values.
Another area where pre-tax risk management is often used is the Asset/Liability Management process. In the author’s experience this is often conducted on regulatory accounting model basis which, again, usually ignores tax issues. If we take the interest free loan issue mentioned above into account we often find that liability durations go up, liability convexities go down and some asset values can change.

The upshot is that a risk manager who focuses solely on pre-tax GAAP earnings is bound to get a surprise from time to time. So again, what is real money?

The position taken by the author in this paper is that embedded value metrics are the only truly consistent way to think about the issues described above i.e. embedded value is “real money”. This point of view is already consistent with product pricing and the buying and selling of blocks of business but should to be extended to the A/L M process and all other day to day risk management activities. This is not possible in the current IFRS or US GAAP accounting environments.

Given any accounting or regulatory valuation model an insurance enterprise is usually valued, by external investors, as the sum of

1. Assets backing free surplus FS valued at market.
2. The value of inforce business defined as a risk adjusted present value of future distributable earnings (PVDE).
3. The potential value of future new business or franchise value. This is often estimated as the PVDE of 5-10 years of recent new business.

A stylized formula for the PVDE metric can be written as

$$PVDE(0) = \sum_{t=1} [BP_t + RC_{t-1}(1 + r(1 - \tau)) - RC_t] \frac{1}{(1 + r + \pi)^t}.$$ 

The notation is as follows:

$BP_t$ represents the after-tax profits that emerge under the reporting model for the time period $(t - 1, t)$. These are the profits that emerge under the reporting model if assets are equal to the reserves required by that model.

$RC_t$ is the required capital defined by the reporting model at time $t$ i.e. assets required over and above reserves.

$r$ is an assumed interest rate earned on assets backing the required capital. In a market consistent model this would be the relevant risk free rate plus any appropriate liquidity adjustment.

$\tau = \tau(t)$ is the assumed tax rate scenario. This parameter will also play a role in the calculation of $BP_t$.

$\pi$ is a spread defining the after-tax target return to the investor who puts up the required capital. If the assumptions underlying the profit projection pan out the investor will earn a return of $r + \pi$. 


π on their investment. For vanilla insurance liabilities the risk premium is often taken to be \( \pi = 0.06 \) but other values are possible.

The formula assumes distributable earnings are the sum of emerging after-tax profits plus the impact (up or down) of changing capital requirements. That this is a reasonable way to define shareholder value is fundamental to what follows in this paper.

There is purely algebraic reshuffle of the PVDE definition which allows us to write the PVDE as the sum of current required capital plus the mismatch between the emergence of book profits and the cost of capital. The formula is

\[
PVDE(0) = RC_0 + \sum_{t=1}^{\infty} \frac{[BP_t - RC_{t-1}(\pi + r\tau)]}{(1 + r + \pi)^t}.
\]

This is an important formula because it tells us how an accounting model has to be engineered if we want it to tell us what the risk enterprise is actually worth i.e. \( PVDE(0) = RC_0 \). This requires

- Assets and liabilities marked to market
- Required Capital defined in a reasonable way
- The release of risk margins (profits) engineered so that \( BP_t = RC_{t-1}(\pi + r\tau) \). This is usually referred to as the cost of capital approach to risk margins i.e. reserves are equal to best estimate values adjusted by risk margins as defined here.

There are at least three different ways in which the evolving IFRS/US GAAP accounting models fail to meet the standard outlined above

1. Failure to take the appropriate tax issues into account when calculating insurance policy reserves.
2. Failure to allow gains at issue to be recognized at the point of sale.
3. Neither of the evolving standards require the use of the cost of capital method for risk margins at this time although it appears that IFRS guidance is so vague on the risk margin issue that the cost of capital method would be acceptable.

In short, accounting models need not be risk management models. This is unfortunate but nevertheless real. Accounting models that claim to be market consistent don’t necessarily tell risk managers what they need to know.

Having laid down a position on the current accounting environment we ask three questions that should be of interest to risk managers.

1. **For a given insurance liability how much asset do we need to have on the balance sheet in order to mature the obligation?** We will call the answer to this question the Fulfillment Value of the Liability or \( FVL \). A high level formula for the \( FVL \) is that it is the risk neutral present value of

   a. Best Estimate Value Liability cash flows or \( BEL \).
   b. “Appropriate” tax cash flows. We will go into more detail later as to exactly what this means. In broad terms appropriate tax cash flows are those marginal taxes
that any participant in the industry would have to deal with if they sold that product.

c. Risk margins to compensate investors for putting up the necessary risk capital.
Again more detail in the next sections.

One of the technical results derived in the next section is that $FVL$ can be calculated as
the risk neutral present value of after-tax cash flows using after-tax interest rates.

2. **If someone offers to take the liability off our hands for an asset transfer of $X$, should we take the offer?**
Given an answer to question (1) above we will define the Transfer Price of the Liability $TPL$ as the value where we are indifferent between manufacturing
the liability ourselves or paying someone else to do it for us. If the tax base of the
liability is a known quantity $V^{Tax}$ then the total assets required to actually transfer the
liability to a third party are not $TPL$ but the sum of $TPL$ and the marginal tax consequences of executing the transfer i.e.

$$TPL + \tau(V^{Tax} - TPL).$$

Here $\tau$ is the assumed marginal tax rate. We are indifferent then if

$$TPL + \tau(V^{Tax} - TPL) = FVL,$$  or  $$TPL = \frac{FVL - \tau V^{Tax}}{1 - \tau}.$$

This is the definition of transfer price that will be used in this paper. The key point is that
$FVL, TPL$ are the answers to two different questions and, due to tax issues, can be two
different numbers. We will call the difference $\tau(V^{Tax} - TPL)$ the Deferred Tax on Liabilities ($DTol$).

3. **Given the different values available which one should we use?** The answer depends on
the application but it is usually the $TPL$. Here are some examples:

a. Economic Financial Statements: If the actuary gives an accountant preparing an
economic balance sheet the value $TPL$ as the statement reserve and the
accountant then computes a traditional undiscounted deferred tax liability
$\tau(V^{Tax} - TPL)$ then the resulting total balance sheet liability will be the $FVL$.
This is appropriate and is what Canadian actuaries have been doing since 2002 in
the Canadian GAAP financial reporting model$^3$.

b. Risk Management: How much economic capital do we need to hold for a
mortality experience shock $\Delta Q$?

If the death benefit for a life insurance policy is $DB$ then the gross loss resulting
from an experience shock is $\Delta Q(DB - FVL)$ since $FVL$ is the total asset we have

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$^3$Much of this paper can be thought of as the authors’ adaptation of existing Canadian GAAP ideas about tax to a
market consistent economic model. Additional details on current Canadian actuarial practice can be found in the
Canadian Institute of Actuaries’ Consolidated Standards of Practice. Some of this guidance will need to change to
reflect the new rules of IFRS.
on the balance sheet for the contract. However, if the loss can be tax effected, the net after-tax loss will be

$$\Delta Q(DB - FVL) - \tau \Delta Q(DB - V^{Tax}) = (1 - \tau)\Delta Q(DB - TPL).$$

This tells us that the after-tax economic capital required to cover a short term mortality fluctuation is $(1 - \tau)\Delta Q(DB - TPL)$. A risk manager setting standards for risk retention should then use $DB - TPL$ as their key metric for net amount at risk exposure.

c. Risk Management: How much economic capital do we need to hold for a plausible shock to an actuarial assumption e.g. $q \to q + \Delta q$? In most tax jurisdictions changing an economic assumption does not change the liability tax base or have any other immediate tax effect. Changing an assumption can change the $FVL$ so the change $\Delta FVL$ is the required economic capital. In terms of $TPL$ this is

$$\Delta FVL = (1 - \tau)\Delta TPL.$$

The only exception to this rule is when we actually change a tax rate assumption $\tau \to \tau + \Delta \tau$ then it can be shown that

$$\Delta FVL = (1 - \tau)\Delta TPL + \Delta \tau(V^{Tax} - TPL).$$

A high level summary of the discussion above is that, for many day to day risk management purposes, it is useful to think of $TPL$ as the “reserve”. One risk management activity where this may not seem to apply is Asset/Liability Management where $FVL$ would appear to be central to the discussion. As we will see later the $TPL$ turns out to be the very useful quantity even for A/L M.

One question that may be bothering the reader is as follows: If $TPL$ is the price an insurer would be willing to pay get rid of a liability, and that value is different from $FVL$, why would another insurer be willing to accept $TPL$ to take on the liability? The graphic below shows why this can make sense if the tax base $V^{Tax}$ does not change when the block of liabilities is transferred from one insurer to another. This is what actually happens in the US since tax reserves are defined by a combination of statute and regulation, not market prices.
In this simple example, we have $TPL = 100, V^{Tax} = 110, \tau = .35$ and so $FVL = 103.5$. The parties have agreed to a direct asset transfer of 100 which generates a taxable gain of 10 for the seller and a tax liability of 3.5. The buyer receives the 100 asset transfer and then sets up the same tax liability of 110 resulting in a tax loss of 10 and a tax benefit of 3.5. The net result is that the entire $FVL$ has made its way from seller to buyer even though the $DToL$ has gone indirectly via the tax man. It is therefore reasonable for the two parties to agree that 100 is an appropriate transfer price, assuming they agree on all other issues.

This is not what happens when two US insurers trade a bond since the tax base of the bond would reset to its transfer price when traded from one legal entity to another.

There is an important case where the tax base of a bond does not reset to market and that is when we are in going concern mode and a company is effectively selling the bond to itself. This has after-tax A/L M consequences in that an observed market price is not necessarily the right value to use when truing up the asset side of an economic balance sheet to a calculated $FVL$.

Following this introduction this paper has five sections.

- A simple three step model of an income tax structure that will be the foundation for what follows. A key conclusion is that it is useful to think of a tax authority as a shareholder with some complicated financial options that we refer to as the “Tax Man’s Put”.
- Calculating $FVL/TPL$. We show how to compute the $FVL$ as the risk neutral present value of after-tax cash flows discounted using after-tax interest rates. We also show how to compute $TPL$ from first principles and draw some useful risk management conclusions.
- After-Tax Financial Engineering. We define the concept of an after-tax forward interest rate and show how it can be useful. Pre-tax and after-tax forward rates are not the same in the presence of interest rate volatility. After-tax forward rates are usually higher. This is a risk management insight that has implications for both pricing and A/L M.
- After-Tax A/L M where we show how to value assets in a way that is consistent with market consistent liabilities. The key issue is that we need to put a value on future asset tax timing differences since observed market values ignore tax timing issues. The author is aware that many people won’t like this result.
- A short conclusion which basically states that after-tax risk management is possible and practical once the right tools have been put in place.

Finally we close this introduction by letting readers know what is out of scope for this paper. There are many tax practitioners who have to interpret vague tax laws and regulations that apply at either the company or policyholder level. There is always a risk that a working interpretation can be challenged by a relevant authority with adverse consequences. That issue is important but outside the scope of this paper.
High Level Model of an Income Tax Structure

Imagine a world with no income tax at all. We have an insurance entity XYZ Corp. that has determined that it needs $10 of economic capital. XYZ Corp’s economic balance sheet looks like this:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MVA = 100</td>
<td>FVL = 90</td>
</tr>
<tr>
<td>EC = 10</td>
<td>Total = 100</td>
</tr>
</tbody>
</table>

| Total = 100  | Total = 100 |

XYZ’s actuaries have engineered the insurance products so that $1 of profit margin is released each year to pay for the cost of capital which we assume is $\pi = .10$ If the interest rate earned on surplus assets is $r$ then the expected return to shareholders on economic capital is

$$\frac{10r + 1}{10} = r + 10\%.$$

Step 1: A Very Simple Tax Structure

To start, assume the tax man takes 35% of all economic income (plus or minus). At this stage in our model we allow negative income taxes so there is complete risk sharing with the tax man. What are the consequences? The first consequence is that we no longer need to hold $10 of economic capital. Due to the risk sharing $6.50 is now sufficient so $3.50 can be paid out immediately to the shareholder. Assuming this has been done, and the insurance product has not been repriced, the expected return to shareholders is now

$$\frac{(6.5r + 1) \times .65}{6.5} = .65 r + 10\%.$$

The shareholder is, almost, neutral. The impact of the assumed tax structure is to reduce the shareholder’s return by 35% of the interest earned on the pre-tax capital. In the MCEV literature this is referred to as frictional cost.

In order to fully compensate the shareholder for this frictional cost the actuaries would have to increase the product’s profit margin by the interest forgone on the capital which the tax man has implicitly contributed i.e. $3.5r$. Assuming $r = 5\%$ the new risk margin is $1.18 = 1 + .05 \times 3.5$. Note that this is not the same as grossing up the pre-tax profit margin to $1/(1-.35) = 1.54$ as might seem intuitive.

Two high level conclusions at this stage of the argument are

- Income taxes are somewhat like shareholder dividends in that they compensate the tax man for implicitly contributing 35% of the economic capital. For the remainder of this article it will be useful to think of the tax man as a special class of investor.
The frictional cost issue is an example of a bias that favors the tax man at the expense of the common shareholder, unless the company passes the cost through to the policyholder.

**Step 2: The Tax Man introduces his own Accounting System** (but we still allow negative income tax).

In most tax jurisdictions companies must put together tax balance sheets and tax income statements that can be very different from their economic or accounting financial statements. However, in most jurisdictions it is still possible to understand the difference between taxable income and economic income as a combination of temporary differences and permanent differences. A little bit of algebra may help here.

Let’s assume we can calculate income tax as follows (we’ll pick up any shortcomings of this assumption in Step 3 of our tax model).

\[
\text{Income Tax} = \text{Tax Rate} \left[ (ACF + \Delta A_{\text{Tax}} - PD^A) - (LCF + \Delta V_{\text{Tax}} + PD^L) \right]
\]

Here \( ACF \) is the Asset Cash Flow received from invested assets and \( \Delta A_{\text{Tax}} \) is the change in tax base of the company’s assets. These two terms add up to the taxable investment income generated by the assets. The term \(-PD^A\) represents a permanent difference to taxable investment income arising from the assets.

The taxable investment income is offset by an analogous term coming from the liability side of the balance sheet which one could think of as the tax deductible interest along with any relevant liability related permanent differences.

The details of how tax values are determined, and what qualifies as a permanent difference, vary greatly by tax jurisdiction and the legal status of the tax payer. Fortunately, we won’t need to know most of these details but some life insurance examples may help to clarify the discussion. The last example in this list will be important later.

- For many jurisdictions a bond asset is valued at amortized cost for tax purposes. In the United States this rule is used unless the bond was bought at a discount. The US tax regime does not recognize any amortization of purchase discount as taxable income until the bond is sold or matures.

- In most jurisdictions the tax base of an asset resets to market value when the asset is sold.

- In the US, an example of a favorable permanent difference is the Dividend Received Deduction or DRD which allows a portion of the dividends received from assets to be deducted from taxable income.

- In Canada, life insurers must pay a federal investment income tax on behalf of their policyholders. This tax is not deductible when computing the company’s corporate income tax in the province of Quebec. This is an example of an unfavorable permanent difference.

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4 Our sign convention for permanent differences is that a positive amount is favorable to the company.
• In the US, equity investments are generally valued at cost for tax purposes. In Canada they are valued at market on the tax balance sheet.

• In most European jurisdictions the tax base of an insurance liability resets to market if sold from one insurer to another. This is not true in the United States where the tax base of an insurance liability is effectively fixed by formulae defined in the tax code and related regulations.

How does this impact the company’s relationship with the tax man? One way to analyze the situation is to break the income tax payments into three pieces that we will call asset taxes, mismatch taxes and liability taxes in this paper. This is done by adding and subtracting the Economic Investment Income ($Econ II$) and Economic Required Interest ($Econ Req’d I$)\(^5\) from the basic tax equation. We then write

\[
\text{Income Tax} = \text{Tax Rate} \times \left\{ (\text{ACF} + \Delta A^{Tax} - PD^A) - Econ II \right\} \quad \text{Asset Tax} \\
+ [Econ II - Econ Req’d I] \quad \text{Mismatch Tax} \\
+ [Econ Req’d I - (LCF + \Delta V^{Tax} + PD^L)] \quad \text{Liability Tax}
\]

The Asset Tax item captures the difference between actual taxable investment income and the economic income. The last line reflects liability issues while the middle line would, in theory, be zero if assets and liabilities were perfectly matched on an economic basis.

We take the view that an economic balance sheet should include the liability taxes in the calculation of $FVL/TPL$ and an adjustment needs to be made to the observed market price of assets to account for the Asset Tax Line.

Mismatch gains and losses should, on average, be zero. If this is the case then no explicit balance sheet liability is required for mismatch. This means mismatch gains and losses will fall to the bottom line as they occur and will get tax affected since they are not reserved for.

Most economic capital models have a capital requirement for mismatch losses which depends on the current state of mismatch. This capital requirement is not zero on average so it makes sense that some provision should be held in the liabilities for this cost of capital. Some people call this the mismatch budget. Detailed discussion of this issue is beyond the scope of this paper.

The asset tax term is interesting because it can vanish if we assume all of the assets are continuously traded so that the economic and tax values are always identical. Putting a value on the asset taxes therefore requires making an assumption about how frequently assets are turned over (i.e. traded). This issue is discussed more fully in the section on after-tax A/L M.

Since 2002 Canadian actuarial practice has been to use an actuarial projection platform to project future asset reinvestments/disinvestments using assumptions specified by the Appointed Actuary as part of the Canadian Asset/Liability Method (CALM) for valuing insurance contracts. This

\[\text{\tiny In this paper Econ Req’d I includes interest on reserves and required capital.}\]
approach implicitly captures the value of asset tax timing differences and permanent differences and is reported as a component of the insurance liabilities. This makes sense but is will not be acceptable under IFRS.

*Step 3: The Tax Man’s Put Option*

No doubt most readers are ready to point out that the first two steps of the tax model outlined here have missed a significant element. In terms of the tax man as shareholder concept he not only defines his own dividend mechanism (Step 2) but he is usually able to limit his downside participation in the company’s fortunes. Again, the details of how this works vary greatly from one tax jurisdiction to another. We will refer to this limit on the ability of the company to pass risk through to the tax man as the “Tax Man’s Put” option.

Some specific examples of the Tax Man’s Put at work are

- Most tax codes do not allow negative taxes per se. Tax losses can often be carried back to prior years or carried forward to future years. There are usually well defined limits on how much of this can be done.

- In Canada, non-capital tax losses can be carried back three years and forward indefinitely. Capital losses can be carried back 3 years and forward indefinitely but can only be applied against capital gains.

- In the United States capital losses on some asset sales can only be used to offset capital gains on similar assets.

This kind of rule puts some constraints on a company’s ability to manage the asset taxes described in Step 2.

Interestingly this is not entirely a one way street. It is the author’s experience that tax specialists in many tax jurisdictions are fully aware of tools and transactions that can manage the potential impact of the “Tax Man’s Put”. This is often a significant activity within a company’s tax department.

The Tax Man’s Put is very much an entity specific issue so it should not be included in policy liabilities. If we think the asset or liability values described so far take too much credit for tax issues then a special “Tax Man’s Put” liability on the economic balance sheet would be appropriate.

**Calculating $FVL/TPL$**

In the introduction we defined $FVL$ as the risk neutral present value of

- Best estimate liability cash flows such as claims and expenses less gross premiums
- Risk loads for the cost of economic capital. The introduction showed that if these loads have the form $(\pi + r\tau)EC$ then the expected after-tax return to the shareholder will be $r + \pi$. In practice risk loads are often calculated using just $\pi EC$ so that the actual after-
tax return to shareholders on risk capital is then \( r(1 - \tau) + \pi \). This section will compute risk loads using the formula \((\pi_0 + \varepsilon r \tau)EC\) where \( \varepsilon = 0 \) or \( \varepsilon = 1 \).

- “Appropriate” taxes which we now take to be the liability taxes identified in the previous section.

We start by working through the ideas for a very simple life insurance policy with no lapses, a deterministic interest rate environment, a constant tax rate and the only risk margin is for contagion risk. This is enough to communicate the main ideas. Subsequent discussion will relax these simplifying assumptions.

**Simple Case**

Notation for this section:

- \( F(t) \) is the FVL,
- \( V(t) \) is the TPL,
- \( \mu(t) \) is a traditional deterministic force of mortality
- \( r, e, g \) are the interest rate, expense rate and gross premium rate respectively
- \( \pi = \pi_0 + \varepsilon r \tau \) is the cost of capital rate

The first step is to write down the basic differential equation which states that \( F \) increases with interest and persistency and then decreases as cash flows are paid out i.e.

\[
\frac{dF}{ds} = (r + \mu)F - [\mu D + e - g + \pi(1 - \tau)\Delta Q(D - V)]
- \tau \left[rF + g - e - \mu D - PD^L - \left(\frac{dV^{\text{Tax}}}{dt} - \mu V^{\text{Tax}}\right)\right].
\]

The first square bracket above consists of death benefits \( \mu D \), expenses \( e \), gross premiums \( g \) and cost of capital risk margins of \( \pi(1 - \tau)\Delta Q(D - V) \). This last item is follows from the fact that the economic capital for a mortality contagion event was shown to be \((1 - \tau)\Delta Q(D - V)\) in the introduction.

The second square bracket above is our model of tax cash flow. We are assuming the interest earned on \( F \) is fully taxable as are gross premiums. We also assume death claims and expenses can be deducted from taxable income along with any increase in the tax base. The term \( PD^L \) refers to any liability permanent differences between cash flow and taxable income as discussed earlier. Our sign convention is that \( PD \) is positive if it reduces taxable income and negative if not.

The equation written above is fine as a statement of first principles but is not immediately useful if you want to actually compute a number. We will now work through a sequence of algebraic reshuffles that result in something you can could actually implement in a computer program.

The first step is to collect some like terms in the formula above and rewrite it as
\[
\frac{dF}{ds} = (r(1 - \tau) + \mu)F - (1 - \tau)[\mu D + e - g] + \tau PD^L + \tau \left( \frac{dV^{\text{Tax}}}{ds} - \mu V^{\text{Tax}} \right) - \pi(1 - \tau) \Delta Q(D - V).
\]

This form of the equation is often interpreted by saying that \( F \) is the present value of after-tax cash flows using after-tax interest rates. This is useful for theoretical understanding but not for practical calculation since the risk margin cash flow depends on \( V \).

One approach is to use the relation \( V = \frac{F - \tau V^{\text{Tax}}}{1 - \tau} \) in the equation above and then rearrange again to get

\[
\frac{dF}{ds} = (r(1 - \tau) + (\mu + \pi \Delta Q))F - (1 - \tau)[(\mu + \pi \Delta Q)D + e - g] + \tau PD^L + \tau \left( \frac{dV^{\text{Tax}}}{ds} - (\mu + \pi \Delta Q)V^{\text{Tax}} \right).
\]

The effect has been to change \( \mu \rightarrow (\mu + \pi \Delta Q) \) while eliminating the risk loading term. This is an equation we can actually use to calculate \( F \) if we want to. If \( F(T) \) is the fulfillment value at some future date (e.g. contract maturity) then a practical calculating formula is

\[
F(t) = e^{-\int_t^T (r(1 - \tau) + (\mu + \pi \Delta Q))dv} F(T) + \int_t^T e^{-\int_t^s (r(1 - \tau) + (\mu + \pi \Delta Q))dv} \left[ (1 - \tau)[(\mu + \pi \Delta Q)D + e - g] - \tau PD^L - \tau \left( \frac{dV^{\text{Tax}}}{ds} - (\mu + \pi \Delta Q)V^{\text{Tax}} \right) \right] ds.
\]

If we actually did this calculation then the transfer price could be calculated using \( V = \frac{F - \tau V^{\text{Tax}}}{1 - \tau} \).

In practice it is common to compute \( V \) first. We can derive an equation for \( V \) by rearranging to get the expression

\[
\frac{dF}{ds} - \tau \frac{dV^{\text{Tax}}}{ds} = (r(1 - \tau)F + \mu(F + \tau V^{\text{Tax}}) + \pi(1 - \tau)\Delta QV - (1 - \tau)[(\mu + \pi \Delta Q)D + e - g] + \tau PD^L.
\]

Now divide both sides of the equation above by \( (1 - \tau) \) and note that the left hand side will become \( \frac{dV}{ds} = \frac{\frac{dF}{ds} - \tau \frac{dV^{\text{Tax}}}{ds}}{1 - \tau} \). The result is

\[
\frac{dV}{ds} = rF + (\mu + \pi \Delta Q)V - [(\mu + \pi \Delta Q)D + e - g] + \tau PD^L/(1 - \tau).
\]

The final algebraic step is to use \( F = V + \tau(V^{\text{Tax}} - V) \) in the result above to get an equation that can be solved for the transfer price \( V \) i.e.
\[ \frac{dV}{ds} = [r(1 - \tau) + (\mu + \pi \Delta Q)]V - [(\mu + \pi \Delta Q)D + e - g] + \frac{\tau PD_L}{1 - \tau} + r\tau V^{T\text{ax}}. \]

If we know the transfer price at a future date \( T \), for example contract maturity, then the solution to the equation above can be written as

\[ V(t) = e^{-\int_t^T (r(1 - \tau) + (\mu + \pi \Delta Q))dv} V(T) + \int_t^T e^{-\int_t^s (r(1 - \tau) + (\mu + \pi \Delta Q))dv} \left[(\mu + \pi \Delta Q)D + e - g - \frac{\tau PD_L}{1 - \tau} - \tau V^{T\text{ax}}\right] ds. \]

We will call this the “calculation” formula because there are no circular elements. This result could be used to write a computer program, once an appropriate numerical integration scheme has been chosen. Given that the transfer price has been calculated we can then calculate the fulfillment value using \( F = V + \tau(V^{T\text{ax}} - V) \).

The calculation formula uses after-tax interest rates \( r(1 - \tau) \) and risk loaded mortality \( \mu + \pi \Delta Q \).

Readers can be forgiven if they are scratching their heads wondering what the calculation formula above really means. One last algebraic reshuffle allows us to write the result in a way that is much easier to interpret. We call this the “presentation” formula.

Rewrite the differential equation defining \( V \) in the following mathematically equivalent but circular form

\[ \frac{dV}{ds} = [r + \mu]V - [(\mu D + e - g) - \pi \Delta Q(D - V)] + \frac{\tau PD_L}{1 - \tau} + r\tau(V^{T\text{ax}} - V). \]

The solution to this equation can be written in a way that is easy to interpret

\[ V(t) = e^{-\int_t^T (r + \mu)dv} V(T) + \int_t^T e^{-\int_t^s (r + \mu)dv} \left[(\mu D + e - g) + \pi \Delta Q(D - V) - \frac{\tau PD_L}{1 - \tau} - r\tau V^{T\text{ax}} - V\right] ds. \]

The calculated transfer price can now be interpreted as the sum of

1. Best estimate liability cash flows (claims + expenses – premiums) using pre-tax interest rates and best estimate persistency. This is what most people would call the best estimate value.
2. The next term is clearly the pre-tax risk margin which is the present value of grossed up planned payments to shareholders for the cost of capital. More on this below.
3. The impact of permanent tax differences, again grossed up to be pre-tax.
4. The present value of interest on the undiscounted deferred tax liability \( \tau (V^{T\text{ax}} - V) \).

Practical implementations of the theory outlined above are usually engineered to produce the kind of decomposition outlined here. Variations are possible.

Some additional comments are warranted.
The risk margin term above should be thought of as
\[ \pi \Delta Q(D - V) = \frac{(1 - \tau)(\pi_0 + \varepsilon \tau)\Delta Q(D - V)}{1 - \tau}. \]

The fact that two \((1 - \tau)\) factors cancel shows that the risk loadings don’t change very much when going from a no tax model to a model with income taxes. The risk loadings changed only because we added the term \(r \tau\) to \(\pi_0\). We saw this happen in the example used in the high level tax model where the impact of taxing the risk margin release was offset by the reduction in required capital.

Many people interpret the term \(r \tau(V^{\text{Tax}} - V)\) as the interest on an interest free loan from or to the tax man depending on the sign of the deferred tax liability. If \(V^{\text{Tax}} > V\) this is a loan from the tax authority to the insurer. Pricing actuaries have been aware of this issue for decades since it often works in a company’s favor.

Clearly if \(V^{\text{Tax}} < V\) then the company is making an interest free loan to the tax man and the cost of that loan is reflected in the transfer price. A common situation where this can happen is for older blocks of SPIA business in the US which have statutory and tax valuation rates that are high by today’s (2017) interest rate standards. Standard statutory formula reserves for such blocks often turn out to be inadequate when subjected to cash flow testing analysis.

**A More Complex Example – Assumption Changes**

In this subsection we discuss the issue of assumption changes for two reasons:

1. Holding capital and risk margins for plausible assumption changes is common in most internal economic capital models and are required by some regulatory models such as Solvency II in Europe and the pending LICAT model in Canada.

2. It forces us to consider time varying tax rate scenarios \(\tau = \tau(t)\).

The simple model discussed in the prior section showed that if holding economic capital \((1 - \tau)\Delta Q(D - V)\) for contagion risk was our only risk issue then we could build in the cost of holding that capital by using a risk loaded mortality assumption of the form \(\mu + \pi \Delta Q\).

Suppose we now hold capital for plausible assumption changes \(\mu \rightarrow \mu + \Delta \mu, \tau \rightarrow \tau + \Delta \tau\). We will briefly describe two approaches to handling this kind of issue. The first approach is motivated by Solvency II while the second approach was developed by one of the authors.

If we take a Solvency II type approach to risk margins we would calculate a best estimate value \(F_0(t)\) using best estimate assumptions and then compute two shocked values \(F_{1,\mu}(t)\) and \(F_{1,\tau}(t)\) using shocked mortality or tax rate as appropriate. Economic capital for a mortality/tax assumption change is then calculated as \(F_{1,\mu} - F_0\) or \(F_{1,\tau} - F_0\). Risk margins are calculated by

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6 Manistre, B.J. “Down but Not Out A Cost of Capital Approach to Fair Value Risk Margins” presented at the 2014 ERM Symposium and is available on the SOA website.
projecting the capital requirements into the future, under best estimate assumptions, and then computing the present value of the cost of capital.

\[
M_\mu(t) = \int_t^\infty e^{-\int_t^s [r+\mu]dv} \pi(s) [F_{1,\mu}(s) - F_0(s)] ds,
\]
\[
M_\tau(t) = \int_t^\infty e^{-\int_t^s [r+\mu]dv} \pi(s) [F_{1,\tau}(s) - F_0(s)] ds.
\]

Here \(\pi(s)\) is the cost of capital rate. This is what most people consider the cost of capital method to mean. The process is conceptually straightforward but can be computationally expensive because of the need to project capital requirements at all future time points.

An alternative, and less computationally expensive, method is to work with risk loaded mortality and tax rates of the form \(\mu(t, s) = \mu(s) + \beta_\mu(t, s) \Delta \mu(s)\), \(\tau(t, s) = \tau(s) + \beta_\tau(t, s) \Delta \tau(s)\). Here the best estimate assumptions \(\mu(s), \tau(s)\) have been augmented by dynamic risk loadings \(\beta_\mu(t, s) \Delta \mu(s), \beta_\tau(t, s) \Delta \tau(s)\). The quantities \(\beta_\mu(t, s), \beta_\tau(t, s)\) are known as margin variables which are zero in the real world (P Measure) but evolve according to certain rules when we enter the valuation world (Q measure). If we choose the risk neutral evolution rules properly we can get results for capital and risk margins that are very similar to the Solvency II type calculation described earlier.

One example of an evolution rule set is to assume \(\beta_\mu(t, t) = 0, \beta_\tau(t, t) = 0\) and for \(s > t\) roll the margin variables forward in time using the dynamics

\[
\frac{d\beta_\mu(t, s)}{ds} = \pi_0 + \epsilon r \tau(s) - \beta_\mu(t, s) \Delta \mu(s),
\]
\[
\frac{d\beta_\tau(t, s)}{ds} = \pi_0 + \epsilon r \tau(s) + \beta_\tau(t, s) \Delta \tau(s).
\]

Having developed these two sets of risk loaded decrement and tax rate assumptions we do the following calculations:

1. Calculate a base fulfillment value \(F(t)\) using the previously defined risk loaded assumptions.
2. Calculate a mortality shocked value \(\hat{F}_\mu(t)\) using the base tax rate \(\tau(s) + \beta_\tau(t, s) \Delta \tau(s)\) and shocked mortality \(\mu(s) + \Delta \mu + \beta_\mu(t, s) \Delta \mu(s)\).
3. Calculate a tax shocked value \(\hat{F}_\tau(t)\) using the shocked base tax rate \(\tau(s) + \Delta \tau + \beta_\tau(t, s) \Delta \tau(s)\) and base mortality \(\mu(s) + \beta_\mu(t, s) \Delta \mu(s)\).

Capital for tax rate risk is then calculated as \(\hat{F}_\tau(t) - F(t)\) and capital for a mortality assumption change is given by \(\hat{F}_\mu(t) - F(t)\). The base value \(F(t)\) can be thought of as the best estimate value \(F_0\) plus sufficient margin to pay for holding both capital amounts i.e. \(F(t) = F_0(t) + M\).

Unfortunately there is not enough space in this document to derive the conclusions stated above in detail. Most of the necessary details can be found in footnote [6].
Once the base and shocked fulfillment values $F, \hat{F}_\mu, \hat{F}_\tau$ have been calculated as above it is straightforward to calculate the related transfer price quantities $V, \check{V}_\mu, \check{V}_\tau$ using the relation $V = \frac{F - \tau V_{Tax}}{1 - \tau}$ derived earlier. This is usually the path of least resistance to get at $V, \check{V}_\mu, \check{V}_\tau$.

It is possible to compute a transfer price from first principles but a time dependent tax rate assumption creates a new technical wrinkle. If the tax rate assumption $\tau(s)$ is smooth enough that the time derivative $\dot{\tau}(s) = \frac{d\tau}{ds}$ is continuous the first principles presentation formula for $V$ is

$$V(t) = e^{-\int_t^T (r+\mu)dv} V(T) + \int_t^T e^{-\int_s^T (r+\mu)dv} \left[ \mu D + e - g + \pi \Delta Q (D - V) - \frac{\tau PD}{1 - \tau} - (r \tau + \dot{\tau})(V_{Tax} - V) \right] ds.$$

The key point is that this is not the same as the formula derived earlier but with a time dependent tax rate. The authors invite the reader to derive the corresponding calculation formula for $V$ and then decide for themselves whether implementing that approach is worth the effort.

**After-Tax Financial Engineering – Stochastic Interest Rates**

The purpose of this section is to briefly indicate what changes if we decide to go with a model where interest rates are stochastic. The main issue is that taking tax timing differences into account can turn an otherwise simple, and deterministic, valuation problem into one requiring a fully stochastic approach. The good news is that some problems are still simple enough that they can be valued using deterministic methods, once we have the concept of an after-tax forward rate in hand.

We start by returning to the simple life insurance example we have been using but now we compute the fulfillment value using a risk neutral expectation i.e.

$$F(t) = EQ \left\{ e^{-\int_t^T (r(1-\tau)+(\mu+\pi \Delta Q))dv} F(T) + \int_t^T e^{-\int_s^T (r(1-\tau)+(\mu+\pi \Delta Q))dv} \left[ (1 - \tau)[(\mu + \pi \Delta Q)D + e - g] - \tau PD \right. \right.$$

$$\left. - \tau \left( \frac{dV_{Tax}}{ds} - (\mu + \pi \Delta Q)V_{Tax} \right) \right] ds \right\}.$$

If the tax rate $\tau$ and cost of capital rate $\pi$ are constant (which they often are in practice) then the only stochastic element in the calculation above is in the after-tax discount factor $e^{-\int_t^T (r(1-\tau))dv}$.

We now define the after-tax forward rate $f_k(t)$ by

$$e^{-\int_s^T f_k(t)(1-\tau)dv} = EQ \left[ e^{-\int_s^T (r(1-\tau))dv} \right].$$
An equivalent definition is $f_\tau(s) = \frac{e^{\int_t^s f_\tau(t)(1-\tau)dv}}{1-\tau} \frac{d}{ds} \mathbb{E}^Q \left[e^{-\int_t^s (r(1-\tau)dv)}\right]$. If $\tau = 0$ this reduces to the standard definition of forward rate.

Under our stated simplification ($\tau, \pi$ are constant), we can take the risk neutral expectation operator through the curly brackets above to write

$$F(t) = e^{-\int_t^T (f_t(1-\tau)+(\mu+\pi\Delta Q))dv} F(T) + \int_t^T e^{-\int_t^s (f_t(1-\tau)+(\mu+\pi\Delta Q))dv} \left[(1-\tau)[(\mu + \pi\Delta Q)D + e - g] - \tau PD^k \right.$$

$$- \tau \left(\frac{dV^{\text{Tax}}}{ds} - (\mu + \pi\Delta Q)V^{\text{Tax}}\right) ds.$$

This a deterministic calculation which is clearly more practical to implement than a stochastic Monte Carlo approach. A useful implication of this result is that the “presentation” formulas derived earlier continue to apply as long as we use the after-tax forward rates instead of the pre-tax forward rates.

In general the after-tax forward rates will have to be estimated by Monte Carlo simulation. The computational advantage is that this may only need to done once and then applied many times over to different contracts.

If the stochastic interest rate model being used is simple enough it is possible to write down a closed form expression for the after-tax forward rates. As a simple example, assume we are using the well-known Vasicek single factor model $dr = \alpha(\theta(s) - r)ds + \sigma dz(s)$. Here $\alpha, \sigma$ are constants and $\theta(s)$ is a deterministic function used to calibrate the model to a given set of pre-tax forward rates $f_0(s)$. It can be shown that the relationship between after-tax forward rates and pre-tax forward rates, for this model, is given by

$$f_\tau(s) = f_0(s) + \frac{\tau \sigma^2}{2\alpha^2} (1 - e^{-\alpha(s-t)})^2.$$

In this formula the valuation is being performed at time $t$ and $s \geq t$.

For more complex models it is always true that $f_\tau(s) \geq f_0(s)$ and the difference gets bigger as the interest rate model’s volatility assumption is increased.

Even if the computational shortcut described above is not applicable a practical implication of the analysis above is that combining an after-tax valuation model with stochastic interest rates can have a material impact. For most insurance liabilities the effect is to reduce the transfer price.

**After-Tax A/L M**

In practice most North American insurers use a variety of metrics to manage day to day issues. Some examples
1. Insurance Product development/pricing is often done using an after-tax model of the form described in this paper.

2. Earnings Management is often driven by a local accounting standard such as US GAAP or IFRS.

3. Asset/Liability Management is often driven by a cash flow testing model specified by local regulators.

Part of the authors’ rant at the beginning of this paper was driven by the obvious inconsistencies outlined above. There are, of course, practical reasons for the situation summarized above. The purpose of this section is to outline what an A/L M process might look like if it were engineered to be consistent with the model developed here.

Assets are different from insurance liabilities and tax issues are one of the main reasons. Suppose we have a vanilla bond whose observed transfer price in the market is $TPA$. How did the market determine the $TPA$? The answer is that bond traders do not take future tax timing differences into account when calculating transfer prices because the tax base of an asset usually resets to market when traded. They do take permanent differences into account if they apply.

If the bond was purchased at some time in the past it will usually have a tax base $ATax$ which is different from its current transfer price. Most accounting models would assign a total market value of $MVA = TPA + \tau (ATax - TPA)$ to the asset since this is the amount of cash on hand if the asset were sold immediately. A starting point for the A/L M process is then to set the $MVA$ of the assets equal to the fulfillment value of the liabilities i.e.

$$MVA = TPA + \tau (ATax - TPA) = FVL = TPL + \tau (VTax - TPL).$$

So far so good. If by A/L M we mean something like duration matching under a yield curve or equity shock then after-tax A/L M would mean

$$\Delta TPA (1 - \tau) = \Delta TPL (1 - \tau)$$

if $ATax$ and $VTax$ were unaffected by a market shock. The obvious conclusion is that the dollar duration of the $TPA$ should be managed relative to the dollar duration of the liabilities as measured by the $TPL$ not $FVL$.

The only thing wrong with the A/L M model outlined above is that it assumes the assets are being continuously traded so that the asset tax base is always equal to the transfer price. This would become clear to anyone actually managing off the model described above once they started doing a rigorous earnings by source analysis. To the extent assets were not actually being traded on a regular basis the asset tax timing differences would start hitting the economic bottom line as they occurred (plus or minus).

In an after-tax world the asset turnover assumption is an actuarial assumption just like mortality or lapse rates. If we know the asset will be sold immediately then the right way to value the asset is to report the $MVA$ described above. At the other extreme, if we know the asset will be held to its maturity date (like an insurance liability) then the $FVL/TPL$ model developed earlier
in this paper should apply because it takes account of future tax timing and permanent differences.

Whatever asset turnover assumption we make will probably be wrong but using an assumption with a known bias does not make sense from a risk management perspective. There are a number of situations where continuous asset trading is not realistic. Here are two examples:

1. The US GAAP accounting model can result in classifying assets as being either Hold to Maturity (HTM) or Available for Sale (AFS). There can be material accounting penalties of for trading an asset in the HTM bucket. The result is that it is reasonable to assume assets assigned to the HTM bucket will not be traded. For these assets a version of the $FVL/TPL$ valuation model should be used for assets in the HTM bucket.

2. Imagine an insurer has gone to the trouble of engineering the asset cash flows backing a block of insurance liabilities to be an exact after-tax match. This means the asset cash flows received in any time interval are equal to the pre-tax liability cash flows plus all related taxes on the combined asset/liability block. Using what is hopefully a transparent notation\(^7\) this is

\[
ACF_t = LCF_t + \tau[A^{Tax}_t + A^{Tax}_{t-1} - PDA_t - (LCF_t + V^{Tax}_t - V^{Tax}_{t-1} + PD^L_t)].
\]

On rearrangement this becomes the statement that after-tax asset cash flows are equal to after-tax liability cash flows i.e.

\[
ACF_t(1 - \tau) - \tau(A^{Tax}_t - A^{Tax}_{t-1} - PDA_t) = LCF_t(1 - \tau) - \tau(V^{Tax}_t - V^{Tax}_{t-1} + PD^L_t).
\]

As we saw earlier, the present value of after-tax liability cash flows discounted at after-tax interest rates is the fulfillment value of the liabilities $FVL$. It therefore makes sense to define the fulfilment value of an asset $FVA$ as the present value of after-tax asset cash flows using after-tax interest rates. There are practical problems with this idea that would need to be wrestled to the ground before it can be put into practice\(^8\). Once we get past those issues we can define the concept of a going concern value or $GCV$ for an asset by $FVA = GCV(1 - \tau + \tau A^{Tax})$. The $GCV$ for assets would then be calculated using the same principles as the $TPL$ for liabilities. The fundamental A/L M equation then becomes

\[
FVA = GCV(1 - \tau) + \tau A^{Tax} = FVL = TPL(1 - \tau) + \tau V^{Tax}.
\]

Duration matching now means matching the dollar duration of the $GCV$ to that of the $TPL$.

Once we start calculating the present value of tax timing differences on assets we run into a number of the same issues that arose when valuing liabilities. In particular, the combination of stochastic interest rates and income tax increases the discount rate. While this was good news

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\(^7\) Again an equation to be taken seriously and not literally.

\(^8\) One issue, among others, is how to handle credit and liquidity spreads in the process.
for liabilities it could be bad news for assets. An issue that needs to be addressed on a case by case basis.

**Conclusion**

In this paper, we have presented an approach to after-tax risk management that is consistent with current actuarial thinking in a risk neutral setting.

In the preparation for this paper, the authors considered why to discount distributable earnings with \( r + \pi \) instead of \( r(1 - \tau) + \pi \). The former approach seems to be favored by MCEV practitioners and by the American Academy of Actuaries March, 2011 Public Policy Practice Note on Market Consistent Embedded Values and the latter was not discussed here. A proper discussion of this issue could easily be a paper on its own.

We presented a discussion of three risk management questions:

a. balance sheet value  
b. sale/purchase value  
c. which to use

This is followed by a description of a high level model of an income tax structure. The fulfillment value of a liability (FVL) was proposed as the solution to (a); the transfer value of liability (TPL) was proposed as the solution to (b) and it was suggested for (c) that the transfer value is usually the one to use.

We then gave a formal derivation of fulfillment value of the liability and transfer price of the liability. For convenience, continuous assumptions were used (including the assumption that tax reserve at issue is 0). The discussion was generalized to include varying assumptions (like income tax) and stochastic interest rates.

Finally there was a brief section on after-tax asset liability management.

None of the topics discussed was developed in enough detail for the ideas or formulas to be taken literally. They should be taken seriously.